Nucleon-deuteron scattering – predictions & theoretical uncertainties

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Outline

Introduction:

- Nucleon-deuteron scattering
- Theoretical uncertainties in few-nucleon physics

Tools

- Chiral forces - current status
- The OPE-Gaussian potential
- Faddeev approach to 3N scattering

Results:

- **Nd scattering with chiral forces**
- **Chiral N4LO:** dependence on regulators and truncation errors
- **OPE-Gausian and ChiralN4LO:** propagation of potential uncertainties to 3N observables

Summary

Why to study nucleon-deuteron scattering ?
N+d → N+d
N+d → N + N + N

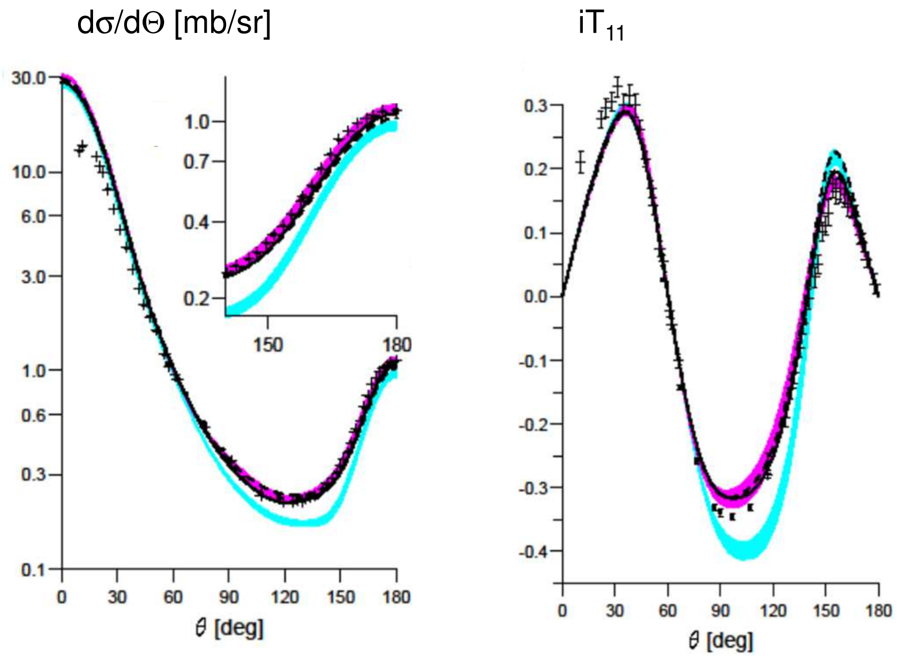
- The simplest reaction beyond 2N system: test for two-nucleon force models (which are usually fitted to **all** 2N data), especially the off-shell part.

- Exact theoretical methods and numerical solutions are available (including three-nucleon force, Coulomb interaction, relativity).
- Many precise experiments (for p+d reactions) also for polarization observables.

- Many observables are sensitive to various terms of interaction.
- For the deuteron breakup there exist many final kinematical configuration sensitive to details of interaction.
- At medium energies the 3N force is important !
- No 4N force or (for neutron-deuteron scattering) Coulomb potential.

→ interesting problems with big impact for nuclear physics

Why to study nucleon-deuteron scattering ?



Elastic Nd scattering at E=135 MeV

Exp.
H.Sakai, et al., Phys. Rev. Lett. 84 (2000) 5288
N.Sakamoto et al., Phys. Lett. B 367 (1996) 60

H.Witała et al., Phys. Rev. C63 (2001) 024007

- In general, the description of observables is very satisfactory,
- however, the puzzling behavior is observed for few observables, like

a) nucleon vector analyzing power at low energies

b) the differential cross section for the SST configuration

Open problems – QFS configuration

- Quasi Free-Scattering configuration – only two nucleons interact
- Reveals problem with the neutron-neutron interaction

neutron-proton

QFSnp: d(n,np)n, En=26 MeV

$\Theta_n = 39°\ \ \Theta_p = 42°\ \ \phi_{np} = 180°$

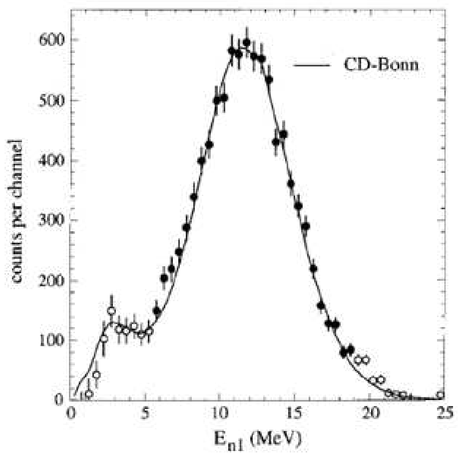


FIG. 2. Data for n-p QFS, projected onto the E_n axis. The solid line is the finite-geometry Monte Carlo prediction, using CD-Bonn for the N-N interaction.

A.Siepe et al., PRC65, 034010 (2002)

neutron-neutron

QFSnn: d(n,nn)p, En=26 MeV

both neutrons measured at Θ_n=42^o

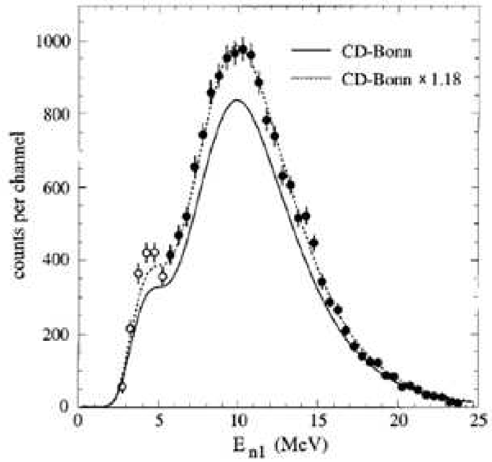


FIG. 4. HE data of Fig. 3, projected onto the E_{n1} axis. The solid curve represents the finite-geometry Monte Carlo prediction using CD-Bonn, the dotted line is the MC result normalized to the experiment by multiplication with a factor of 1.18. Only events with E_{n1} and $E_{n2} > 6$ MeV have been included in the analysis.

Theoretical uncertainties

- Arising from our ignorance of (effective) NN interaction
  - Various models like AV18, CD-Bonn, chiral models, …

- Arising from uncertainty of free parameters of NN forces
  - origin in uncertainties of NN data
  - not taken with much care before
  - progress in this field obtained recently by Granada group (E.Ruiz Arriola, R. Navarro Perez and collaborators) – revision of NN database and new models of NN forces (OPE-Gaussian, TPE-Gaussian). Also recent work by P.Reinert et al. (the Bochum-Bonn group) takes care about parameters' uncertainties.

- Chiral models
  - truncation errors
  - regularization dependence

- Theoretical methods introduce their own uncertainties (small in the Faddeev approach for Nd scattering) and suffer from finite computational accuracy.

Nuclear forces from χEFT

- Starting point is Lagrangian for nucleonic and pionic fields with broken chiral symmetry like in QCD.
- Effective Hamiltonian for nucleons and pions.
- Power counting – perturbative ordering ($(Q/\Lambda)^{\nu}$) of various contributions to two- and many-body forces. Finite number of diagrams at given order.
- Short range interactions are included to the contact terms

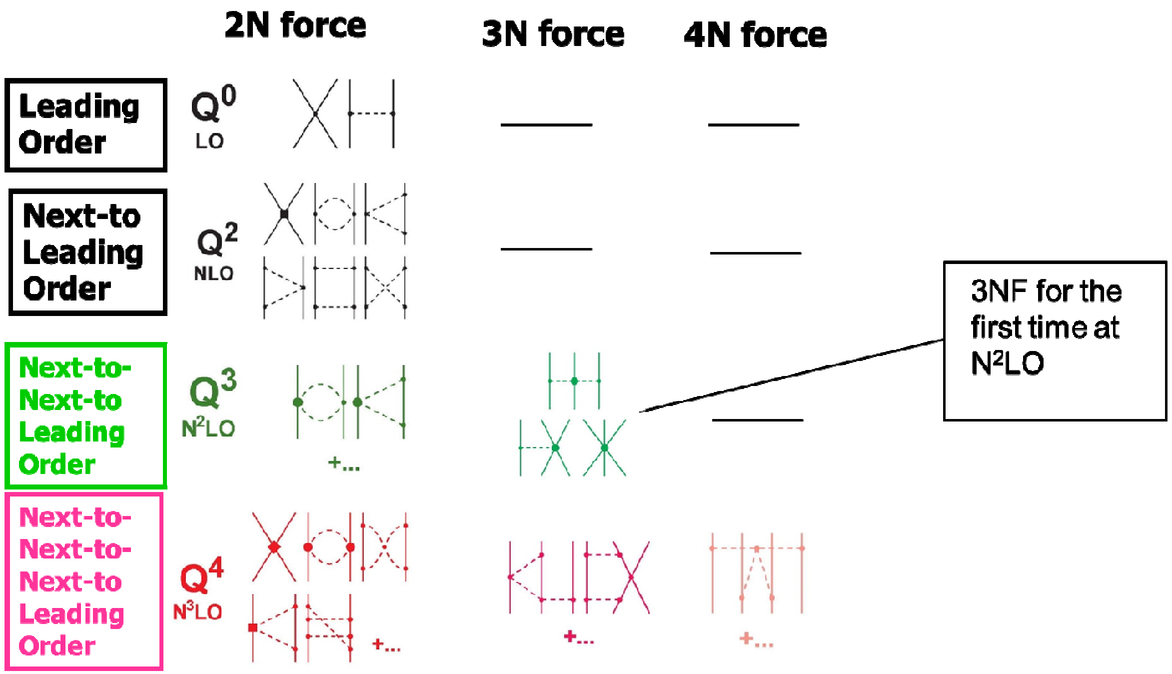


Fig. from R.Machleidt and E.Epelbaum

Nuclear forces from χEFT - regularization

- Chiral forces (2N,3N, …) require regularization to avoid divergences in the Lippmann-Schwinger equation and in $\pi\!-\!\pi$ loops.

Various solutions have been proposed. They are:

- **Nonlocal regularization** (convenient for applications but introduces unwanted artifacts in a long range part of interaction)
- in momentum space V(p',p) → V(p',p)f(p',p) with $f(p',p) \equiv \exp\left(-\left(\frac{p'}{\Lambda}\right)^{2n} - \left(\frac{p}{\Lambda}\right)^{2n} \right)$ where Λ=450-550 MeV and n=2,3,4,…

- **(Semi)local regularization in coordinate space** (2015, Bochum-Bonn (LENPIC)) – by construction long-range physics is unaffected by regularization
  - E.Epelbaum, H.Krebs, U.-G.Meißner, Eur. Phys. J. A51 (2015) 3,26 – up to N3LO
  - E.Epelbaum, H.Krebs, U.-G.Meißner, Phys. Rev. Lett. 115 (2015) 12, 122301 – up to N4LO
  - Local regularization in coordinate space $V_{lr}(\mathbf{r}) \rightarrow V_{lr}(\mathbf{r})f(r)$ with $f(r) \equiv \left(1 - e^{-r^2/R^2}\right)^n$
  - n=6, R=0.8–1.2 fm what corresponds to Λ=330-500 MeV
  - Regularized potential is transformed to momentum space (different effects in different partial waves)

Nuclear forces from χEFT - regularization

- **(Semi)local regularization in momentum space** (2018, Bochum (LENPIC))

P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A 54, 86 (2018).

Modification of the propagators of pions exchanged between nucleons

$$1/(l^2 - m_\pi^2) \rightarrow F(l^2)/(l^2 - m_\pi^2) \quad \text{with} \quad F(l^2) = e^{\frac{-(l^2+m_\pi^2)}{\Lambda^2}}$$

Additional improvements for this state-of-the-art model are

- the pion-nucleon low energy constant are taken directly from the pion sector (recent very precise determination),
- remaining LECs have been fixed directly from data (the Granda database) and not from the Nijmegen PWA,
- redundant operators at N^3LO are removed what improves convergence to χ^2/data minimum,
- some contact terms from N^5LO are also included („N^4LO+ force")
- correlations between parameters are known
- Λ=400-550 MeV $\rightarrow$ R=0.7-1.0 fm

All these result in excellent data description with χ^2/data≈1.0

Mini-summary: nuclear forces from χEFT – AD 2018

- Currently, the most advanced models come from:
- the Bochum-Bonn group (E.Epelbaum, U.-G.Meißner, H.Krebs, P.Reinert, V.Bernard, W.Glöckle†, …) – 2N (up to N^4LO+), 3N (up to N^3LO) and 4N forces (up to N^3LO)

  a) SCS (semilocal coordinate space) regularization

  b) SMS (semilocal momentum space) regularization

- the Moscow(Idaho)-Salamanca group (R.Machleidt, D.R.Entem, Y.Nosyk) N^4LO

  D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96, 024004 (2017).

- Other chiral models of nuclear forces, suitable for nuclear physics at not very small energies, base mainly on works of these two groups.

OPE-Gaussian potential

R.Navarro Pérez, J. E. Amaro, and E. Ruiz Arriola, Phys. Rev. C 89 (2014) 064006

- The NN OPE-Gaussian interaction can be decomposed as
$$V(\vec{r}) = V_{short}(\vec{r})\theta(r_c - r) + V_{long}(\vec{r})\theta(r - r_c)$$
where r_c=3 fm.
- The long range part has two parts: OPE part and electromagnetic corrections (for the proton-proton force)
$$V_{long}(\vec{r}) = V_{OPE}(\vec{r}) + V_{em}(\vec{r})$$
- The short range part is
$$V_{short}(\vec{r}) = \sum_{n=1}^{18} \hat{O}_n \left[\sum_{i=1}^{4} V_{i,n} F_i(r) \right], \qquad F_i(r) = e^{-r^2/(2a_i^2)}$$

where O_n are the same operators as in the AV18 + three additional operators;
$V_{i,n}$ and a_i are unknown coefficients to be determined from NN data,
F_i are radial Gaussian functions.

- Authors prepared and used „3σ self-consistent database" to fix free parameters.
- **Finally, they obtained values of all 42 free parameters and their uncertainties (statistically well defined standard deviations and correlation coefficients).**

- The OPE-Gaussian force can be seen as a remastered the AV18 interaction.

Formalism for 3N scattering

- 2N bound state: Schrödinger equation,
- 2N scattering state: Lippmann-Schwinger equation for the t-matrix (interaction + free propagation)

$$t(E) = V + VG_0(E)V + VG_0VG_0(E)V + \dots$$

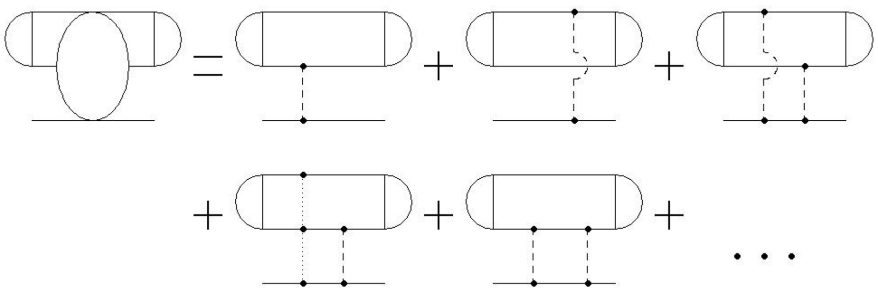
$$G_0(E) \equiv \lim_{\varepsilon \to 0^+} \frac{1}{E - H_0 + i\varepsilon}$$

- 3N: **Faddeev equation**

$$T = tP\phi + (1 + tG_0)V_{123}^{(1)}(1+P)\phi + tPG_0T + (1 + tG_0)V_{123}^{(1)}(1+P)G_0T$$

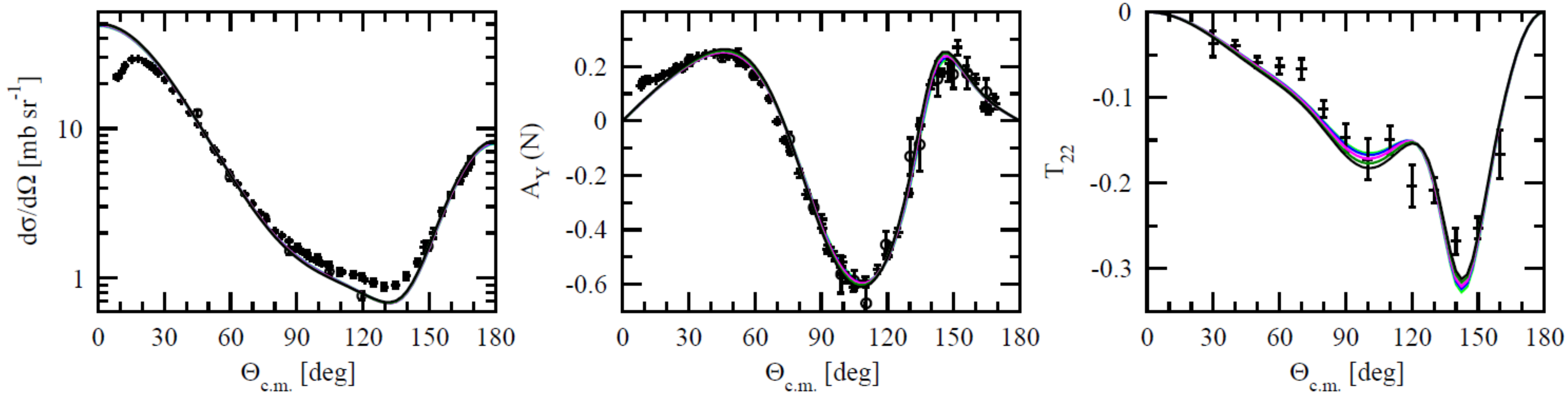
Transition amplitudes

$$U = PG_0^{-1} + V_{123}^{(1)}(1+P)\phi +$$
$$+ PT + V_{123}^{(1)}(1+P)G_0T$$
$$U_0 = (1+P)T$$

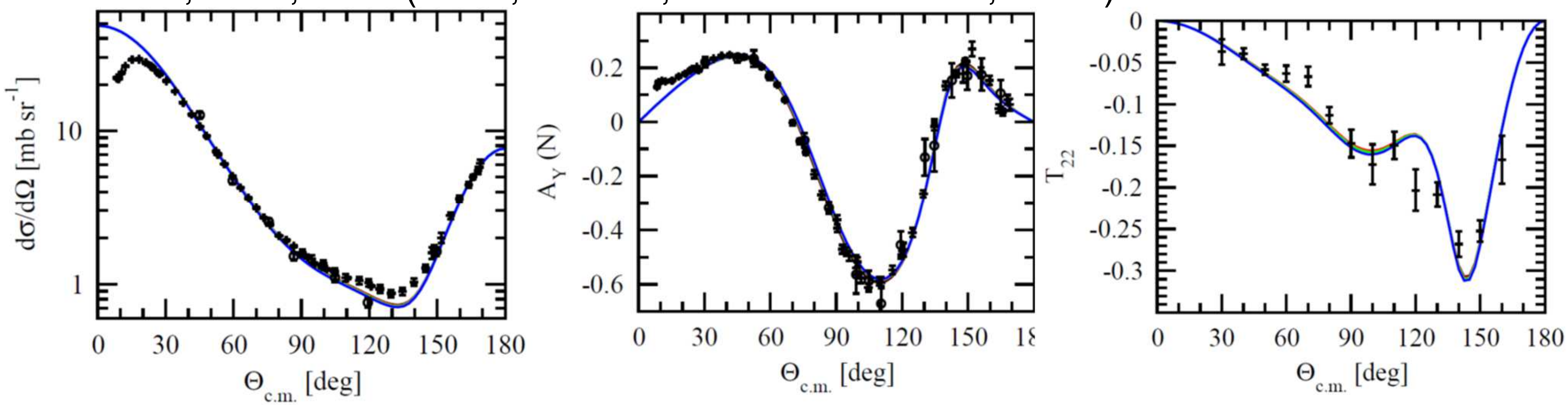


Test: regulator dependence at 65 MeV

E.E.,H.K.,U.-G.M. (SCS, N^{4}LO, R=0.8-1.2 fm)



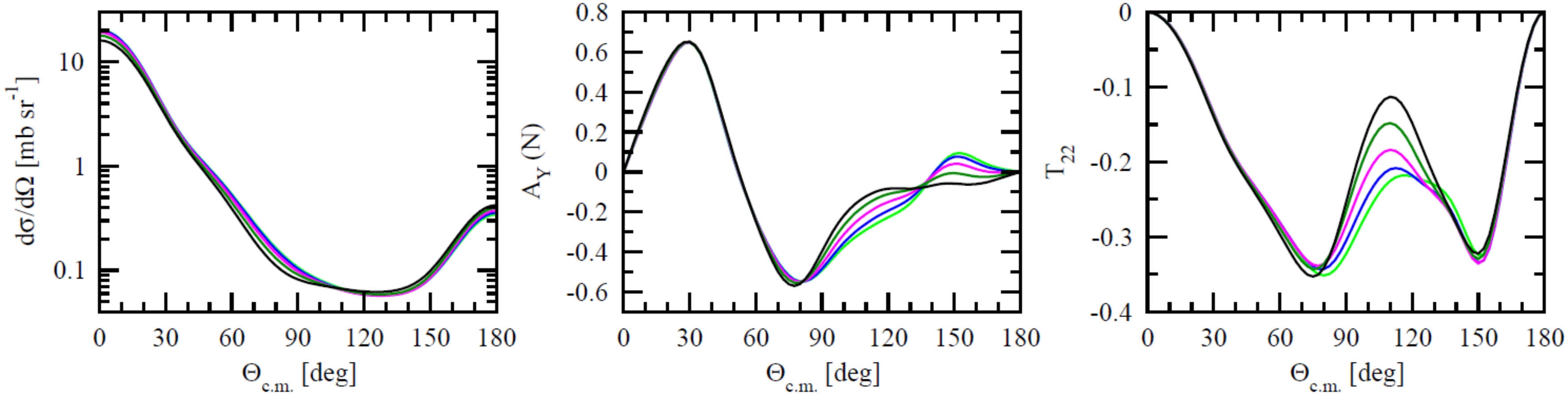
P.R., H.K., E.E. (SMS, N^{4}LO+, Λ=400-550 MeV, 2018)



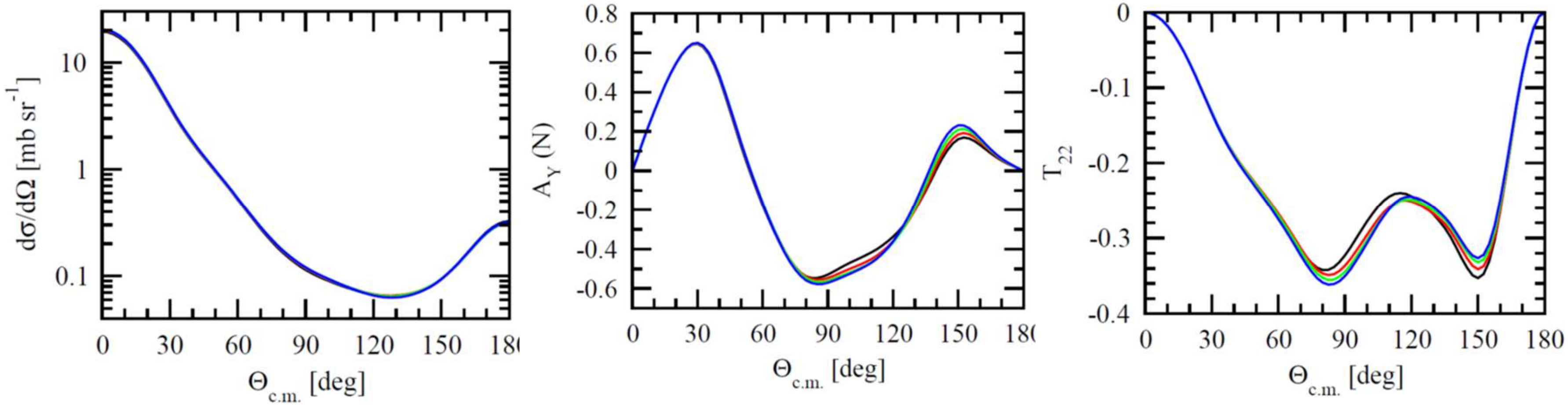
→ both local regularizations lead to very small regulator dependence (at E=65 MeV). Improvement for SMS.

Test: regulator dependence at 200 MeV

E.E.,H.K.,U.-G.M. (SCS N^{4}LO, R=0.8-1.2 fm)



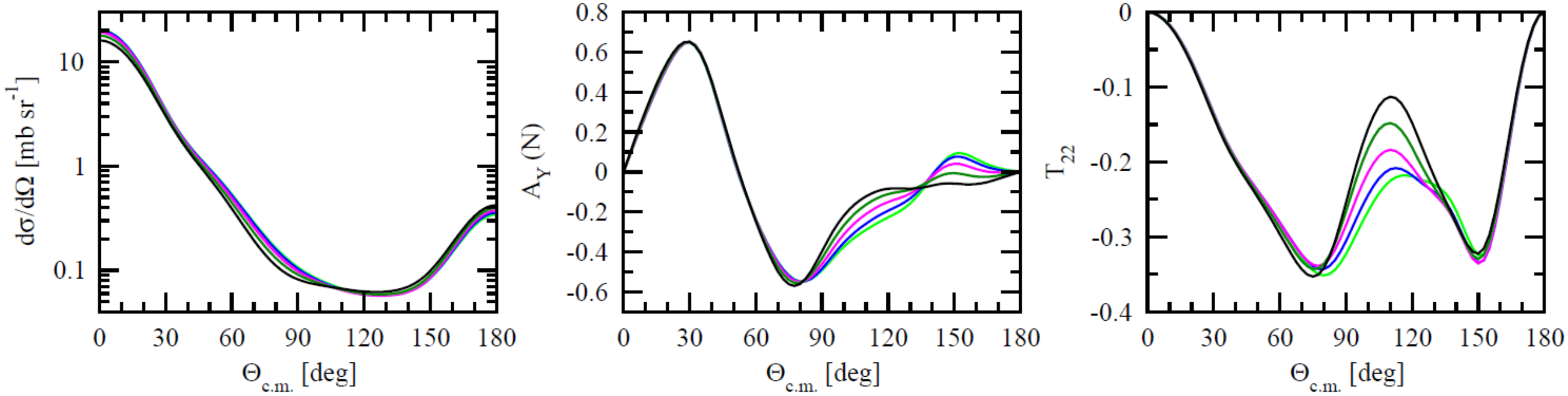
P.R., H.K., E.E. (SMS, N^{4}LO+, Λ=400-550 MeV, 2018)



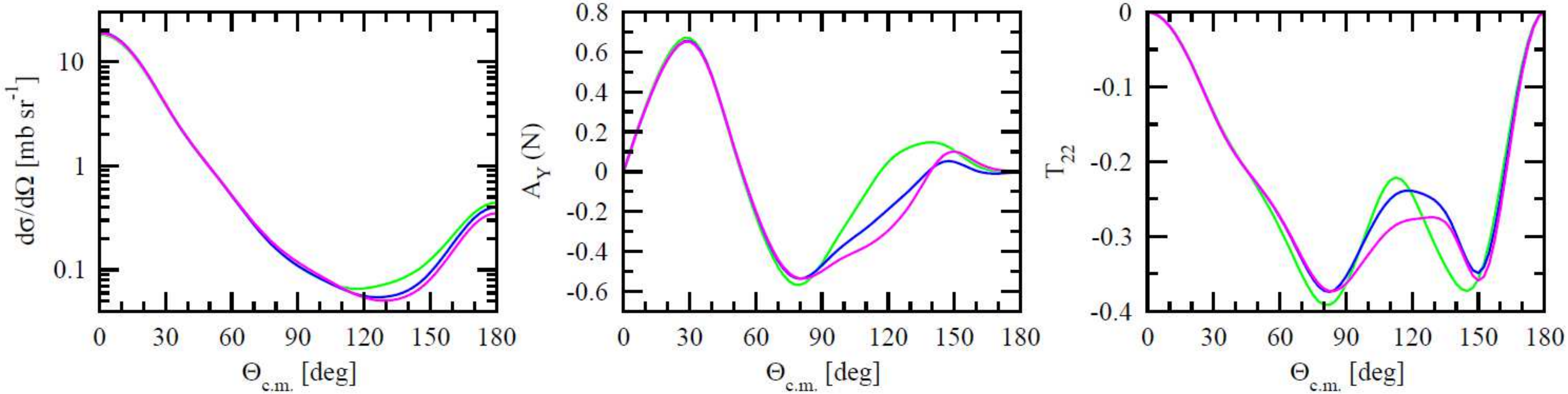
→ substantially smaller regulator dependence for SMS force.

Test: regulator dependence at 200 MeV

E.E.,H.K.,U.-G.M. (SCS N^{4}LO, R=0.8-1.2 fm)



D.R.E.,R.M.,Y.N. (N^{4}LO, Λ=450-550 MeV, 2017)



→ big regulator dependence but energy is at limit of χEFT applicability

Truncation errors

- We use prescription proposed in S.Binder et al., Phys. Rev. C93 (2016) 044002.

$$\Delta X^{(2)} = X^{(2)} - X^{(0)}$$
$$\Delta X^{(i)} = X^{(i)} - X^{(i-1)}$$

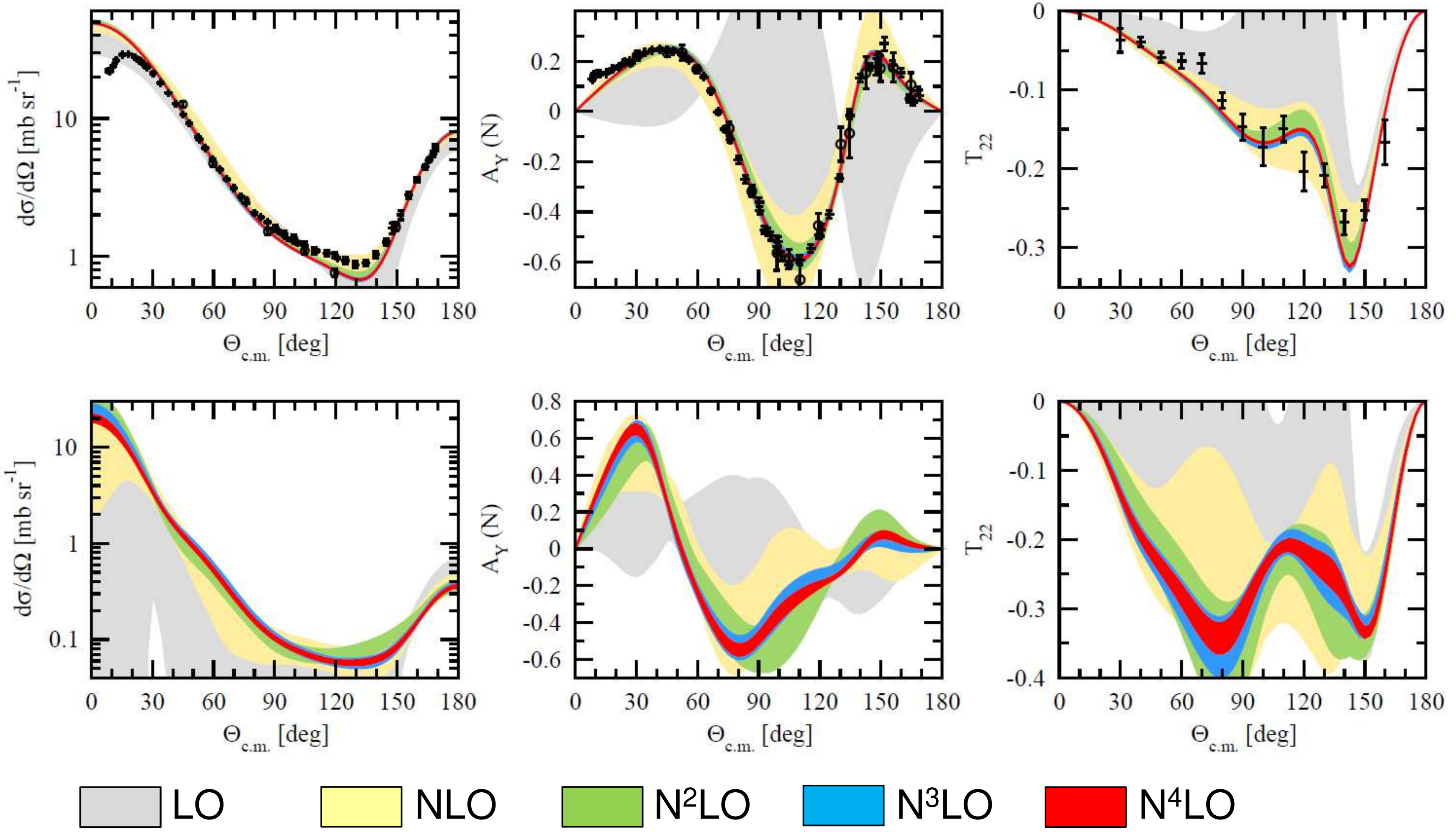
$$\delta X^{(0)} = Q^2 \left| X^0 \right|$$
$$\delta X^{(2)} = \max\left(Q^3 \left| X^0 \right|, Q^1 \left| \Delta X^{(2)} \right|\right)$$
$$i \geq 3: \; \delta X^{(i)} = \max\left(Q^{i+1} \left| X^{(0)} \right|, Q^{i-1} \left| \Delta X^{(2)} \right|, Q^{i-2} \left| X^{(3)} \right|\right)$$

$$Q = \max\left({m_\pi}/{\Lambda_b}, {p}/{\Lambda_b} \right)$$

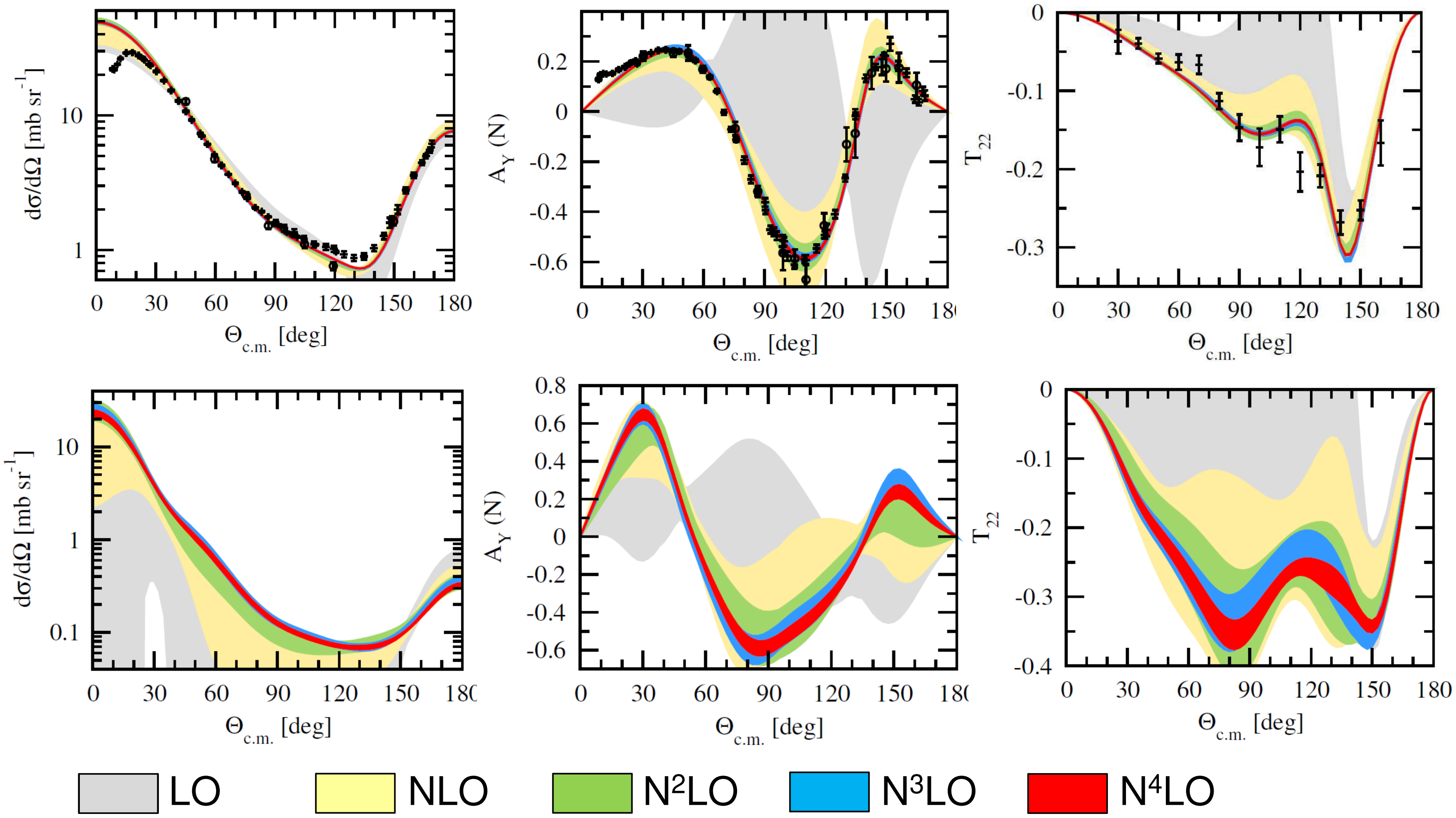
$$\delta X^{(2)} \geq Q\,\delta X^{(0)}$$
$$\delta X^{(i \geq 3)} \geq Q\,\delta X^{(i-1)}$$

- This simple prescription is in agreement with more advanced Bayesian analysis discussed in R.J.Furnstahl et al., Phys. Rev. C92 (2015) 024005.
- Note, E.E, H.K., U.-G.M use Λ_b=600 MeV and D.R.E.,R.M.,Y.N. use Λ_b=1000 MeV.

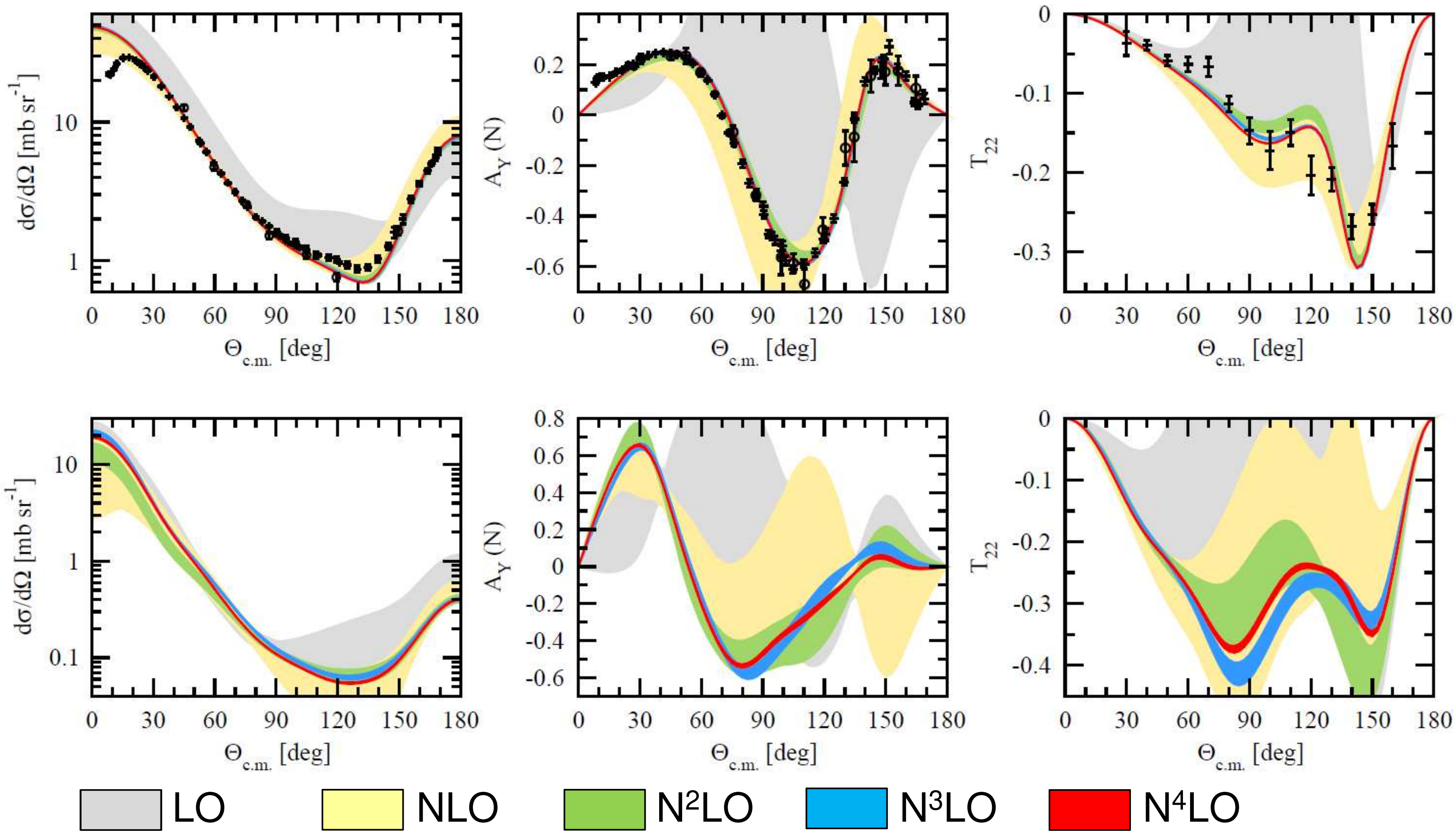
Truncation errors for SCS force at 65 and 200 MeV



Truncation errors for SMS force at 65 and 200 MeV



Truncation errors for D.R.E.,R.M.,Y.N. at 65 and 200 MeV



How to estimate statistical uncertainties ?

- Statistical uncertainties – here uncertainties of 3N observables arising from uncertainties of 2N force parameters

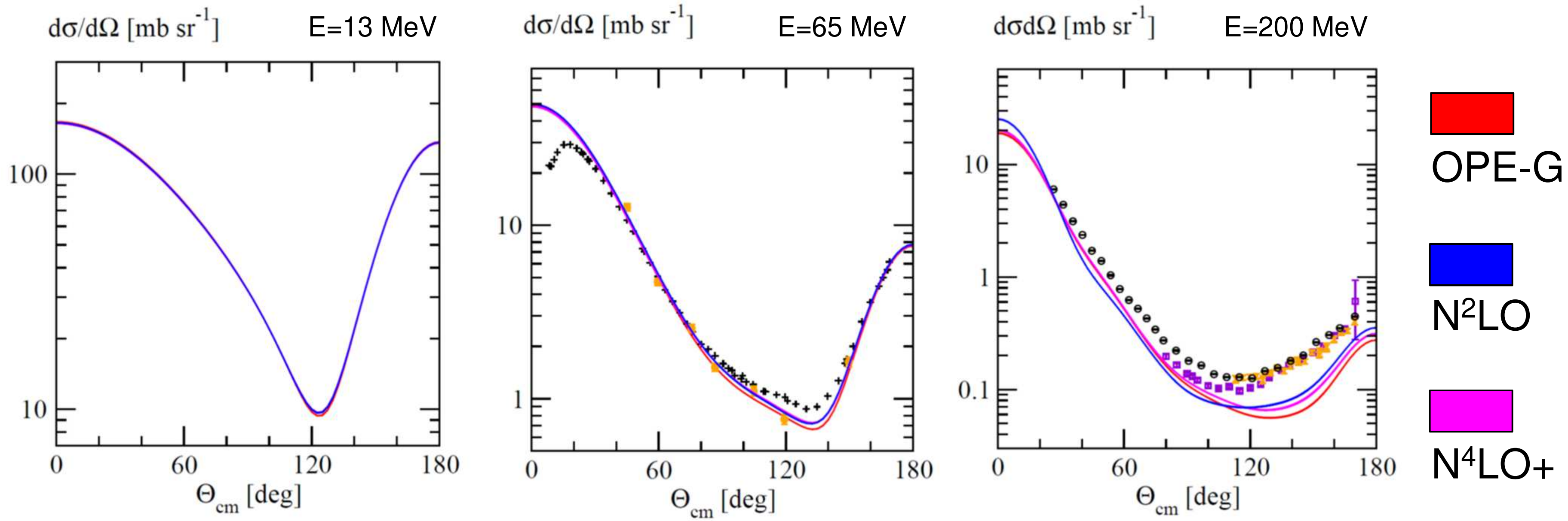
- Knowing 2N force parameters and their correlation matrix we sample many (50) sets of potential parameters

- For each set we solve Faddeev equation and compute 3N observables.
- Thus for each observable (at given energy and scattering angle) we have 50+1 predictions.

- Basing on these predictions we estimate the uncertainty of given 3N observable. This can be done in various ways, which in practice leads to similar results. We use $\Delta_{68\%}$ - a difference between maximal and minimal value of observable taken after neglecting most extreme predictions.

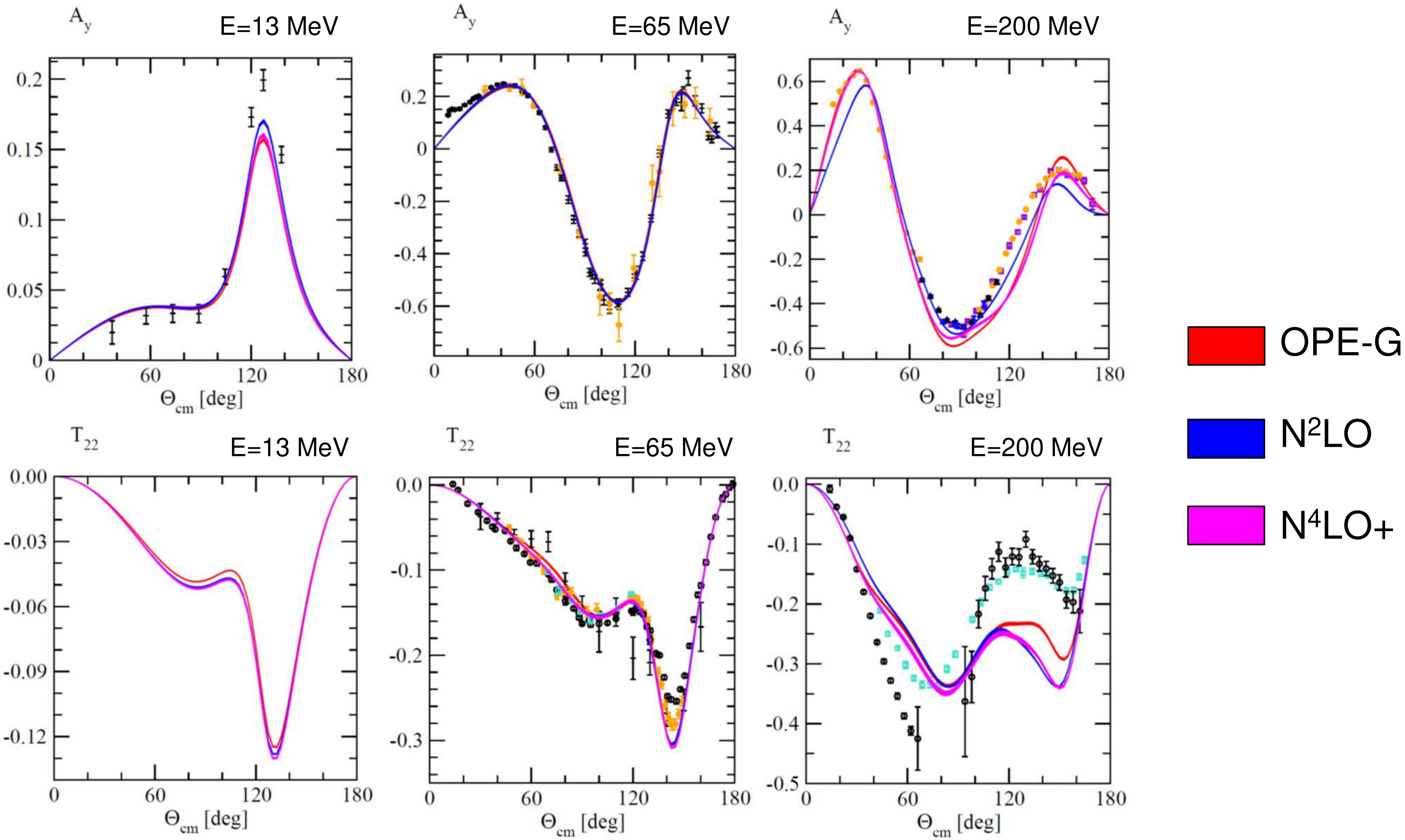
Statistical errors with chiral forces

- New SMS potential also allows to study propagation of uncertainties to 3N system



- Statistical errors for the SMS force are of similar magnitude as the ones for the OPE-Gaussian.
- Similar magnitudes at N^{2}LO and N^{4}LO+.

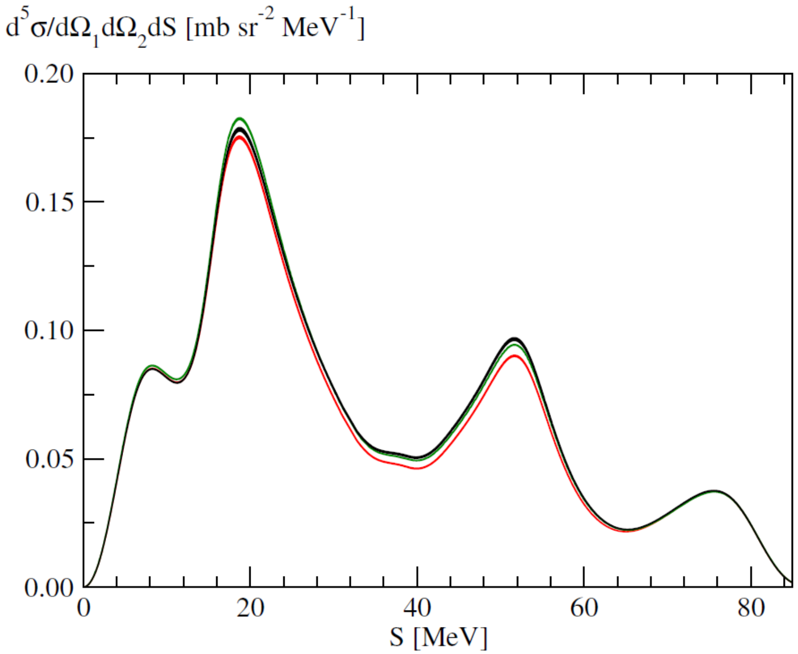
Statistical errors with SMS chiral forces



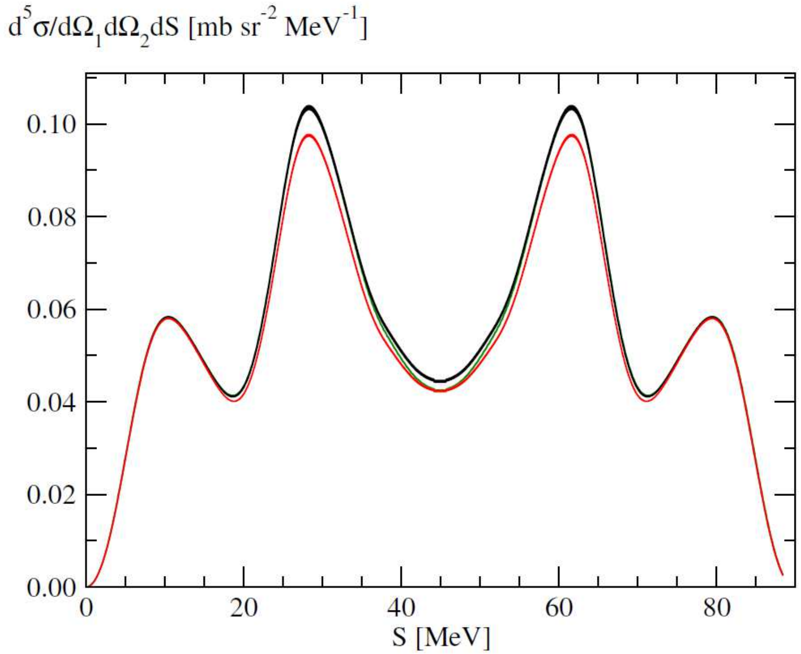
- Statistical errors for the SMS force are of similar magnitude as the ones for the OPE-Gaussian.

Deuteron breakup at E=65 MeV - statistical uncertainties

$\Theta_1=45.0^o$ $\varphi_1=0^o$ $\Theta_2=75.6^o$ $\varphi_2=180^o$



$\Theta_1=59.5^o$ $\varphi_1=0^o$ $\Theta_2=59.5^o$ $\varphi_2=180^o$

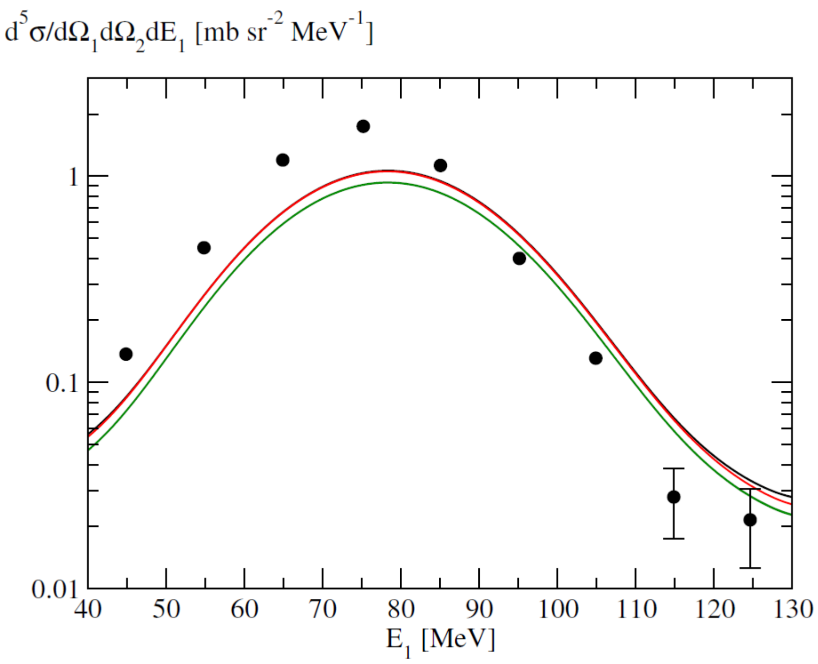


N^2LO SMS N^4LO SMS   OPE-G

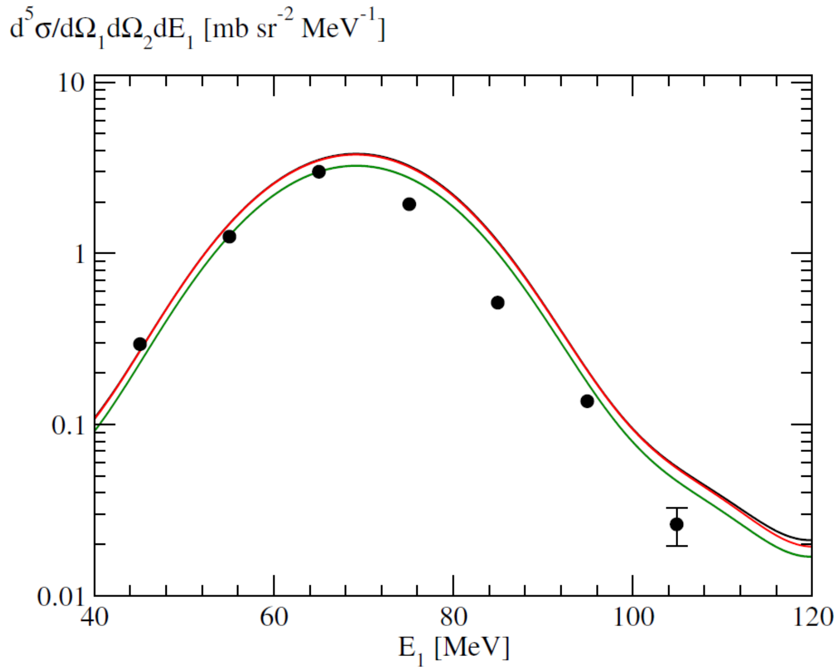
→ Statistical uncertainties are very small at E=65 MeV

Deuteron breakup at E=200 MeV - statistical uncertainties

Θ_1=45.0° φ_1=0° Θ_2=35.0° φ_2=180°



Θ_1=52.0° φ_1=0° Θ_2=35.0° φ_2=180°



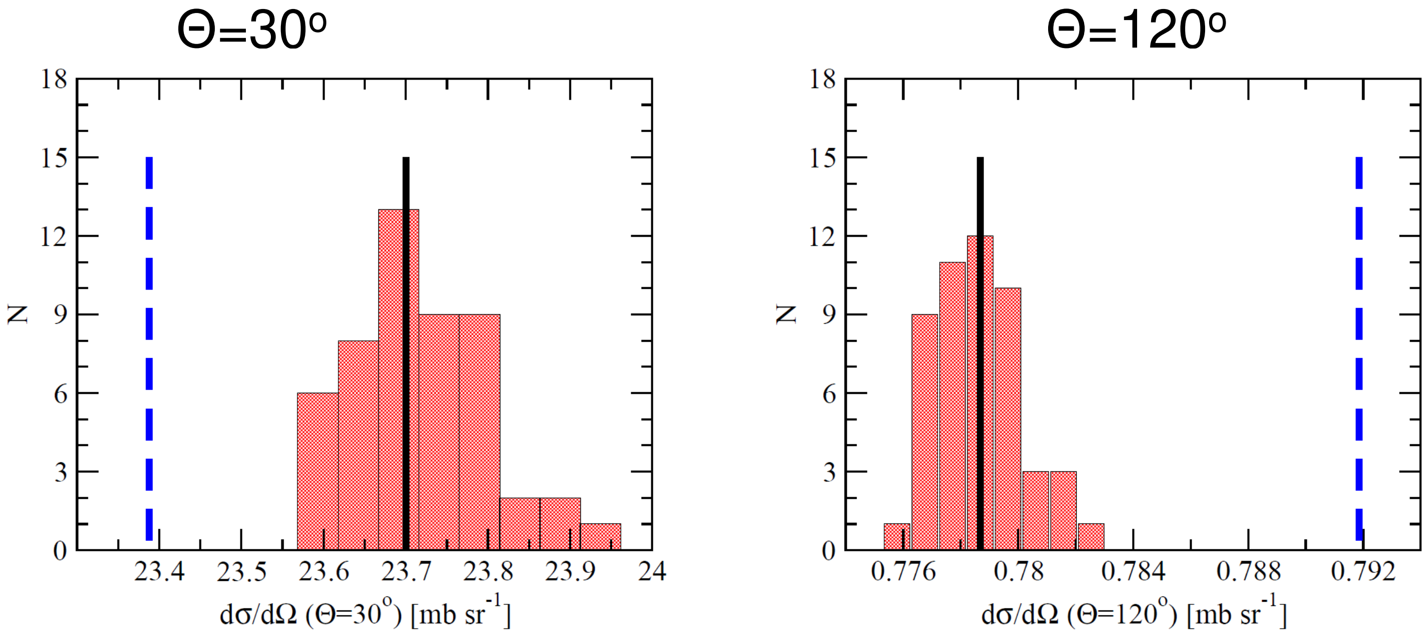
N²LO SMS N⁴LO SMS OPE-G

→ Statistical uncertainties remain small also at E=200 MeV

Exp: W. Pairsuwan, et al.,
Phys. Rev. C52, 2552 (1995).

Cross section distribution at given scattering angle

- E=65 MeV



- Normality test (Shapiro-Wilk test)

at Θ=30° P-value= 0.479

at Θ=120° P-value=0.676

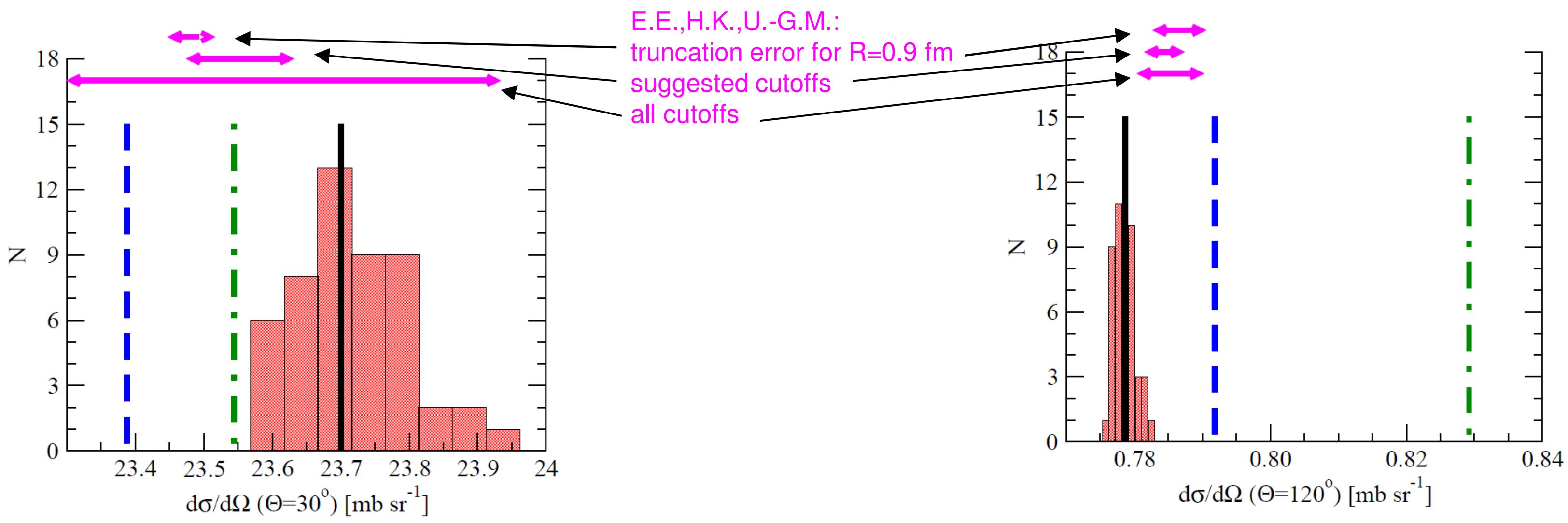
Black solid line : OPE-Gaussian (central values of parameters)

Blue dashed line : AV18

Red histogram : 50 predictions with sampled potential parameters

Summary I

- dσ/dΩ at Θ=30° (left) and 120° (right) at E=65 MeV



Black solid curve : OPE-Gaussian (central values of parameters)

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Green dash-dotted: CD-Bonn

Magenta: Chiral N^4LO, E.E.,H.K.,U.-G.M.

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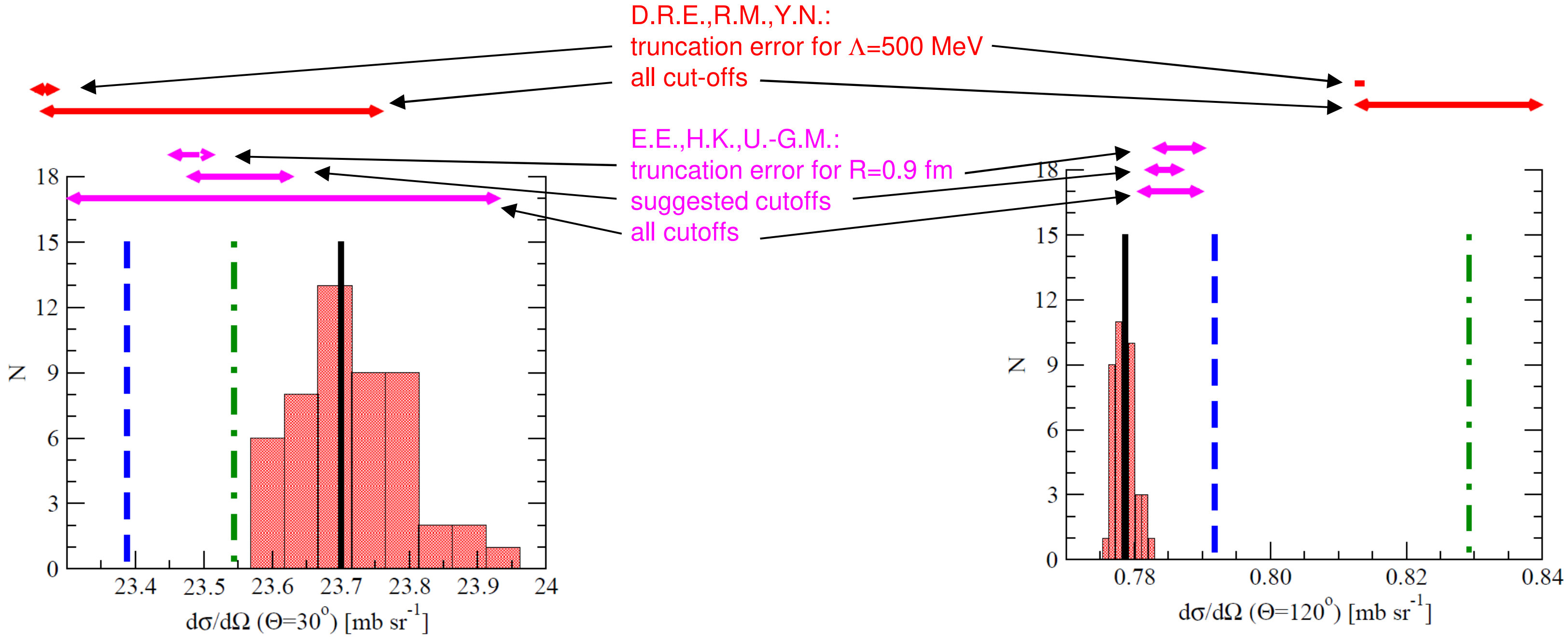

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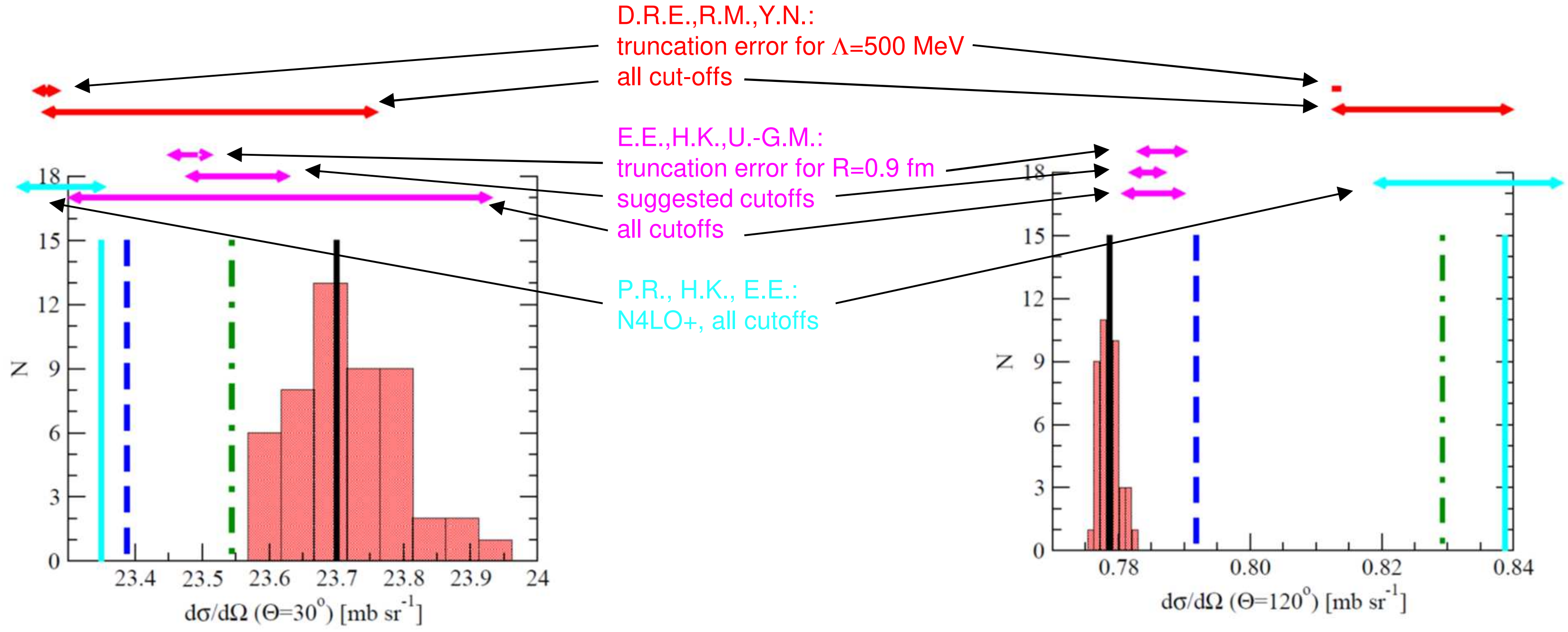

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Red: Chiral N^4LO, D.R.E.,R.M.,Y.N. Cyan N4LO+, Λ=450 MeV, P.R.,H.K.,E.E.

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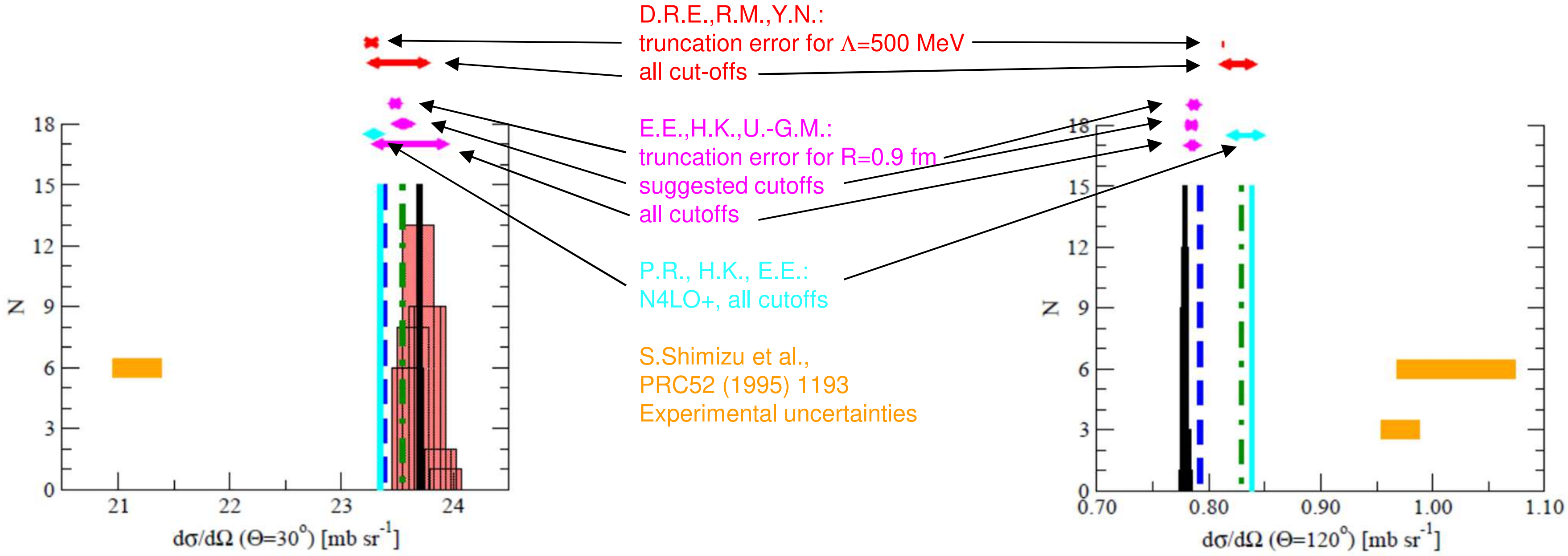

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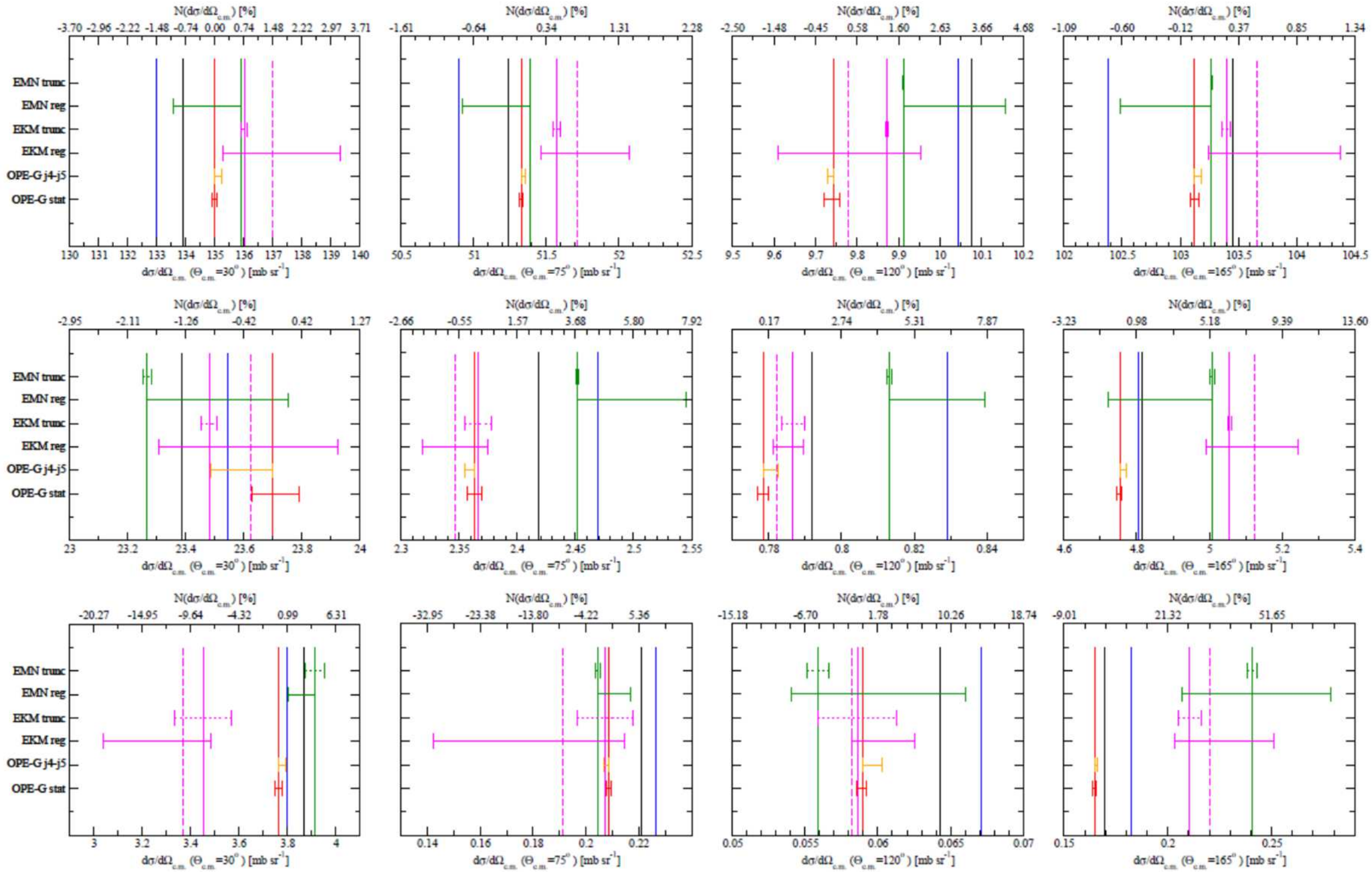
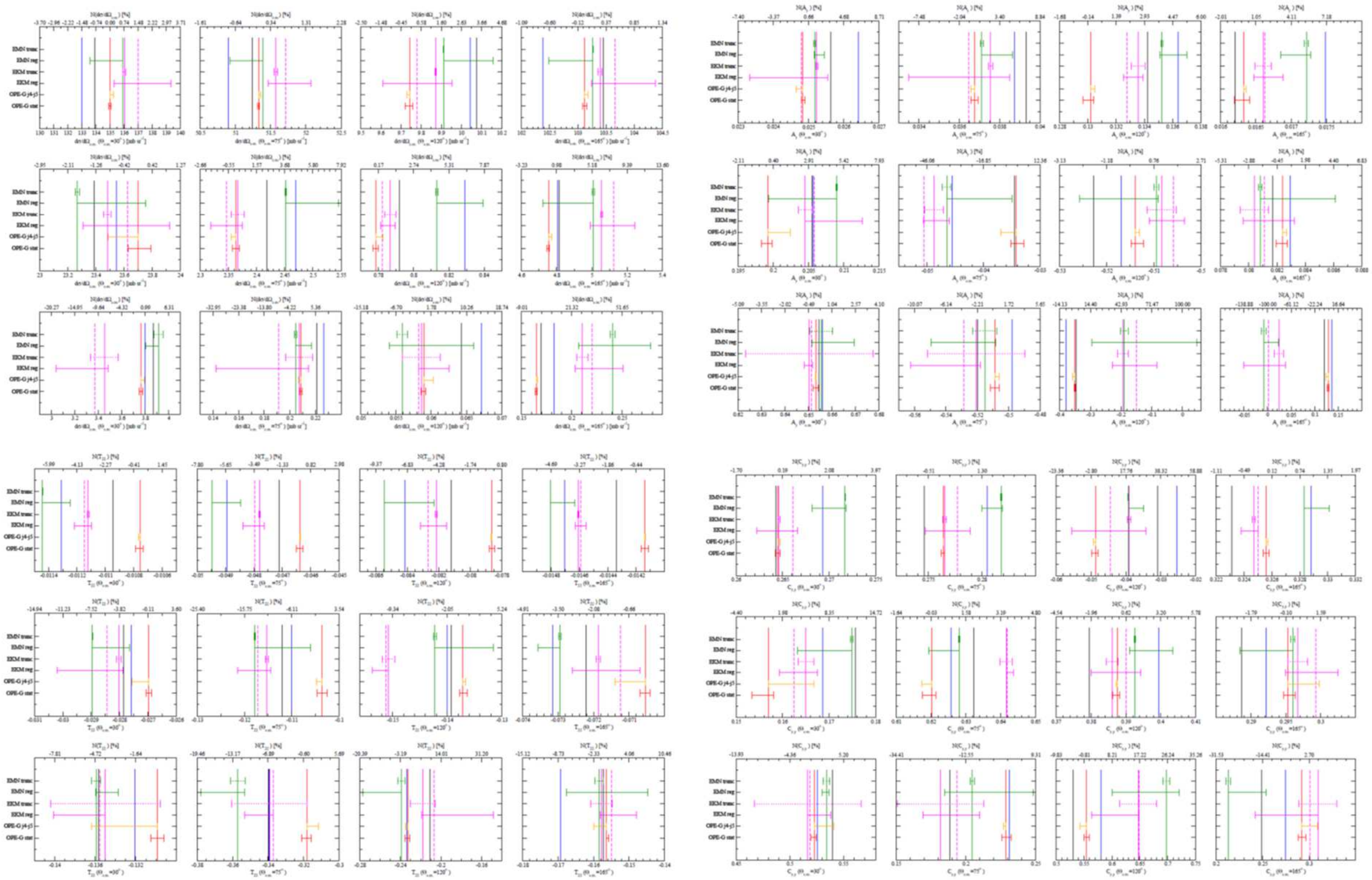
Red: Chiral N^4LO, D.R.E.,R.M.,Y.N.   Cyan N4LO+, Λ=450 MeV, P.R.,H.K.,E.E.

More models of interaction, more angles, more energies …



… and more observables …



Summary I

- All (realistic) forces has the similar quality.
- There is no single model which gives systematically the smallest or the biggest value. There are also no two models, whose predictions for all the cases are close to each other.

- The dominant theoretical uncertainties arise from using various models of the NN interaction.
- The statistical errors are small (and with no practical importance)
- For chiral models the dependence on regularization parameters dominates. Truncation errors become relatively big at higher energies.
- Two chiral models (the SCS Bochum-Bonn and the Idaho-Salamanca) at N^4LO often disagree.
- In general, the theoretical uncertainties remain smaller than the experimental ones.

More discussion in R.S. et al., Phys. Rev. C98, 014001 (2018).

Summary II

- New models of the chiral nuclear forces derived recently have been applied to the nucleon-deuteron scattering up to E=200 MeV.
- We find a **good data description**, **however** at the orders of chiral expansion investigated at the moment (with NN force up to N^4LO and NN+3NF up to N^2LO) **none of three-nucleon puzzles is solved.**

- New semilocal regularizations, both in coordinate as well as in momentum spaces, lead to **significantly smaller cut-off dependence** then older generation of Bochum-Bonn potentials. Especially, for the SMS force this dependence is so weak that **the problem of cut-off dependence (from 3N scattering application perspective) is solved.**
- **Statistical errors** can be also estimated for the SMS chiral interaction. We conclude that resulting uncertainty is **smaller than** truncation errors.
- Consistent SCS NN and 3N N^2LO potentials give predictions of the similar quality as semi-phenomenological interactions.

Thank you for your kind attention !