SPIN HYDRODYNAMICS
IN RELATIVISTIC HEAVY-ION COLLISIONS

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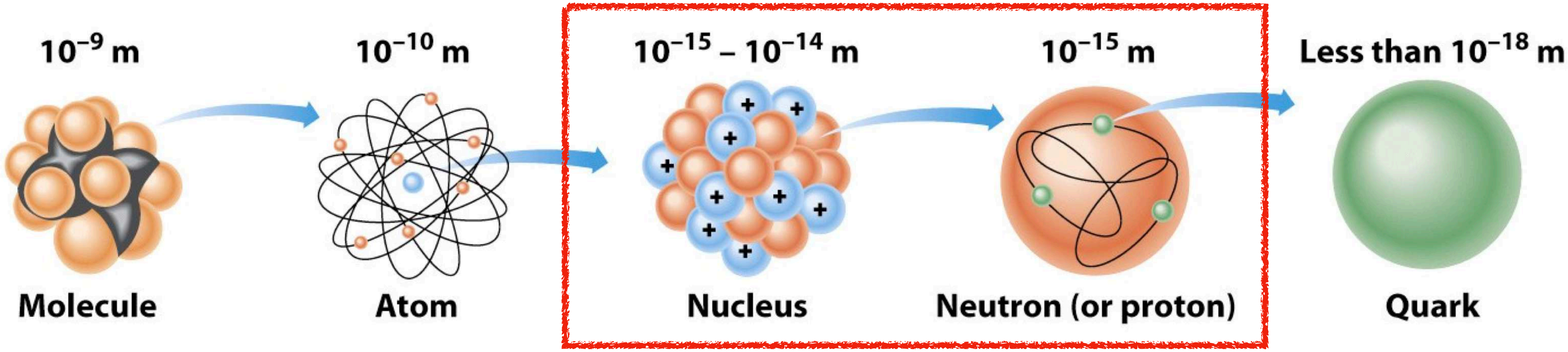
NUCLEAR PHYSICS DIVISION SEMINAR
18 DECEMBER 2025
UNIVERSITY OF WARSAW

THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

MOTIVATION

THE STRUCTURE OF MATTER AT THE FEMTOSCALE



PARTONIC STRUCTURE OF HADRONS

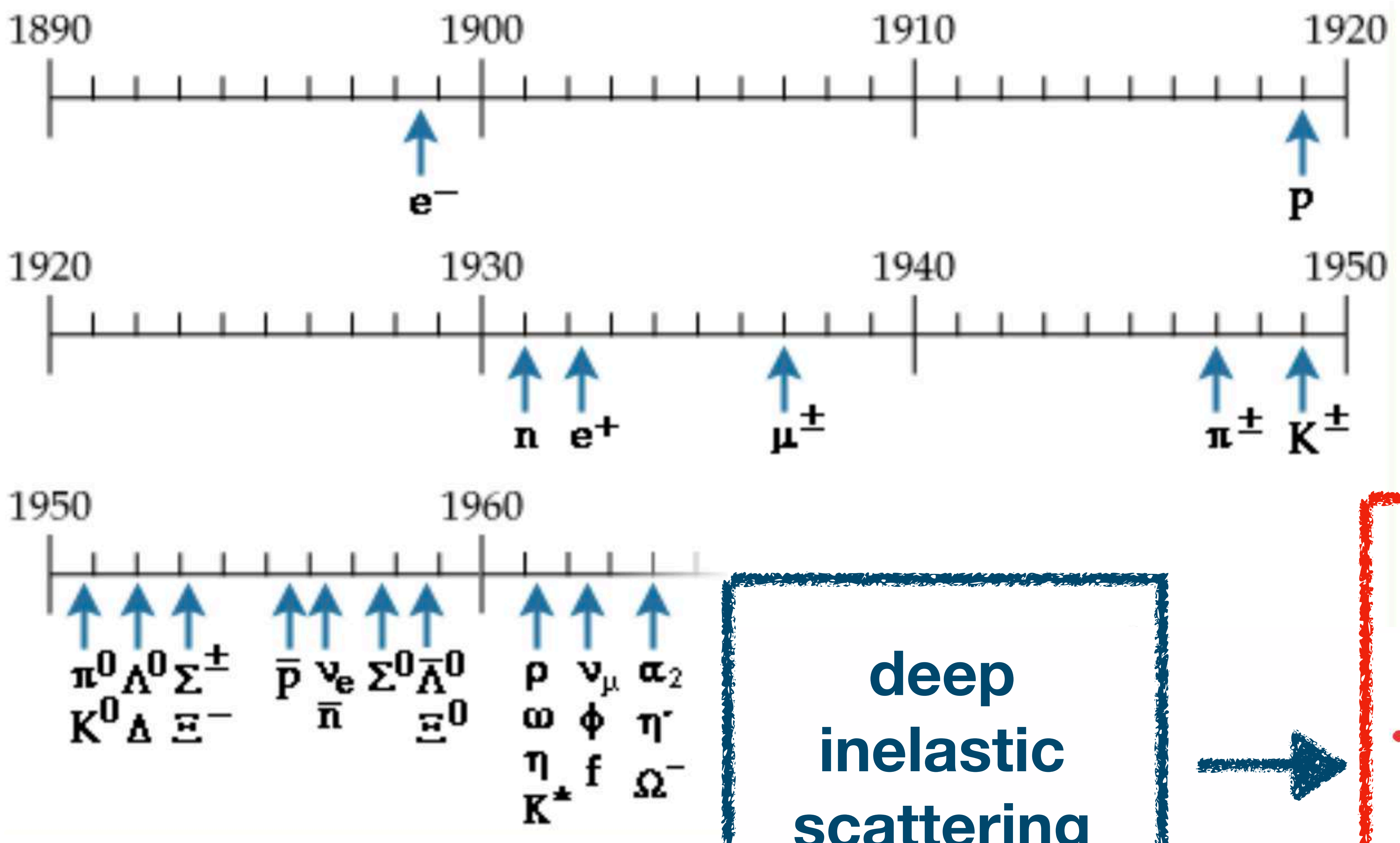
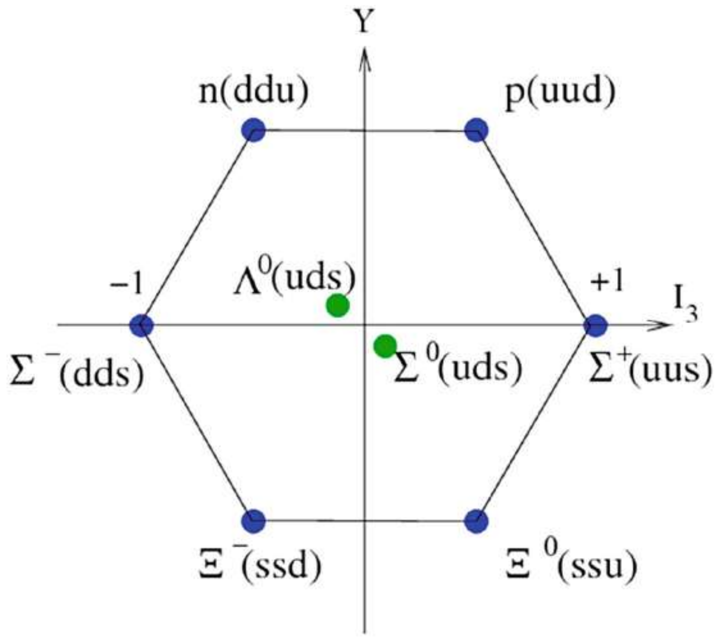


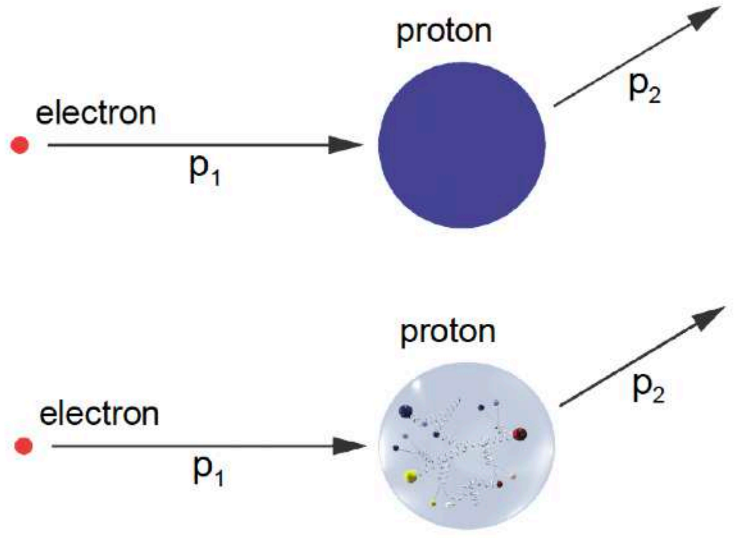
figure: Jianwei Qiu

quark model

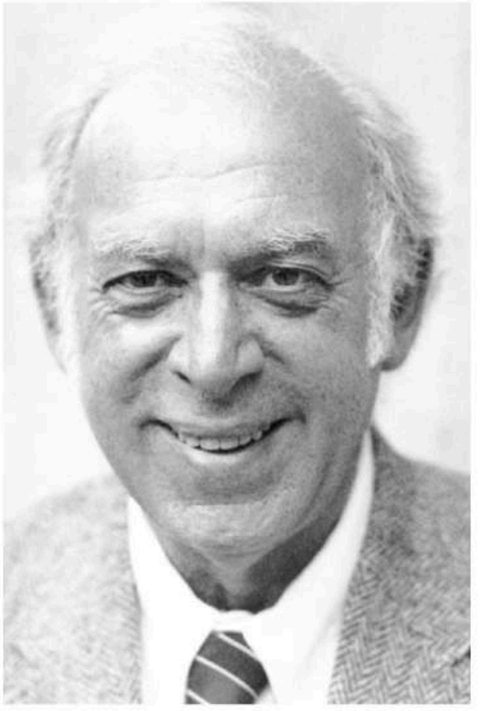


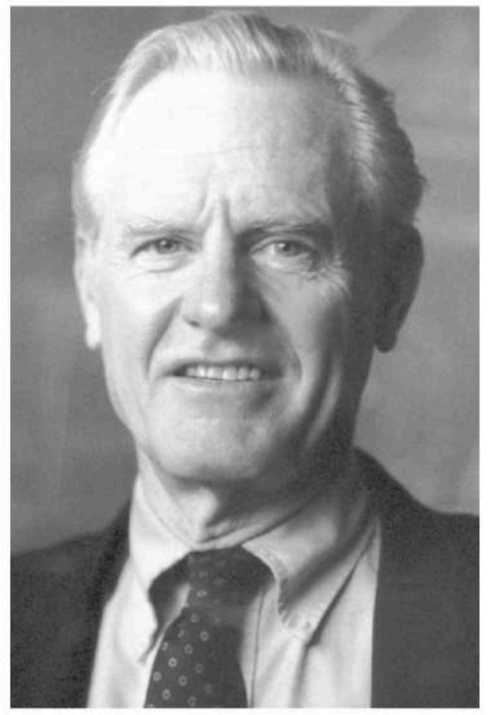
1969

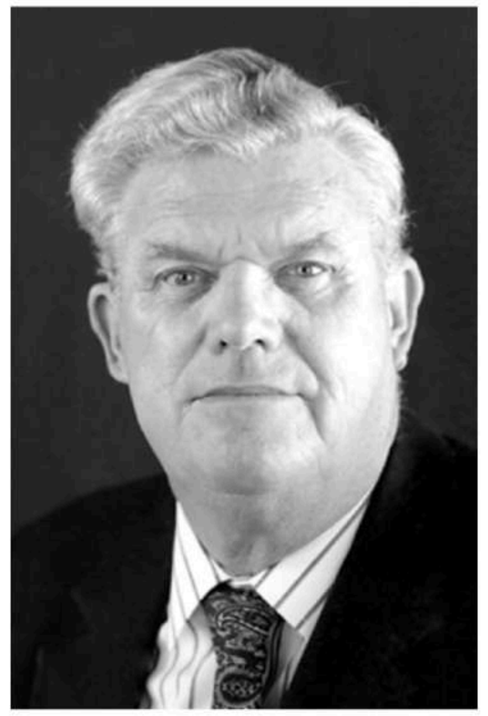
deep inelastic scattering



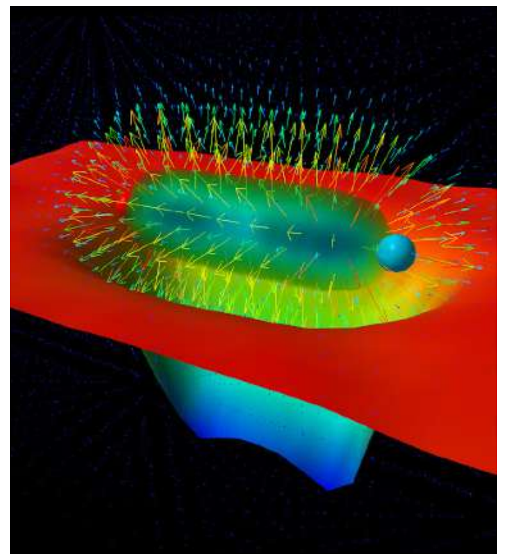
1990



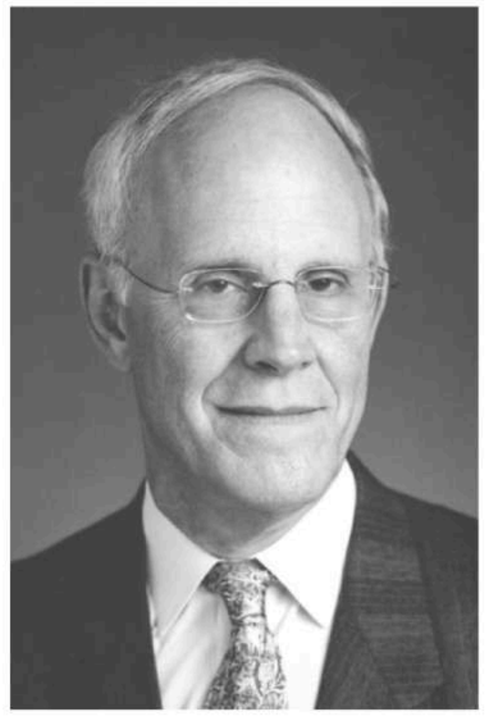


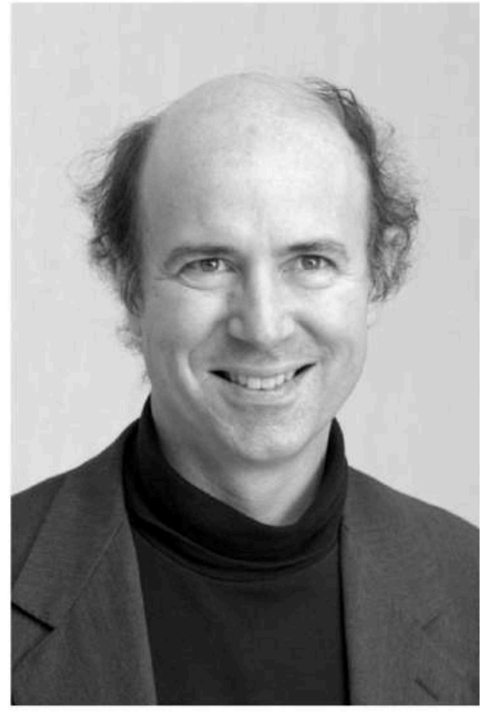


asymptotic freedom

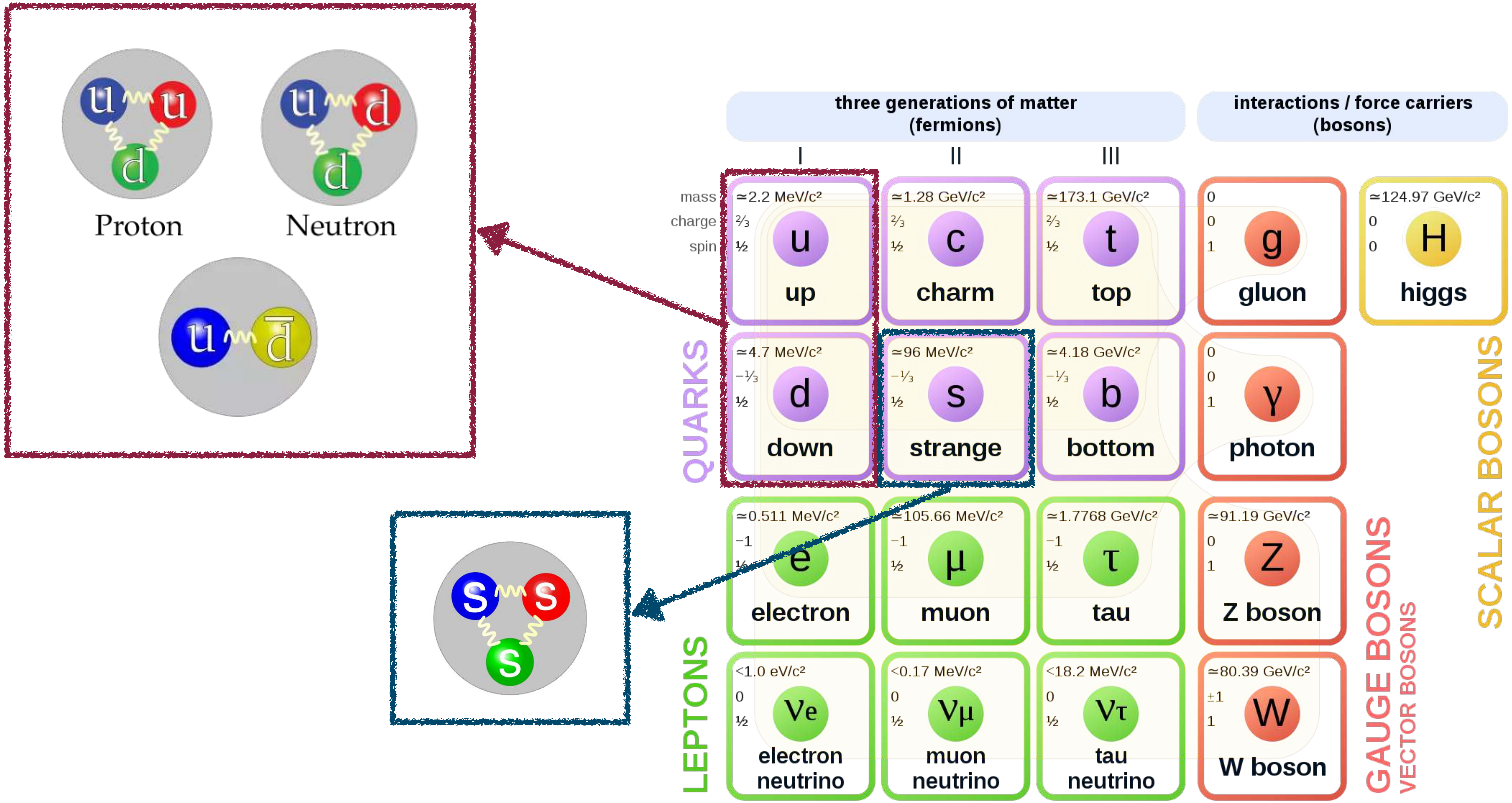


2004

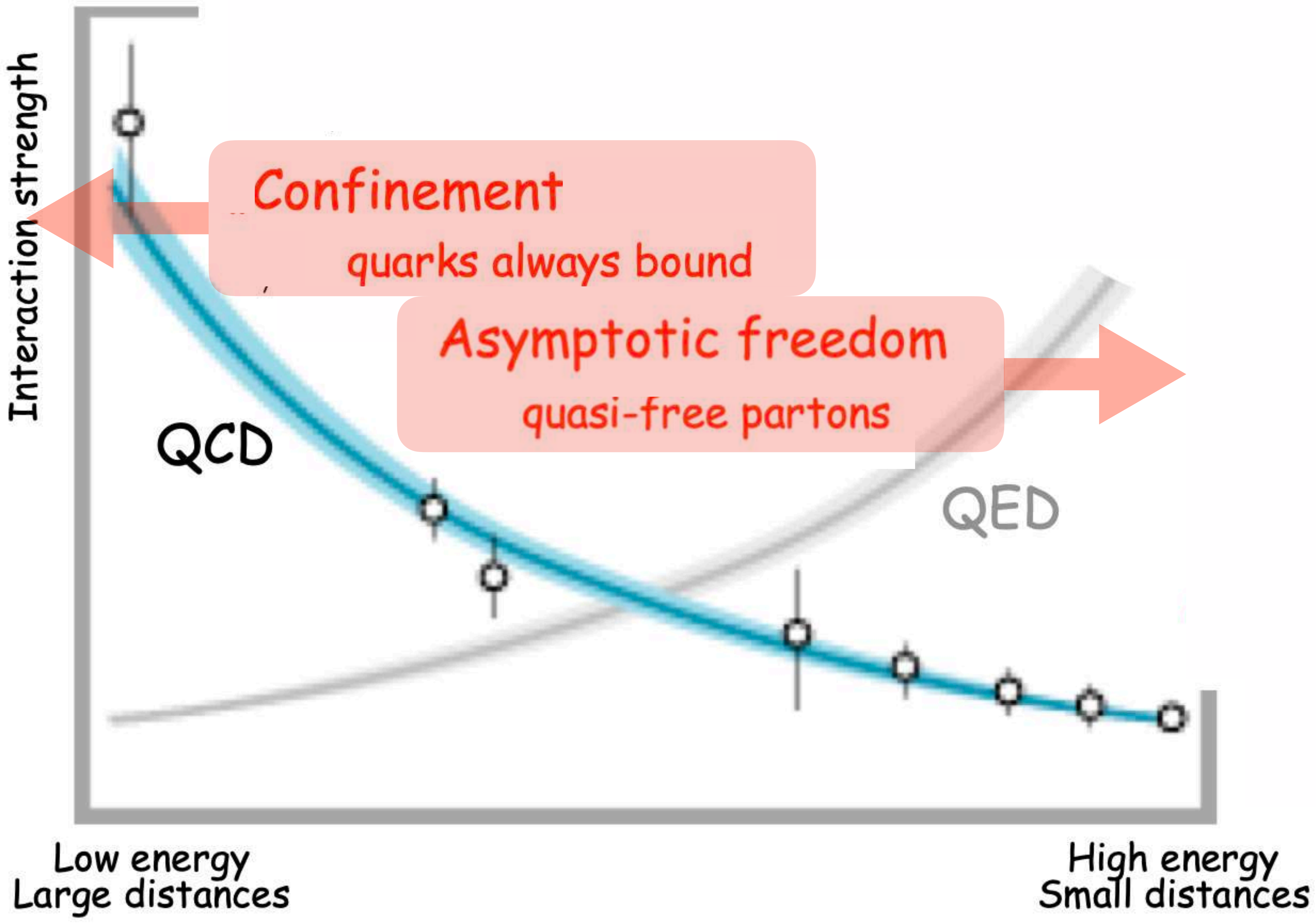




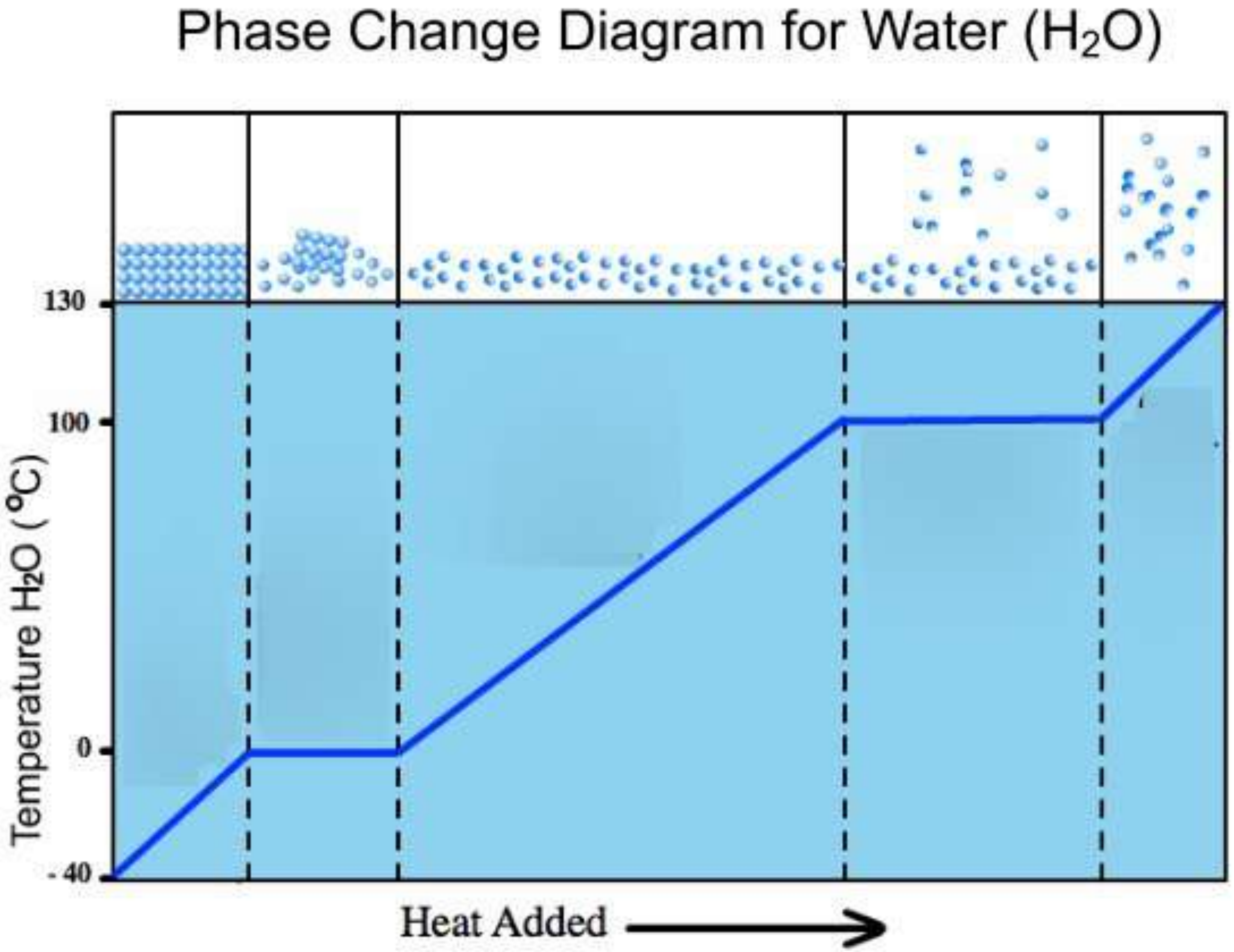
STANDARD MODEL AND QUANTUM CHROMODYNAMICS

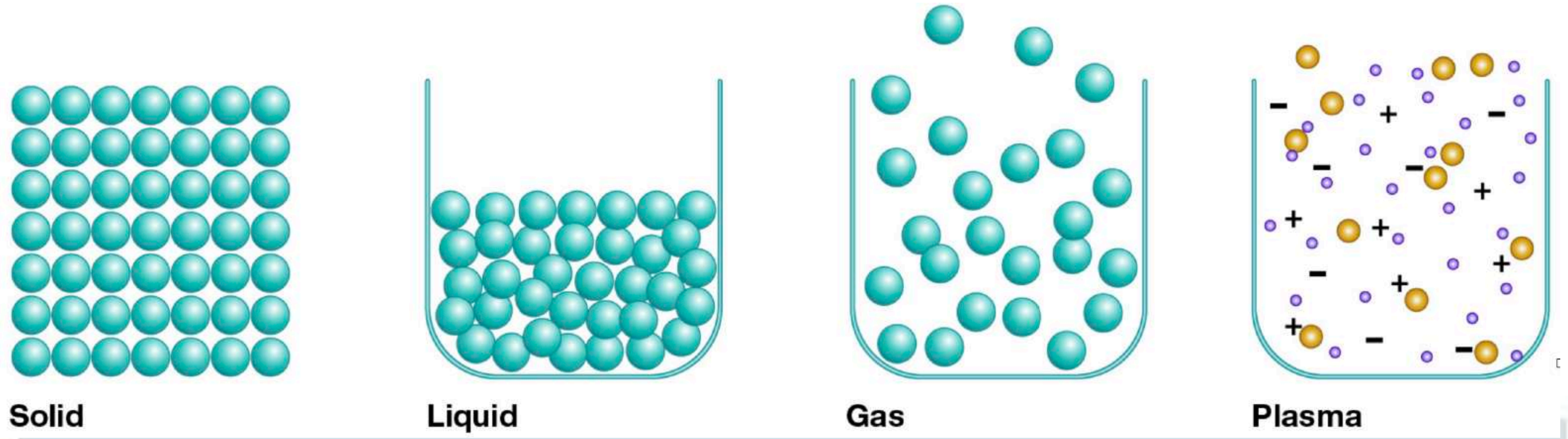


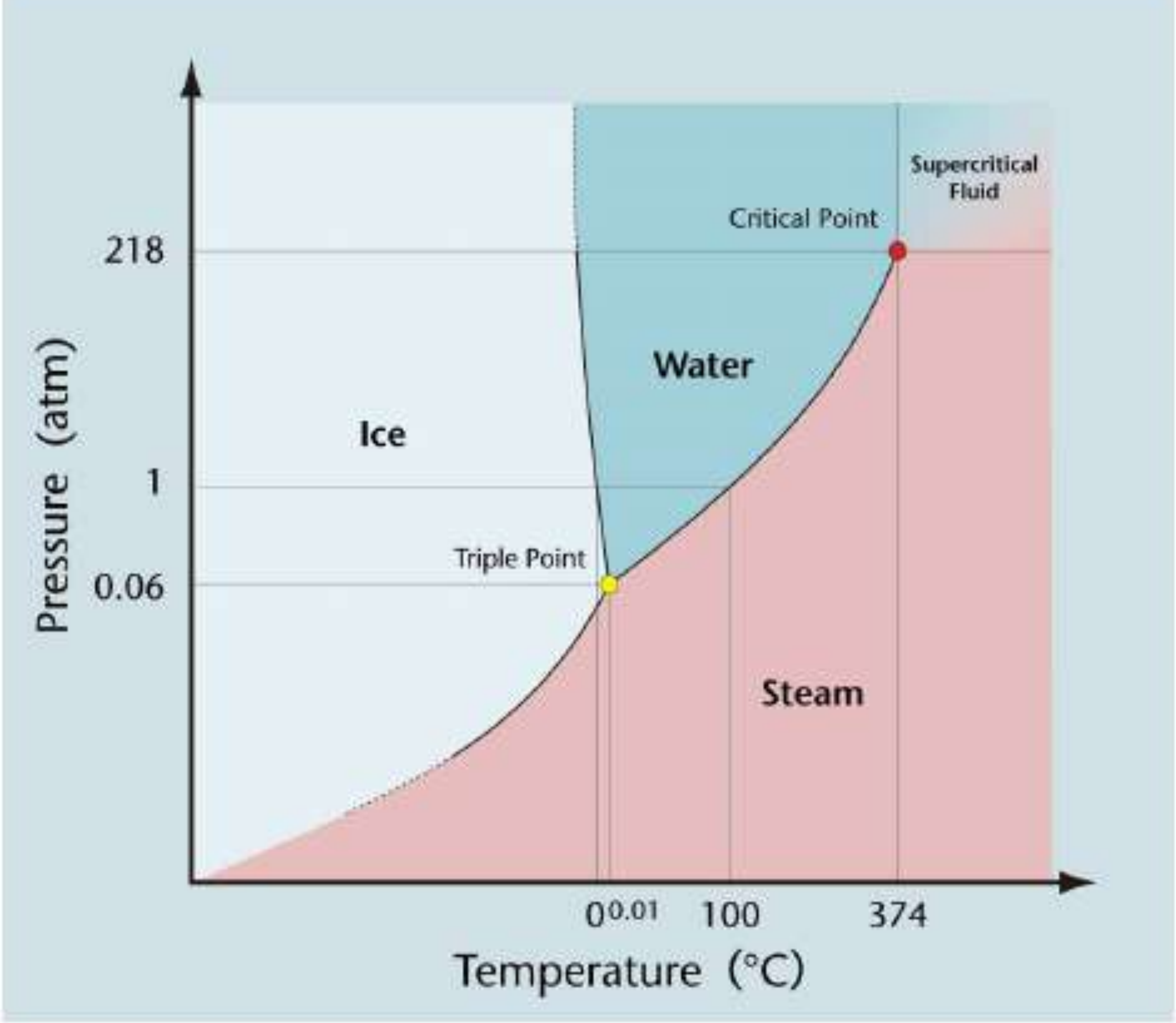
CONFINEMENT VS ASYMPTOTIC FREEDOM



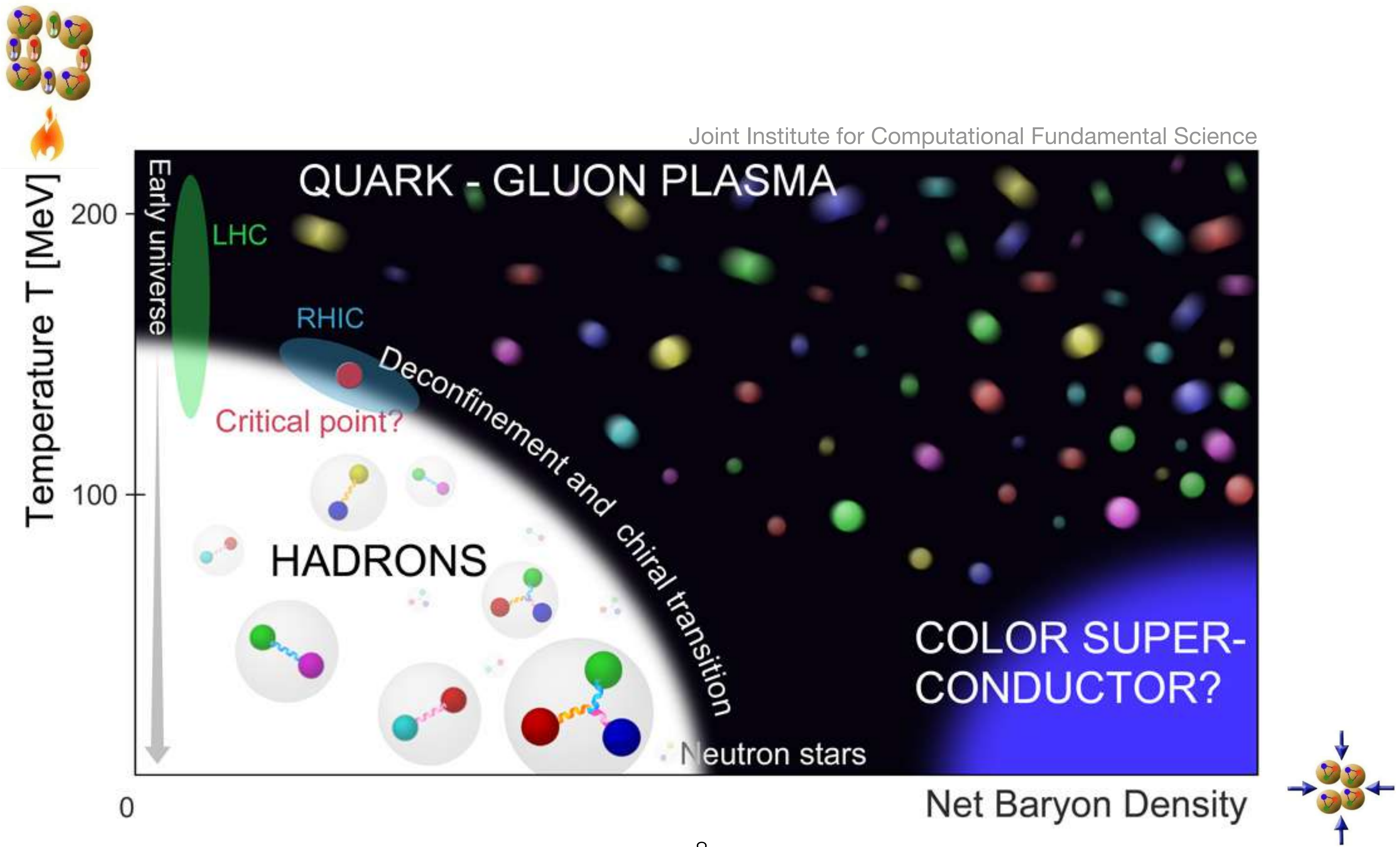
PHASE TRANSITIONS OF WATER







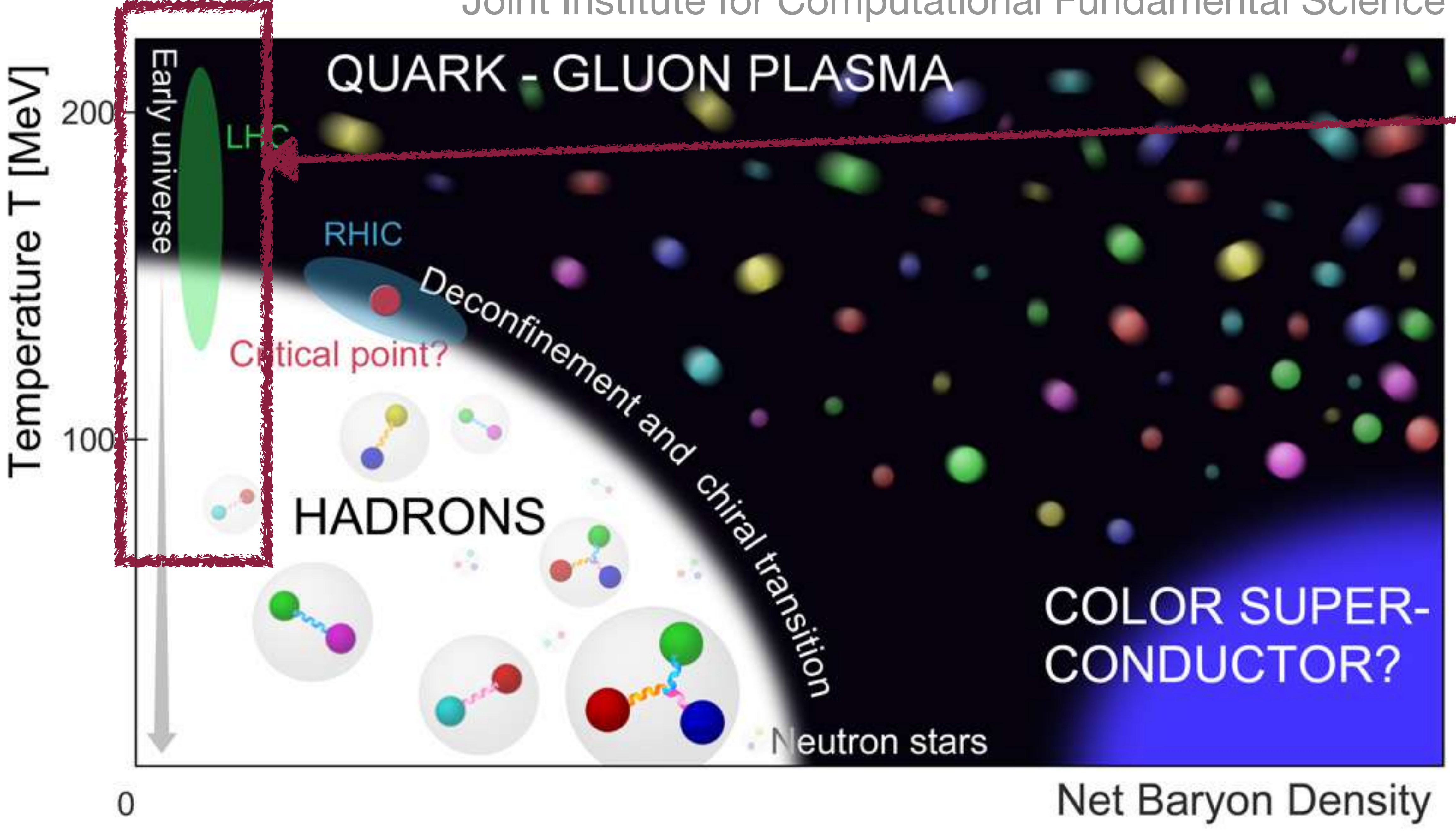
PHASE DIAGRAM OF QCD IN EXTREME CONDITIONS

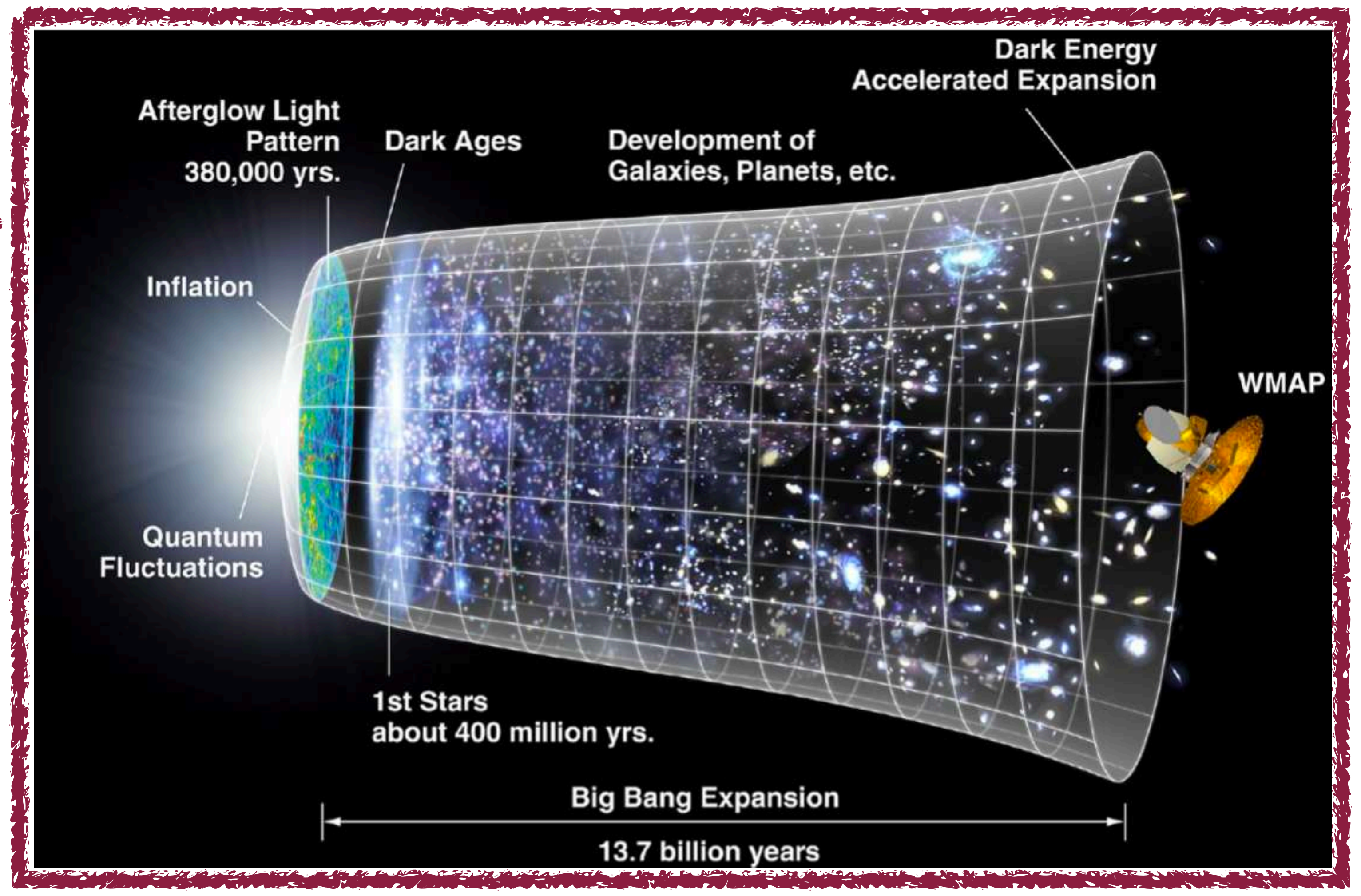


PHASE DIAGRAM OF QCD IN EXTREME CONDITIONS

early Universe

Joint Institute for Computational Fundamental Science

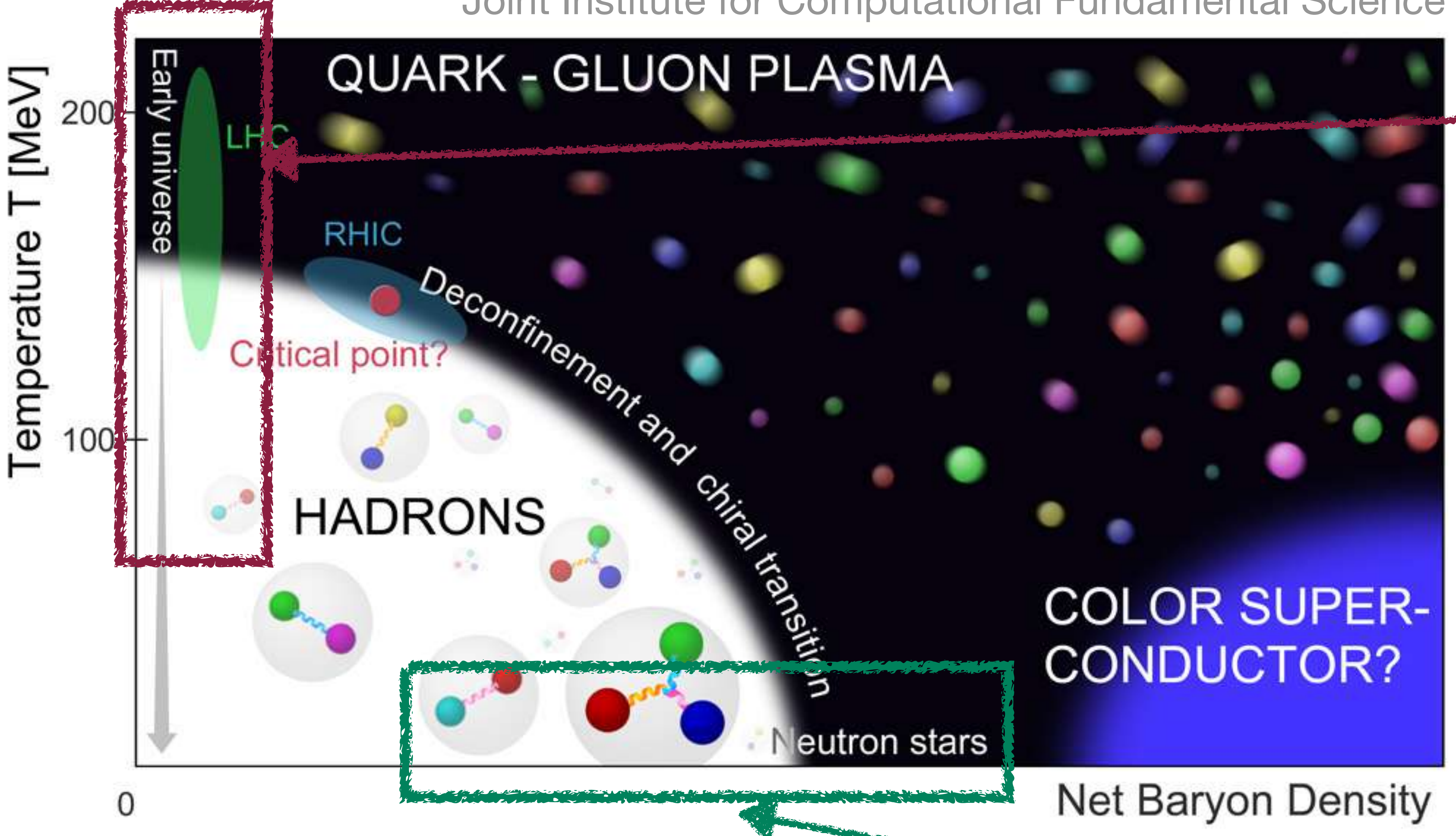




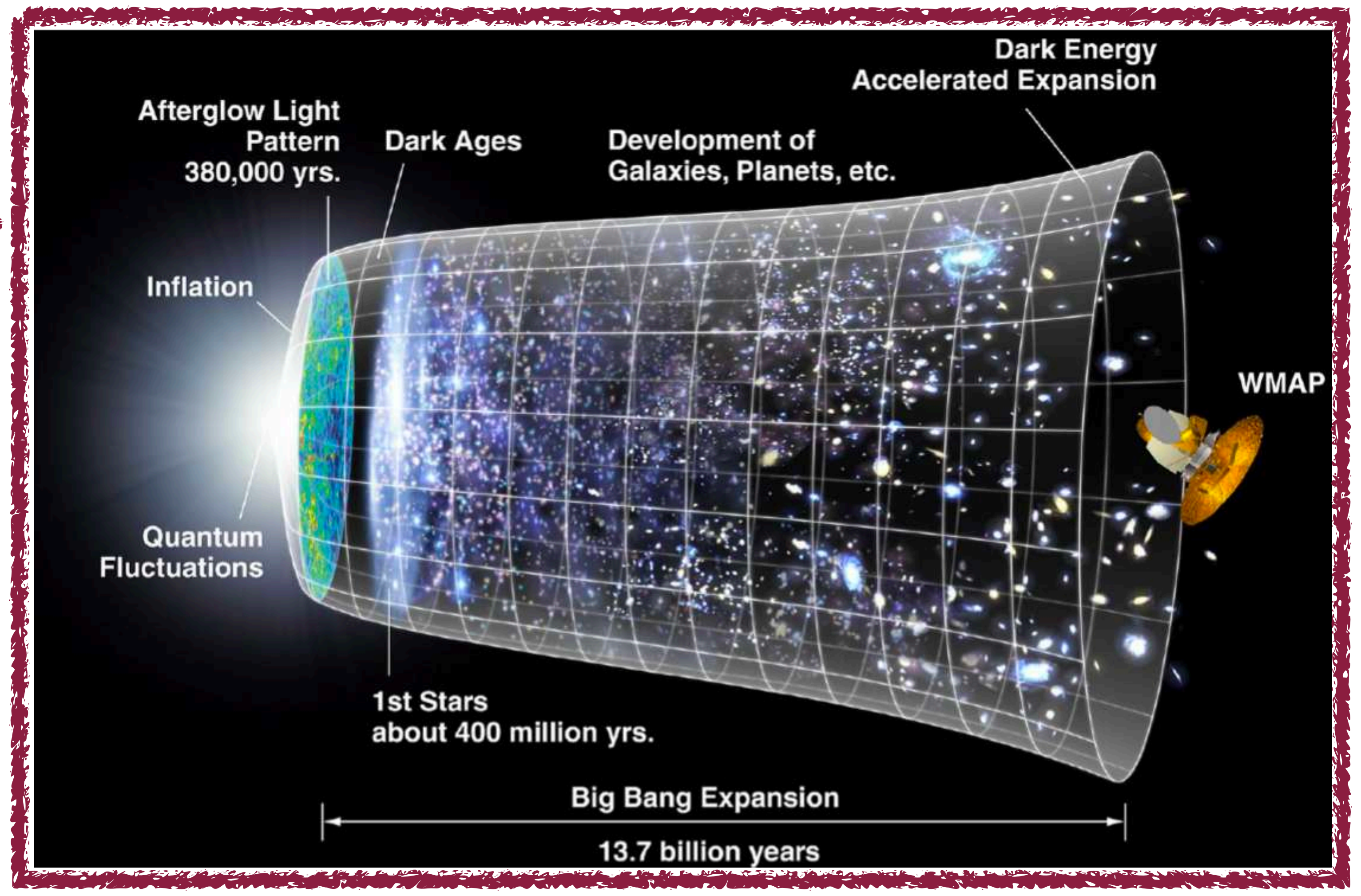
NASA

PHASE DIAGRAM OF QCD IN EXTREME CONDITIONS

Joint Institute for Computational Fundamental Science

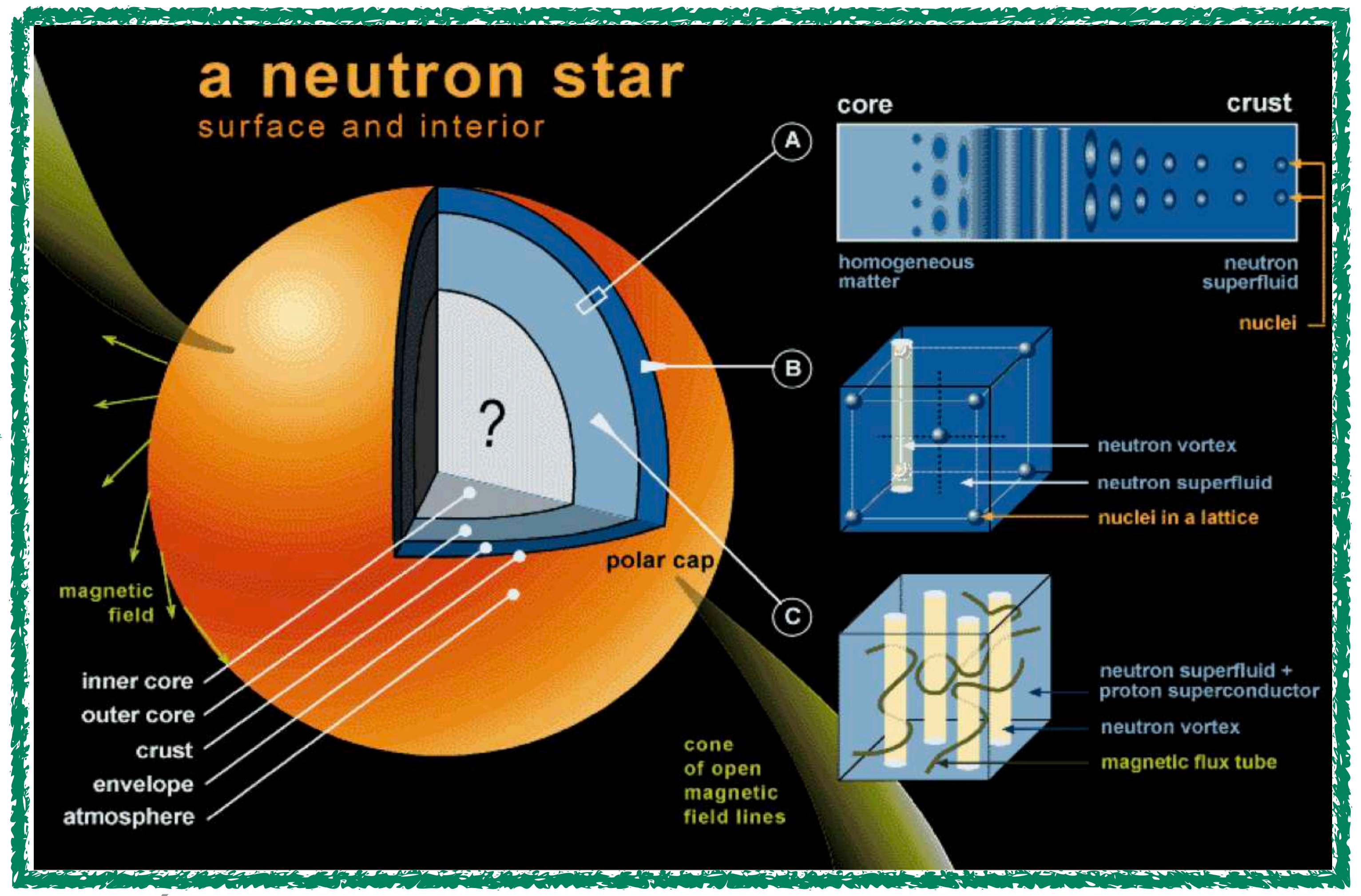


early Universe



NASA

neutron stars

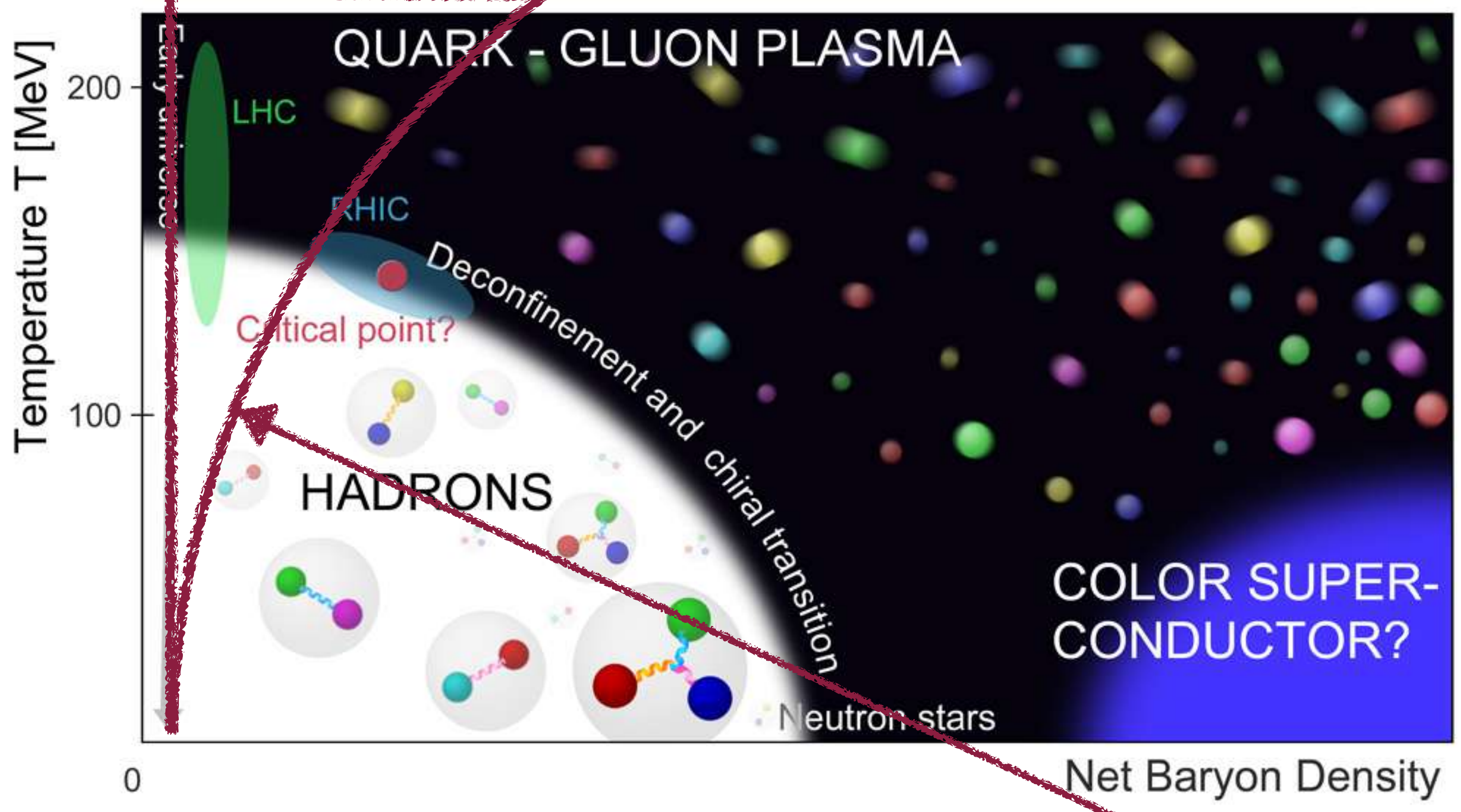


D.E. Á. Castillo, talk @RagTime 22

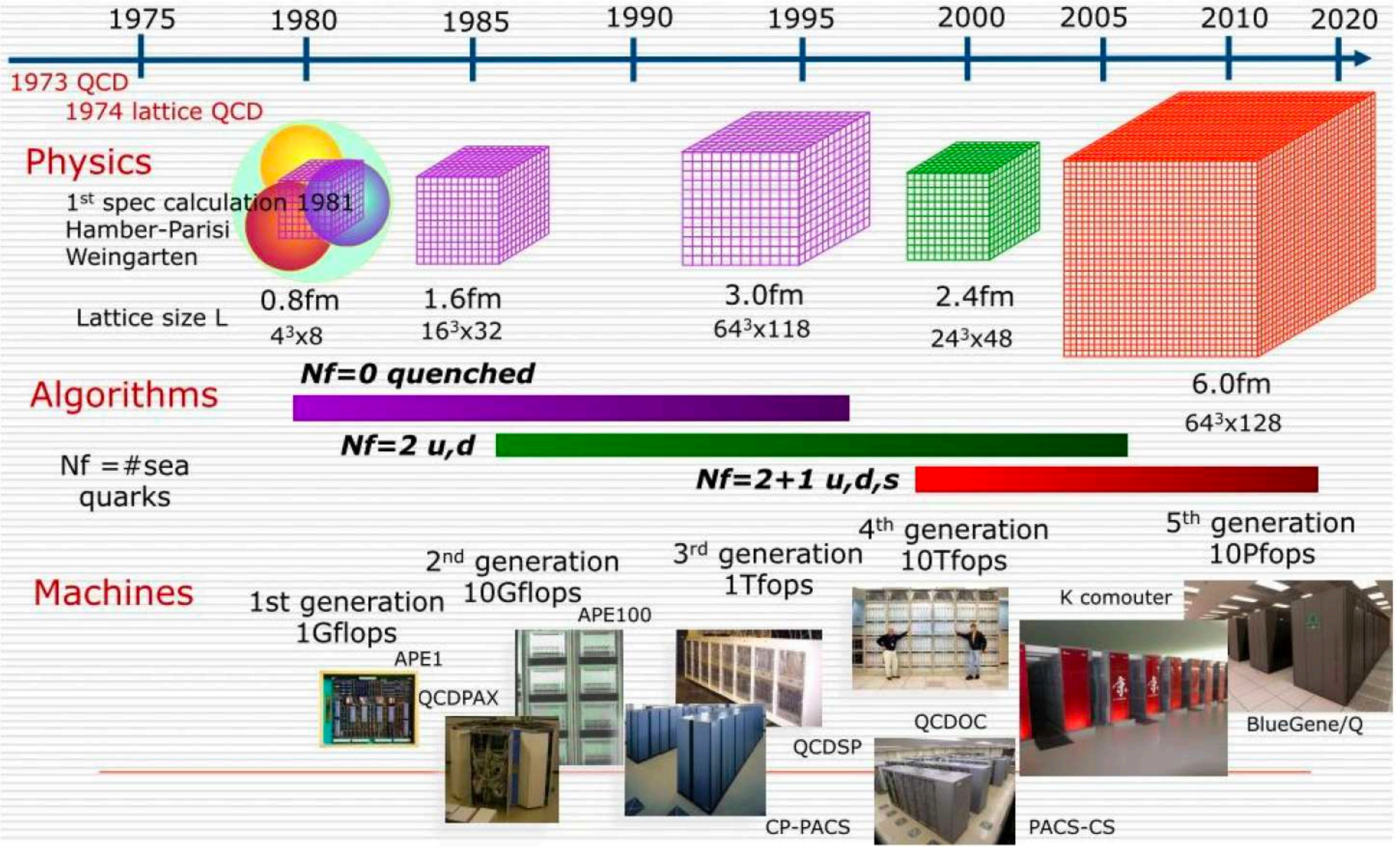
PHASE DIAGRAM OF QCD IN EXTREME CONDITIONS

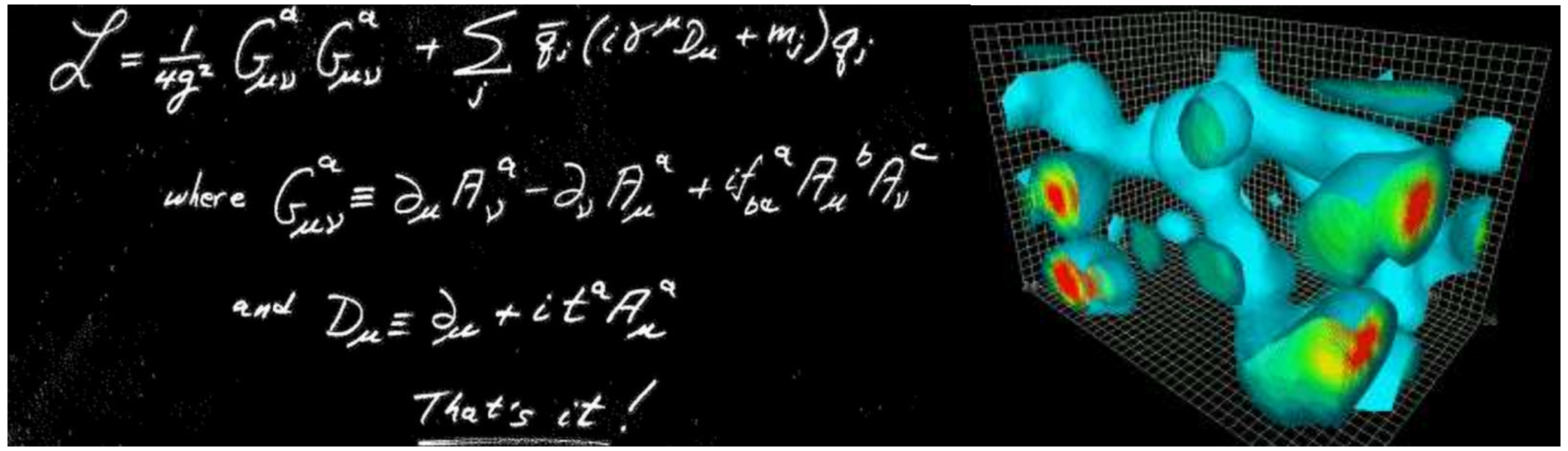
simulations of QCD on the lattice

Joint Institute for Computational Fundamental Science

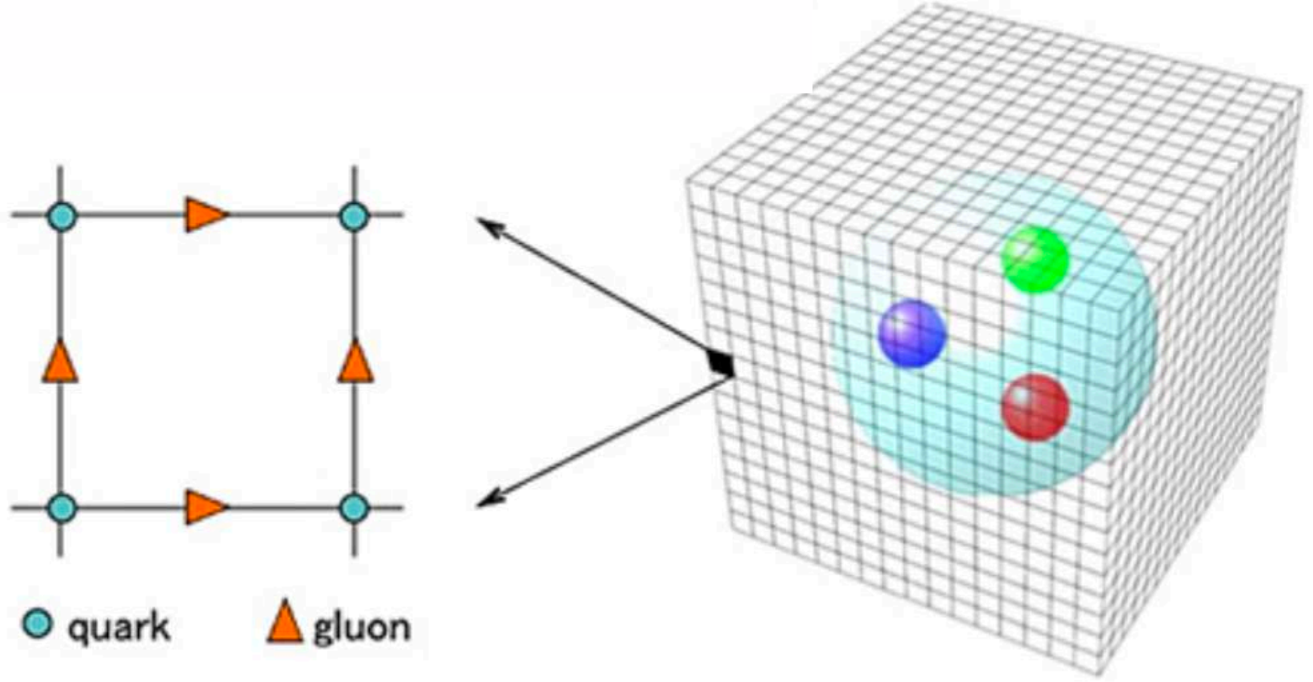


http://clrwww.in2p3.fr





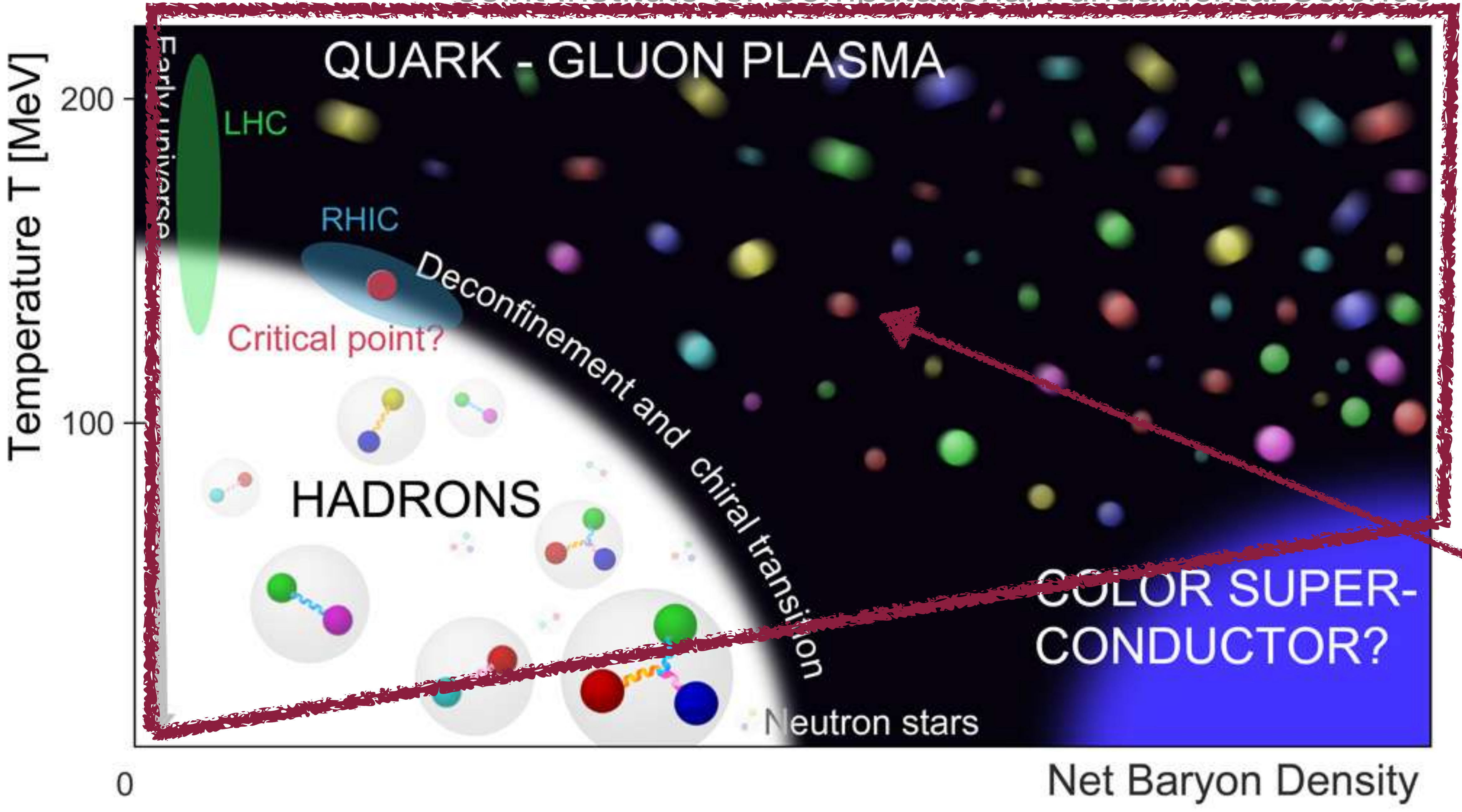
D. Leinweber (www.physics.adelaide.edu.au)



F. Karsch et al., Phys. Lett. B478, 447 (2000)

PHASE DIAGRAM OF QCD IN EXTREME CONDITIONS

Joint Institute for Computational Fundamental Science



heavy-ion collisions

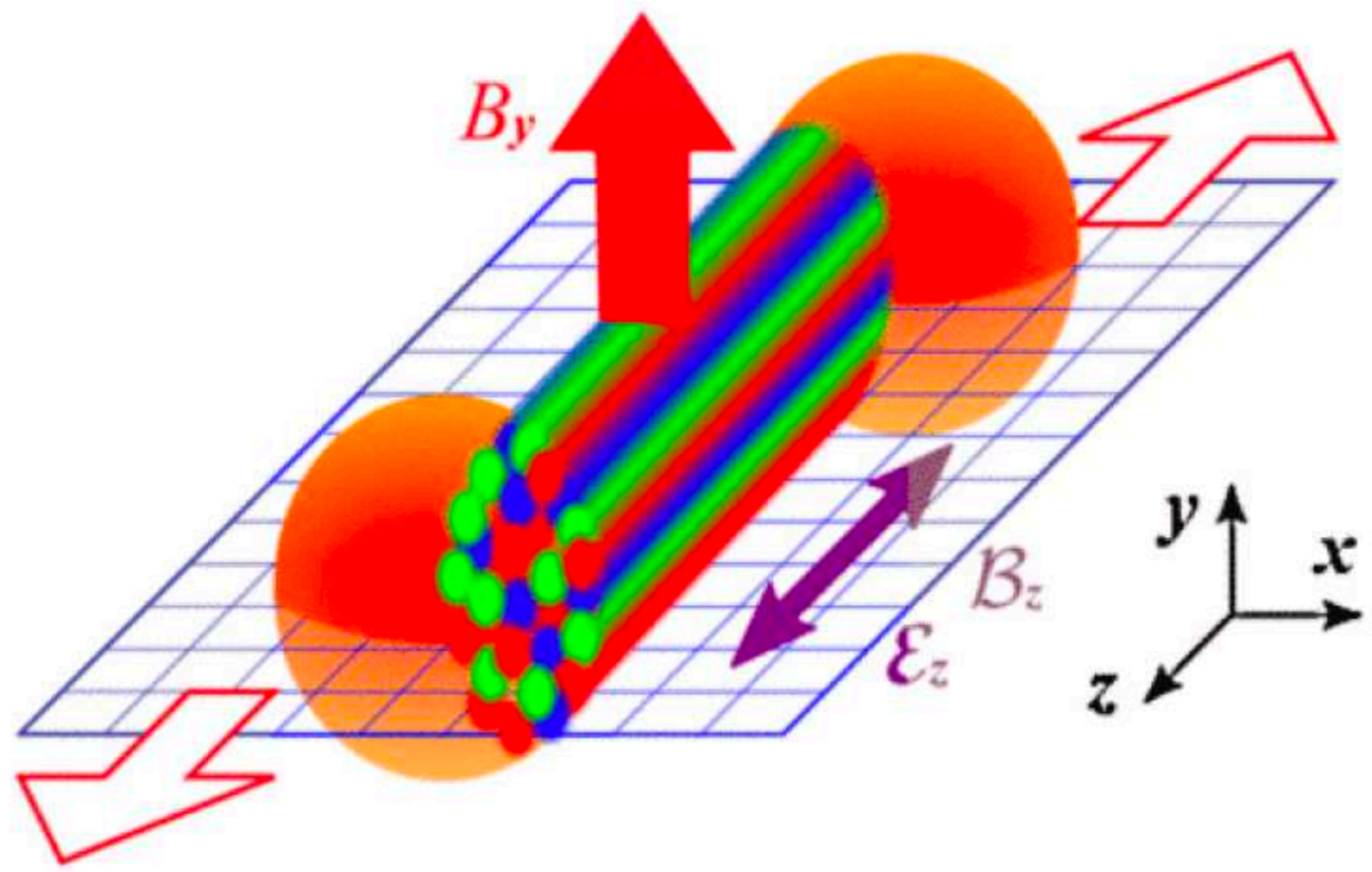
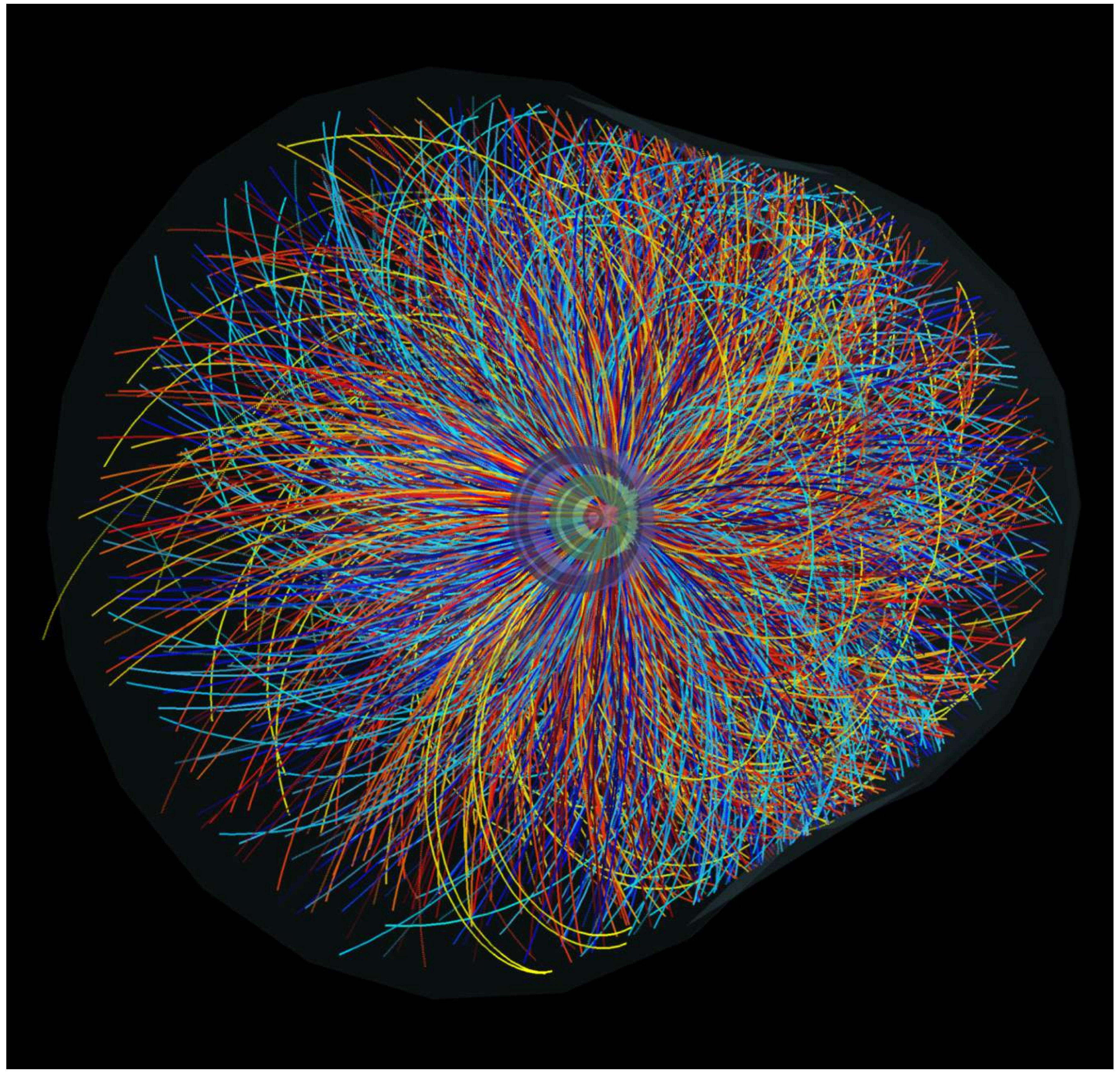


figure: K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRL 104, 212001

PROBING THE MATTER PROPERTIES AND DYNAMICS



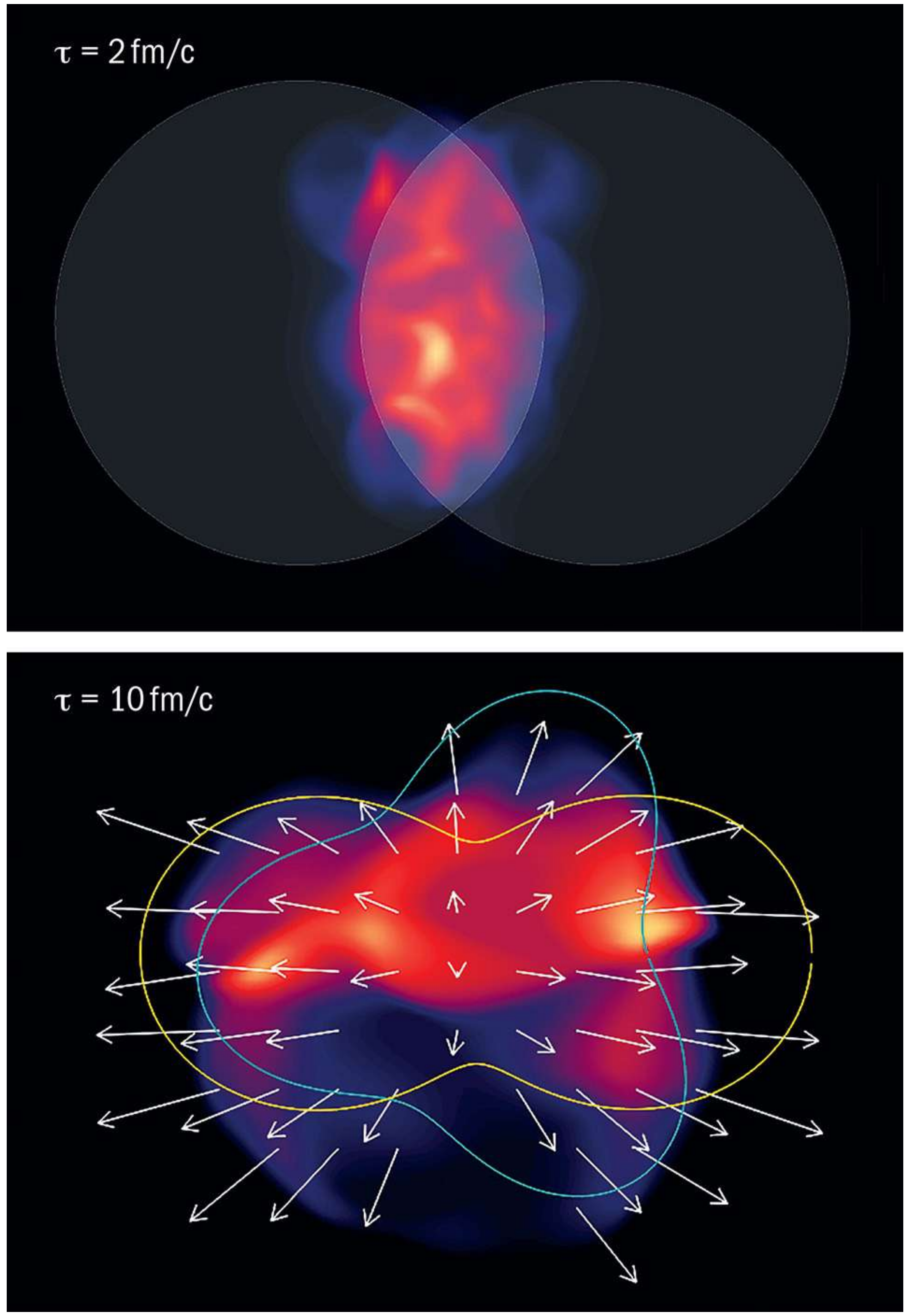
final state

PROBING THE MATTER PROPERTIES AND DYNAMICS

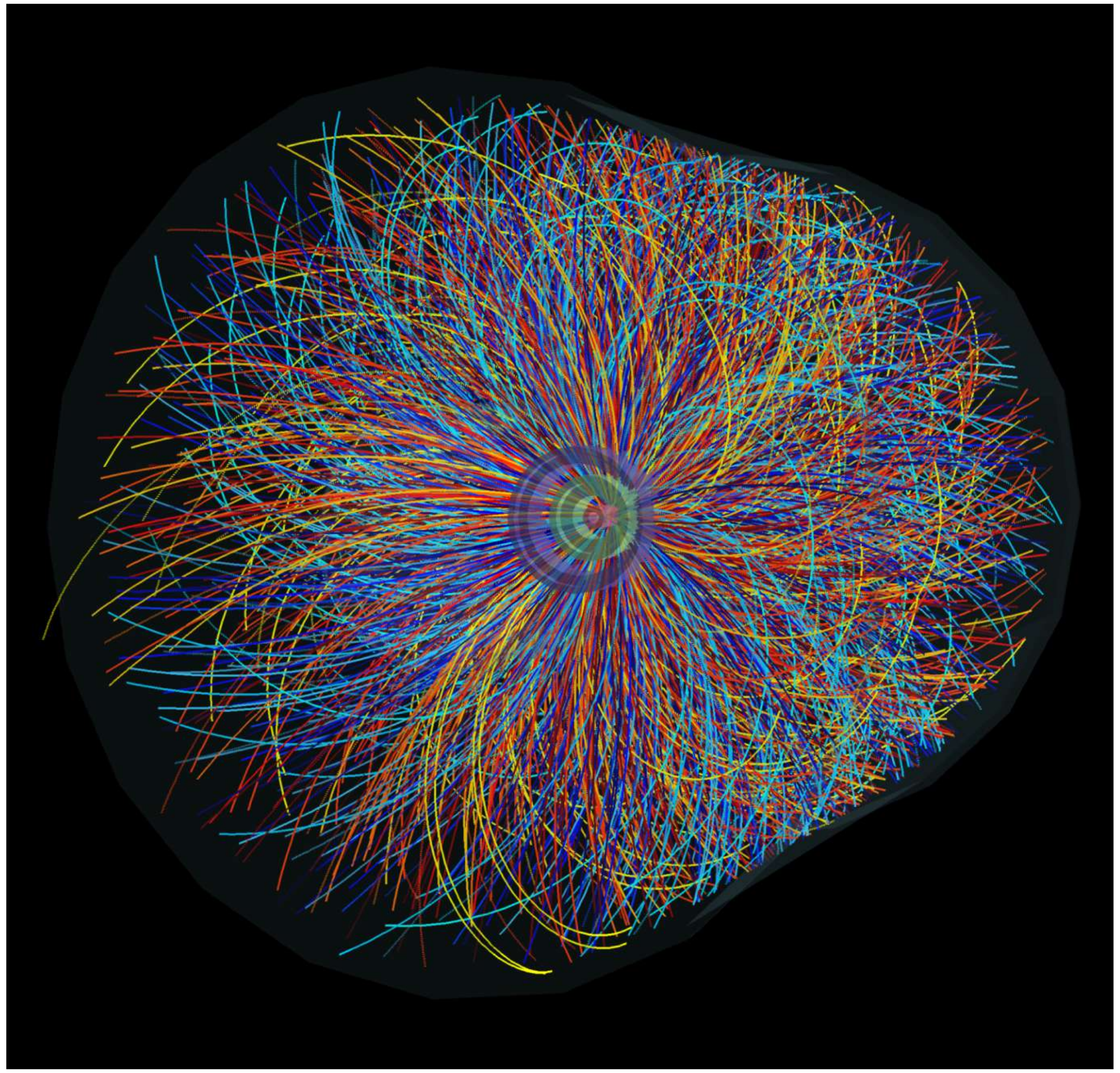
initial state

properties

dynamics



https://cerncourier.com/a/going-with-the-flow/



final state

HOW WE CAN PROBE QGP?

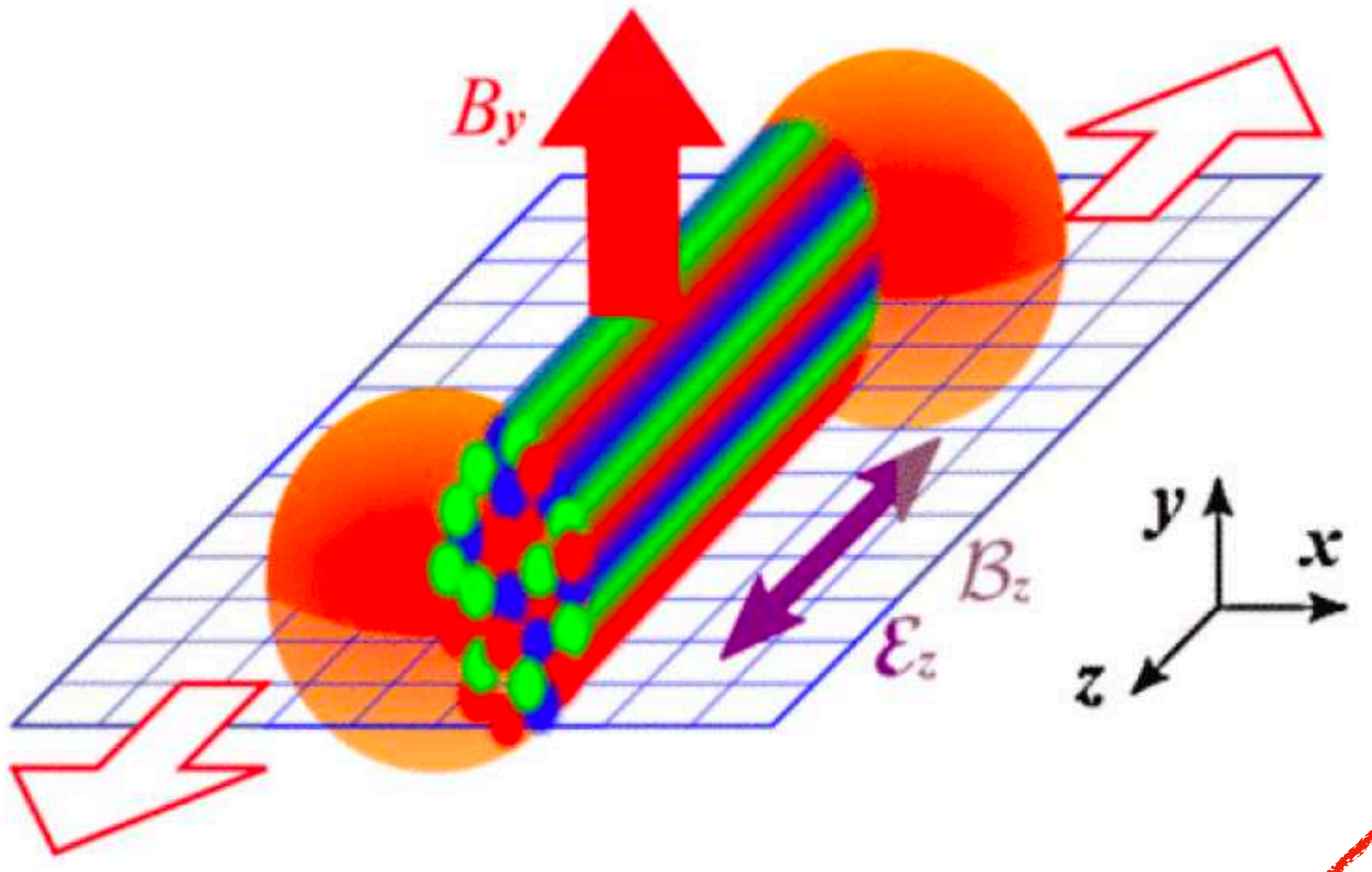


figure: Phys. Rev. Lett. 104, 212001

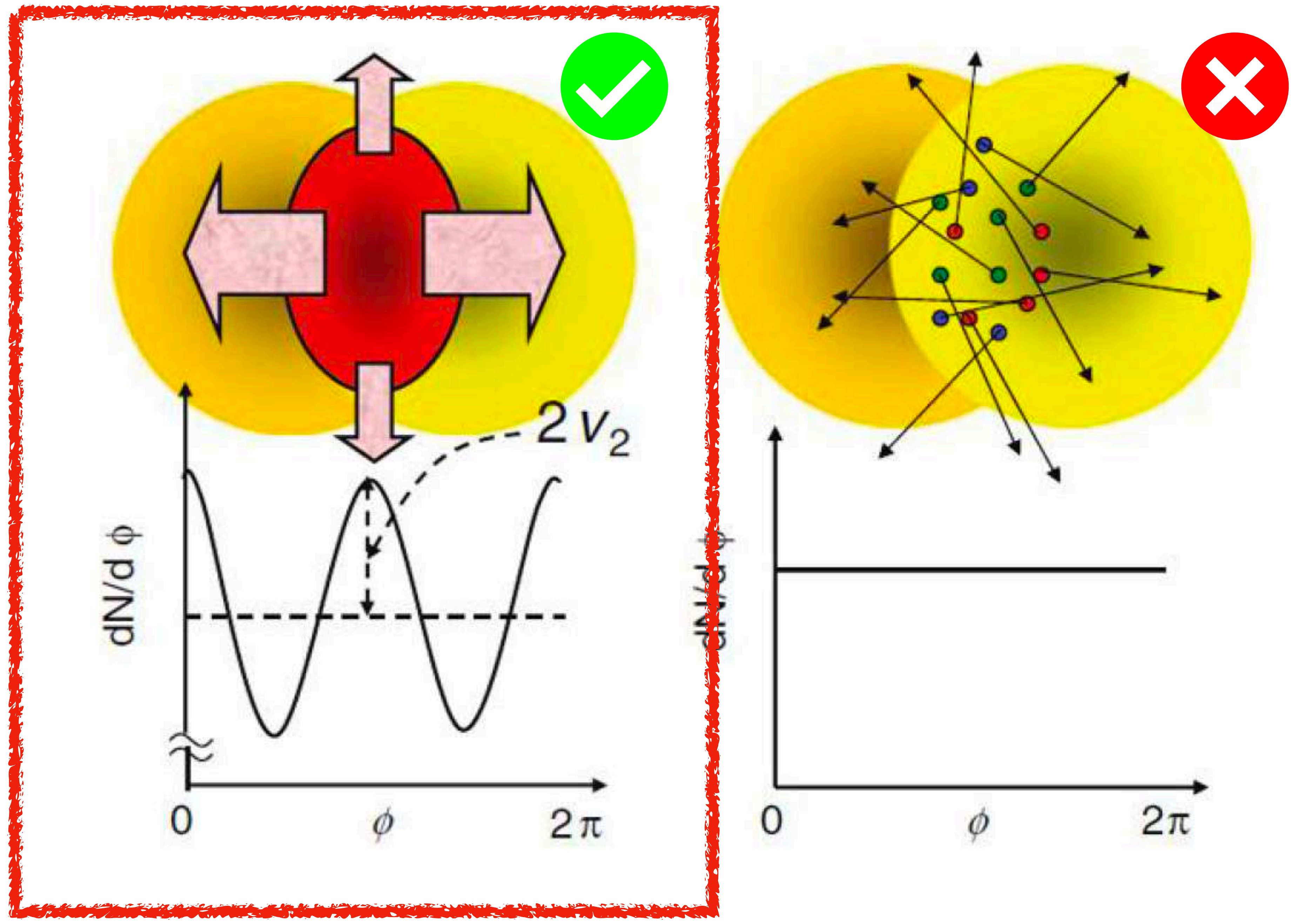
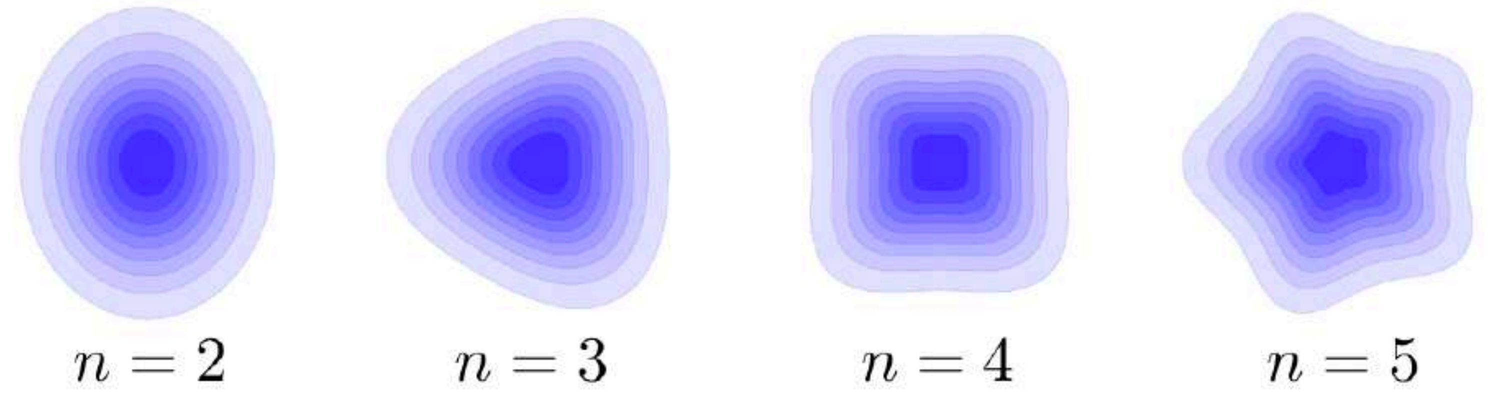


figure: Lect.Notes Phys. 785 (2010) 139-178

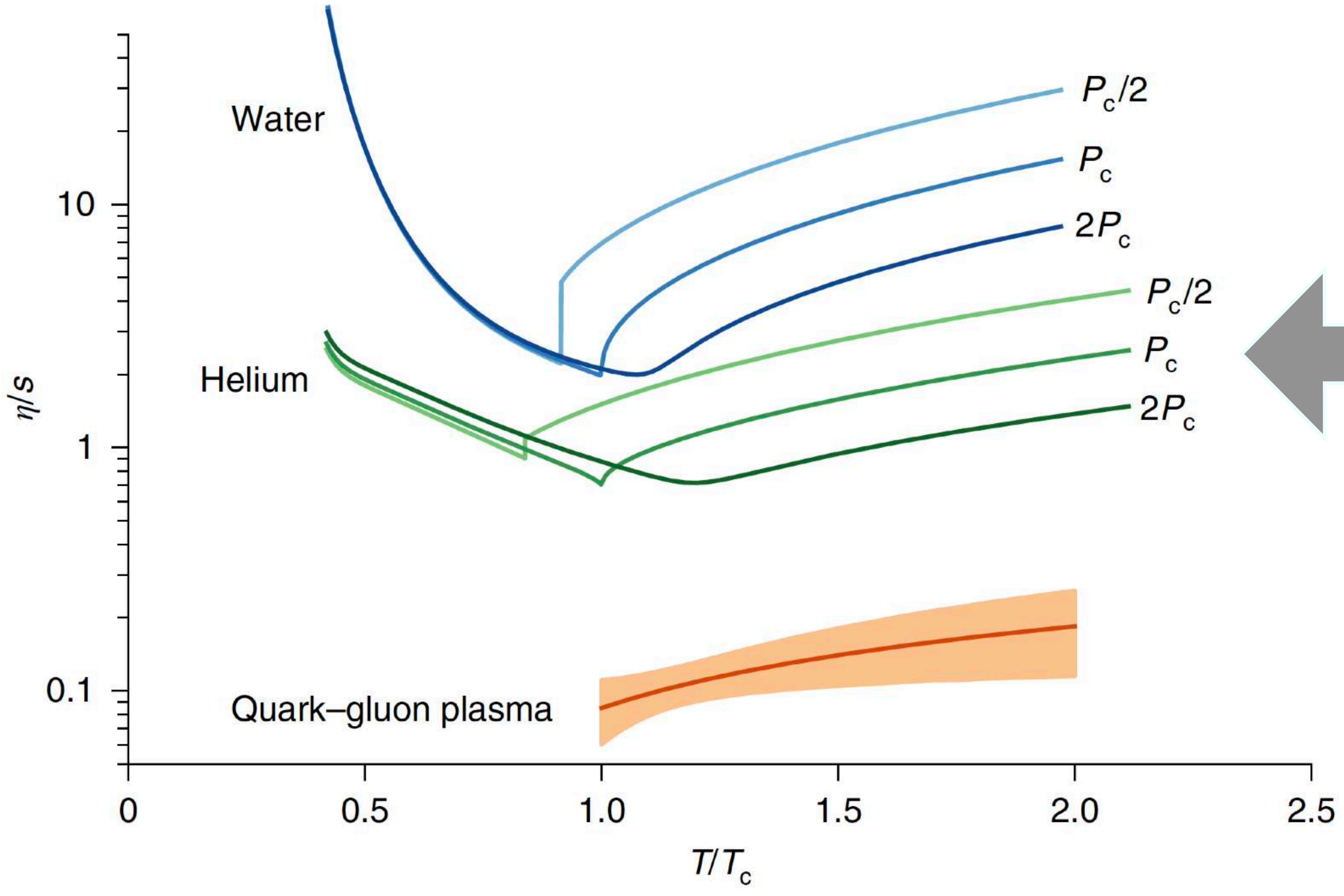
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left[1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + ...\right]$$



strongly-interacting → fluid

QGP IS AN ALMOST PERFECT FLUID

The hot and dense QCD matter behaves as an almost perfect fluid



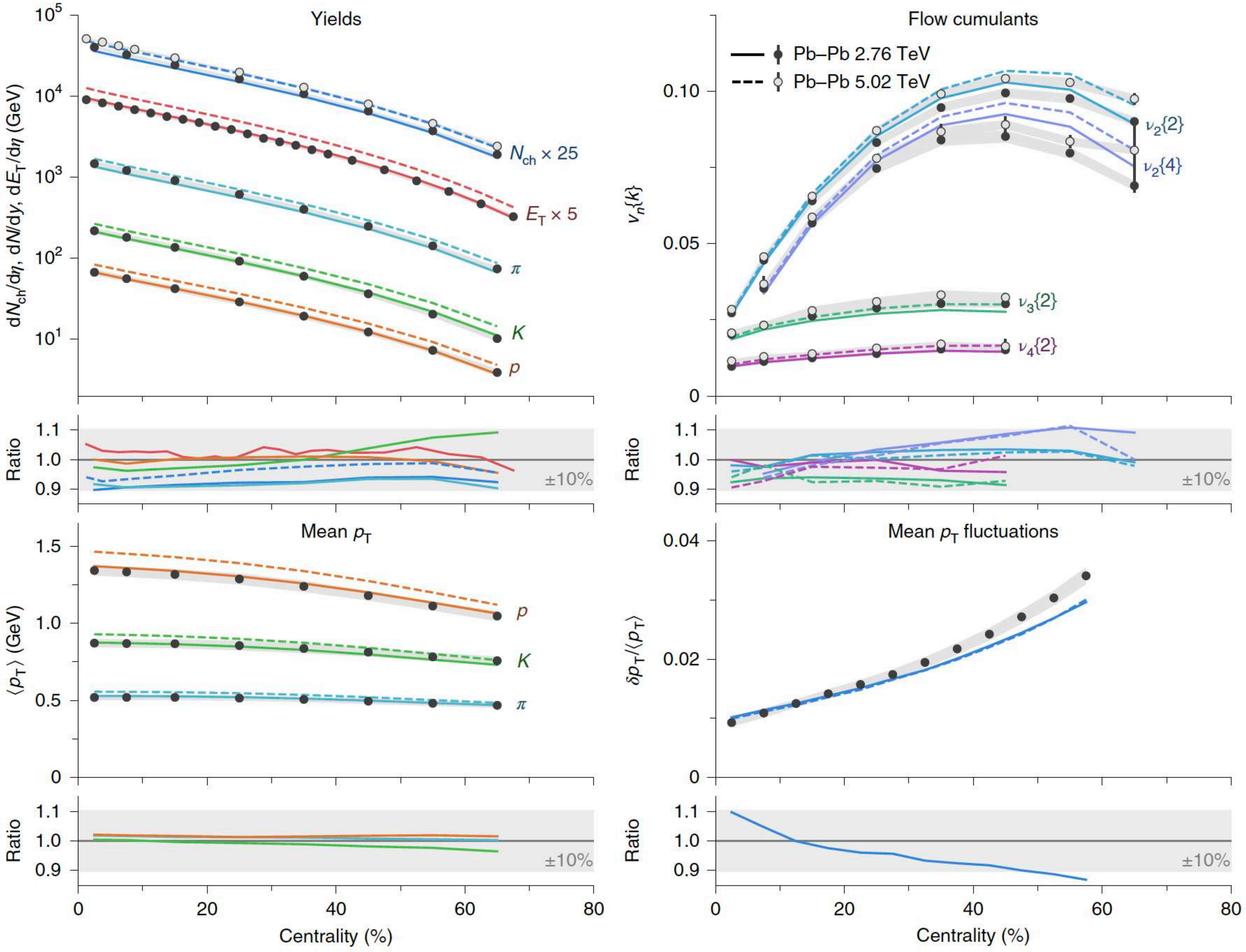
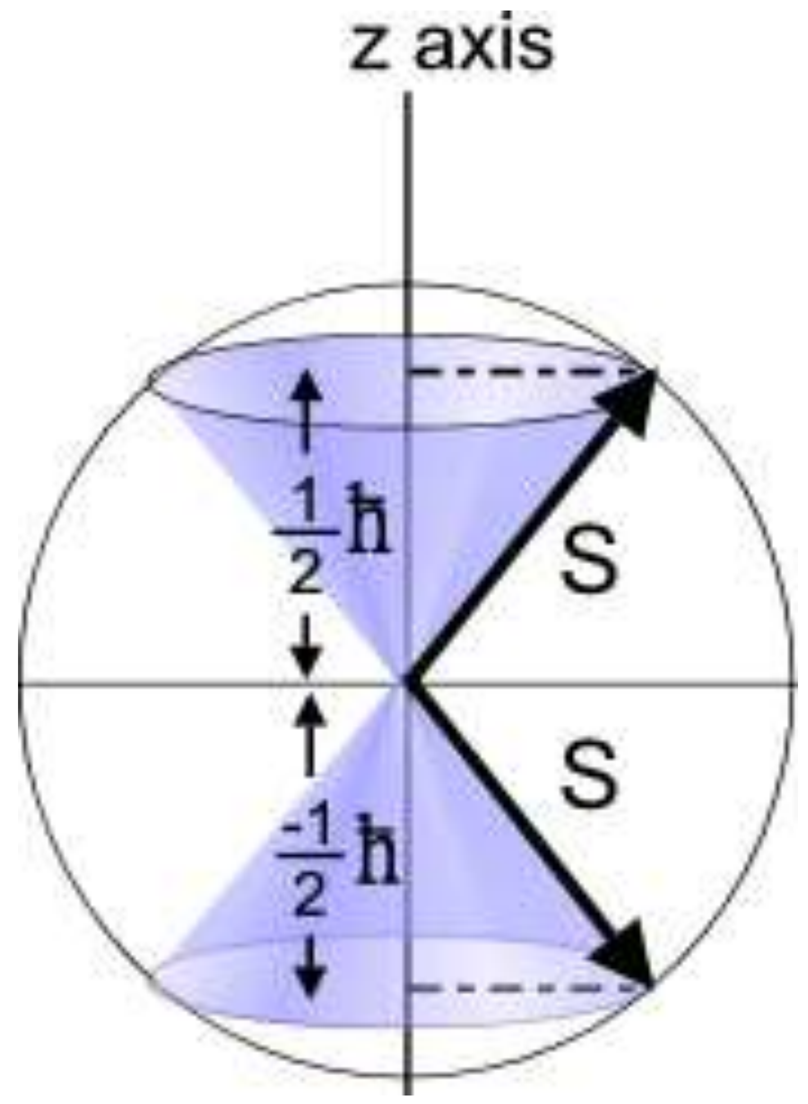


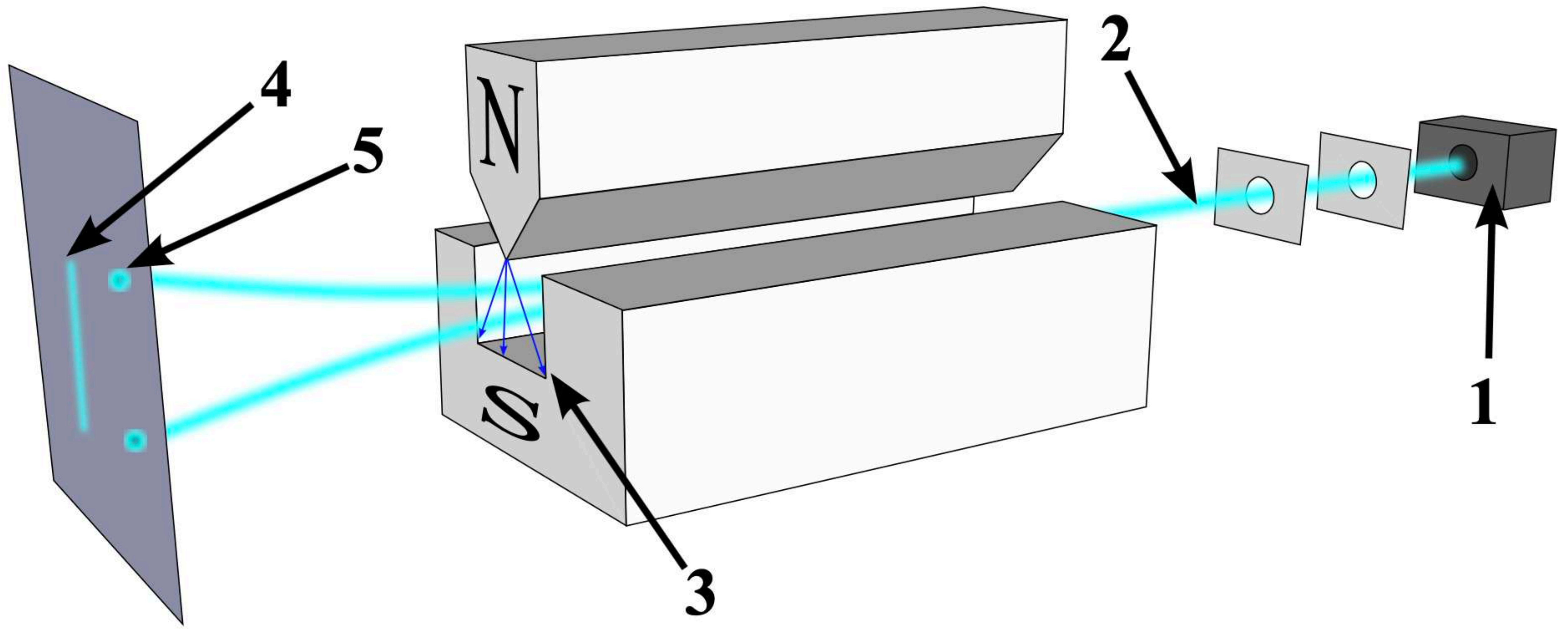
Figure: Nature Phys. 15, 1113–1117 (2019)

SPIN POLARIZATION

THE ROLE OF PARTICLES' SPIN IN QGP?



Stern-Gerlach Experiment with Ag atoms (1922)



ANGULAR MOMENTUM AND MAGNETIC FIELD IN HIC

- Large orbital angular momentum (OAM)

Becattini, Piccinini, Rizzo, PRC 77 (2008) 024906

$$|\boldsymbol{L}_{\text{init}}| = P_{\text{nucleus}} \cdot b \approx A \cdot \frac{\sqrt{s_{\text{NN}}}}{2} \cdot b \approx 10^5 \hbar \ \ (\text{RHIC})$$

orbital

$\boldsymbol{L}_{\text{init}}$

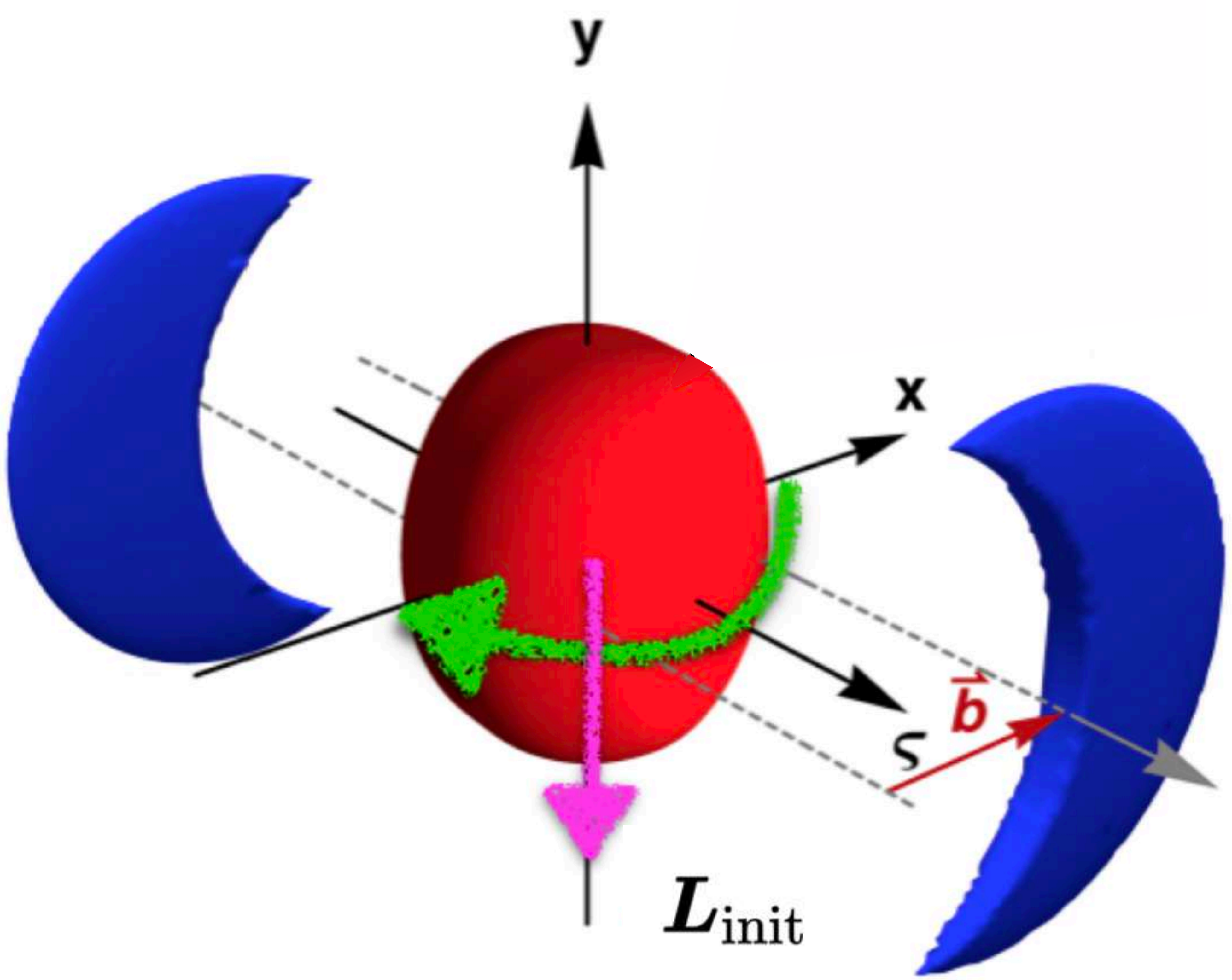


fig: R. R.

ANGULAR MOMENTUM AND MAGNETIC FIELD IN HIC

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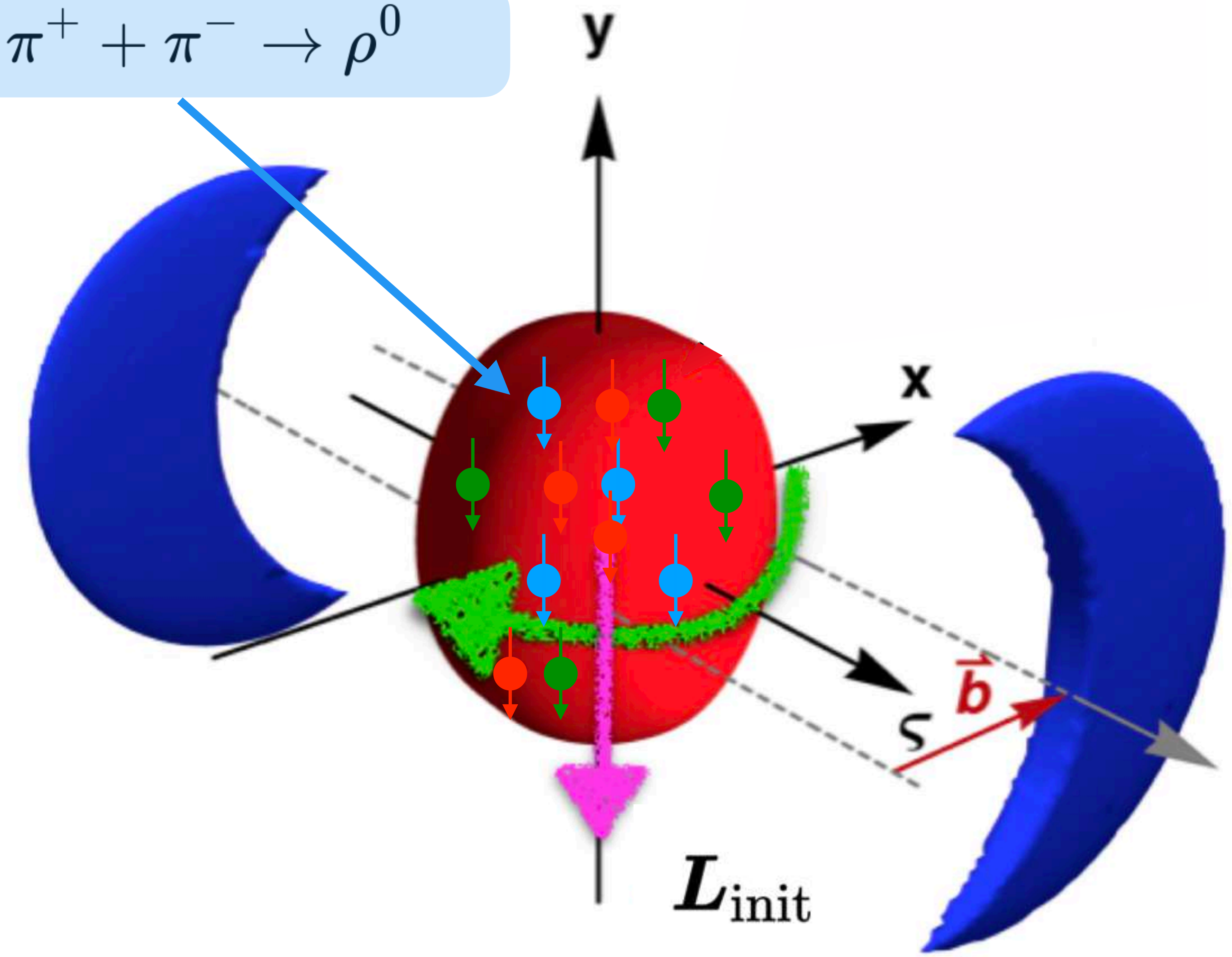
- OAM can be transferred to the spin of QGP

orbital orbital + spin

$$L_{\rm init} \rightarrow L_{\rm final} + S_{\rm final}$$

e. g. $\pi^+ + \pi^- \rightarrow \rho^0$



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  Gao, et al. PRC 77:044902 (2008)
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orbital   orbital + spin

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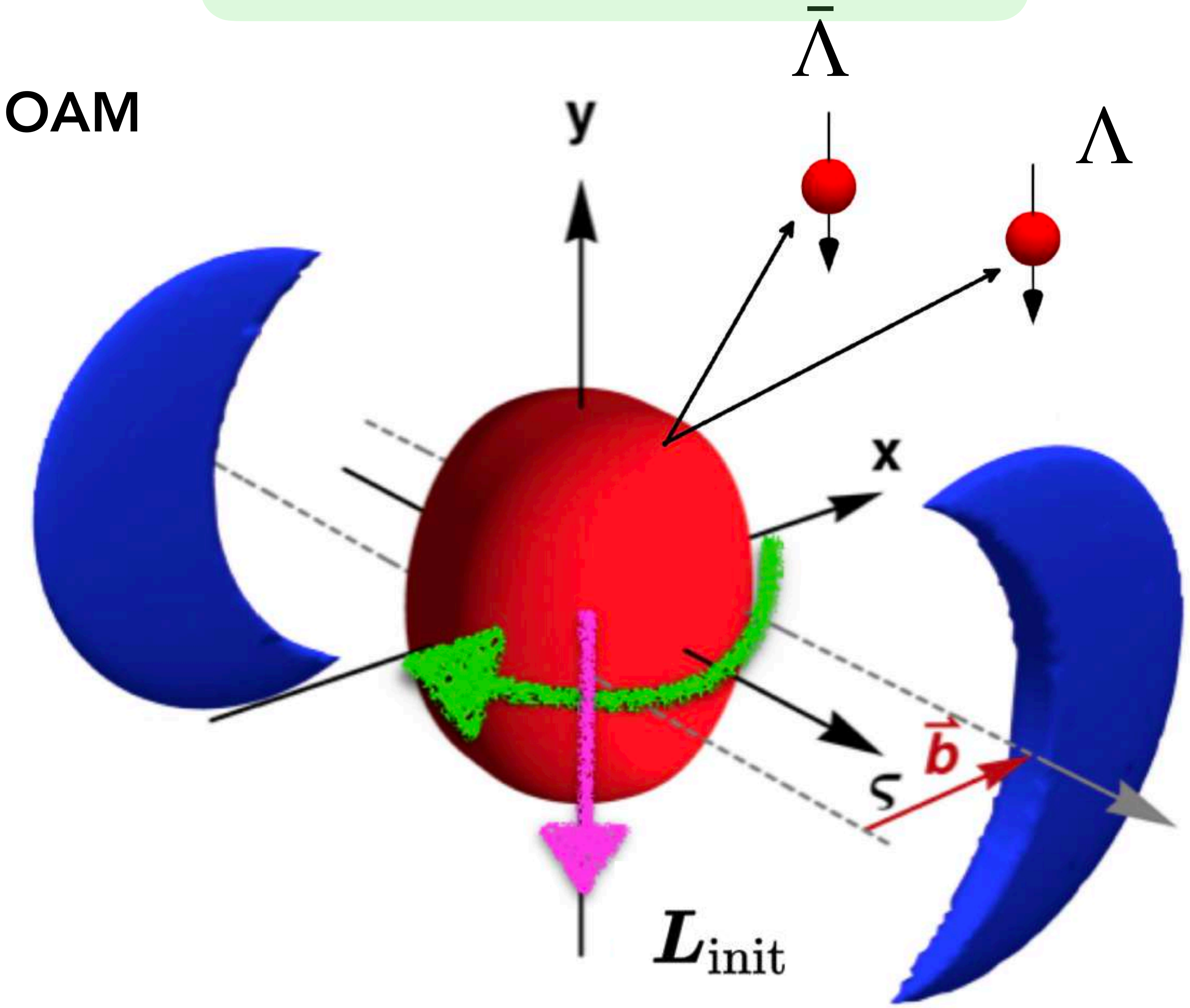

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  Rafelski, Muller, Phys. Rev. Lett. 36, 517 (1976)
  *Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008).*
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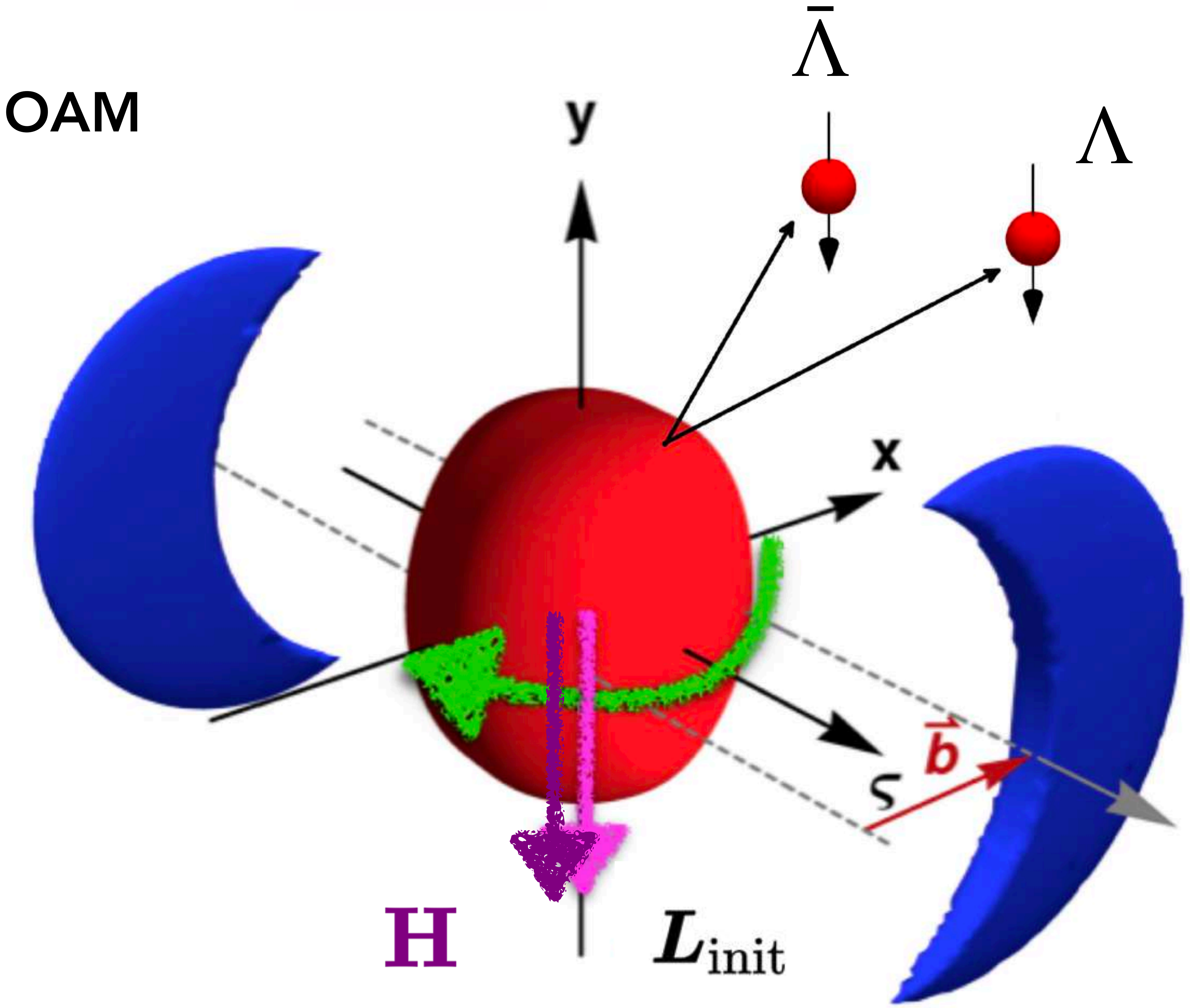

$$eB \sim m_\pi^2 \sim 10^{18}\ \text{G}$$

fig: R. R.

ANGULAR MOMENTUM AND MAGNETIC FIELD IN HIC

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- Alignment of magnetic moments is possible,
  which will split the signal for particles and antiparticles

$$\mu(\bar{\Lambda}^0) = -\mu(\Lambda^0) = +0.613 \pm 0.004 \mu_N$$

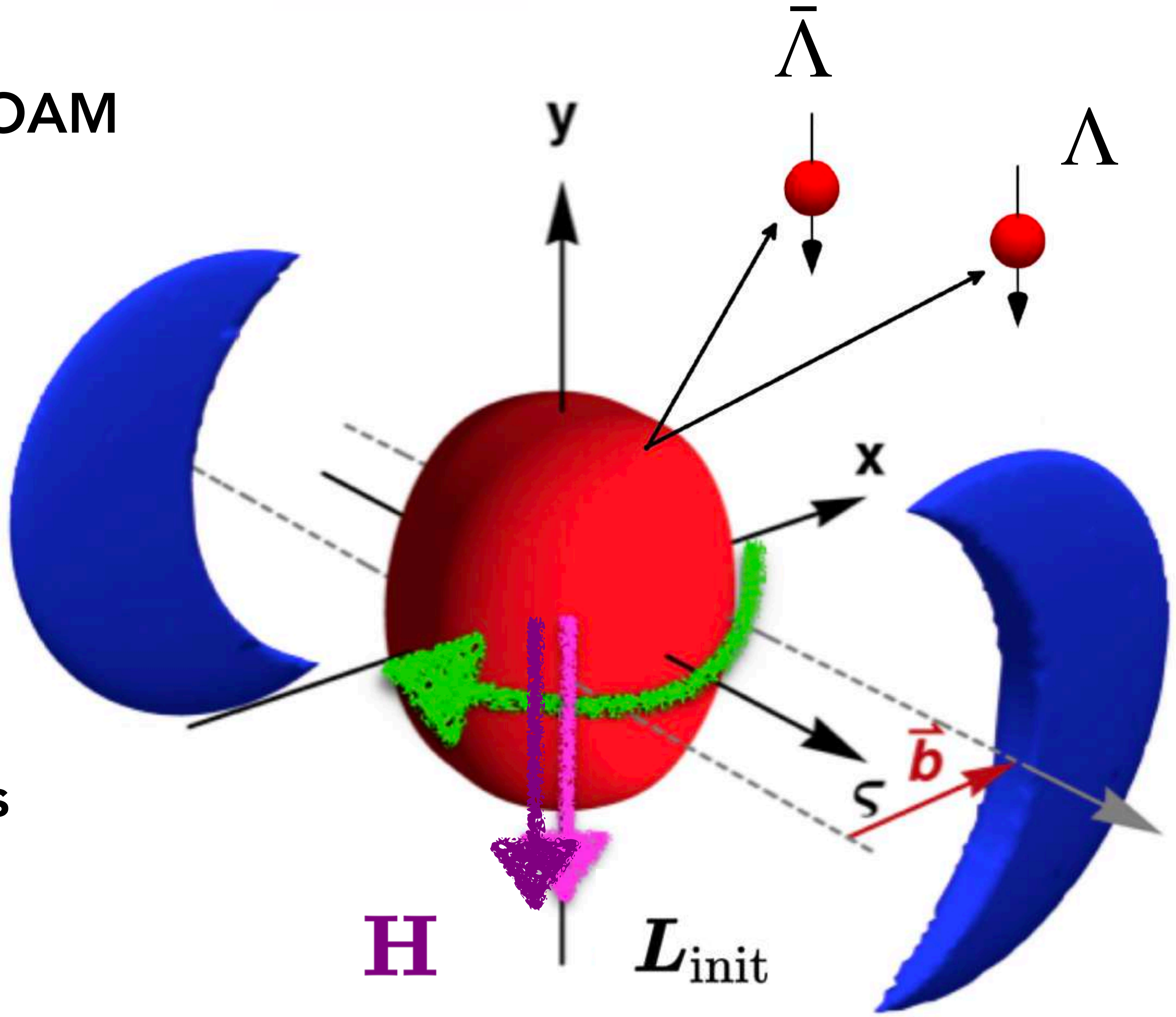


fig: R. R.

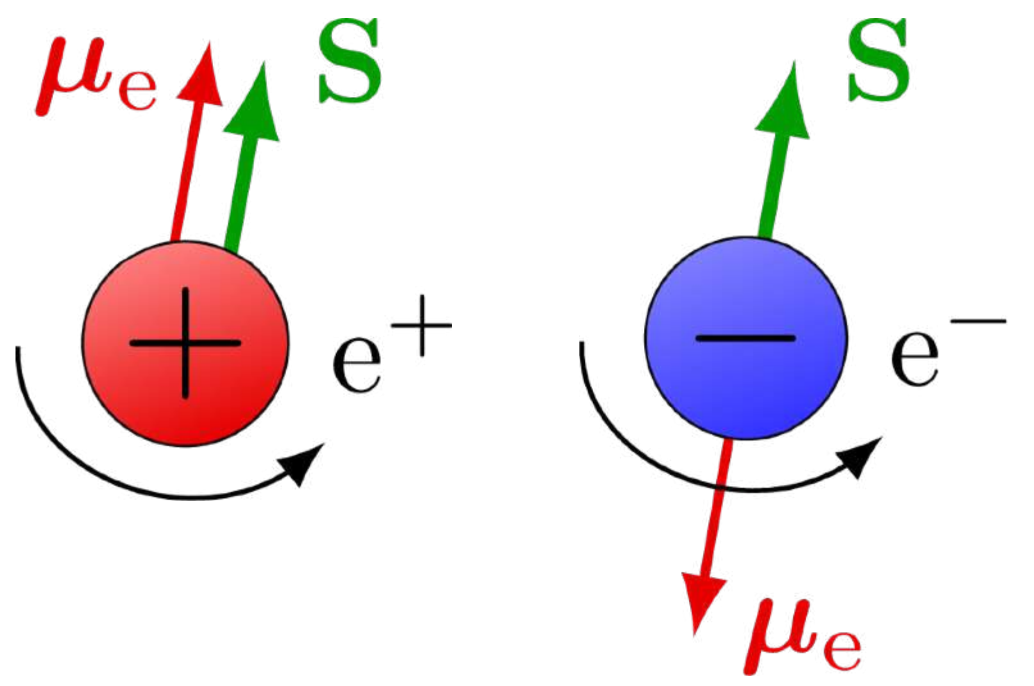
SPIN ALIGNMENT

Rotating frame aligns spins with vorticity

$$S_n = \pm\frac{1}{2}$$

Magnetic field aligns magnetic moments

$$H_{\mathrm{rot}} = -\boldsymbol{\omega}\cdot\boldsymbol{S}$$



$$H_{\mathrm{B}} = -\boldsymbol{B}\cdot\boldsymbol{\mu}$$

$$\boldsymbol{\mu} = \frac{g_s q}{2m}\boldsymbol{S}$$

$$N_\pm \propto e^{-E_\pm/T} \qquad E_\pm = E_0 \mp \frac{\omega}{2} \mp \mu B$$

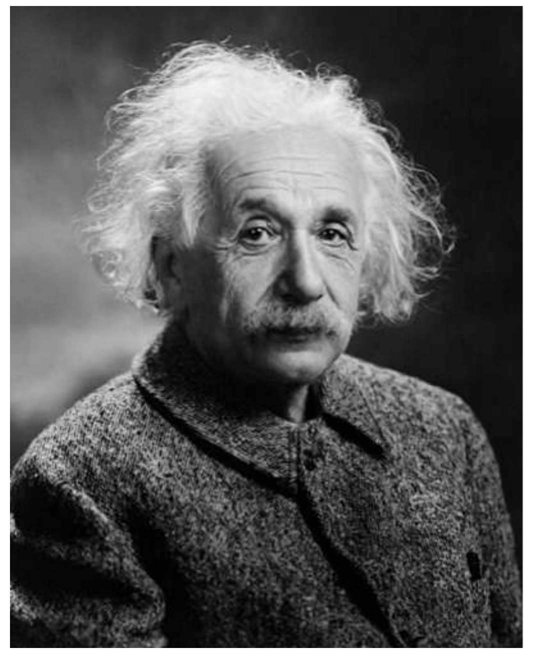
$$\mathcal{P} \equiv \frac{N_+ - N_-}{N_+ + N_-}$$

$$\mathcal{P}_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T} \quad \text{and} \quad \mathcal{P}_{\bar{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$$

ANGULAR MOMENTUM TRANSFER IN SOLID STATE PHYSICS

Einstein-de-Haas effect: magnetization → rotation

Einstein, de Haas, Ned. Akad. Wet. Proc. Ser. B Phys. Sci. 18:696 (1915)



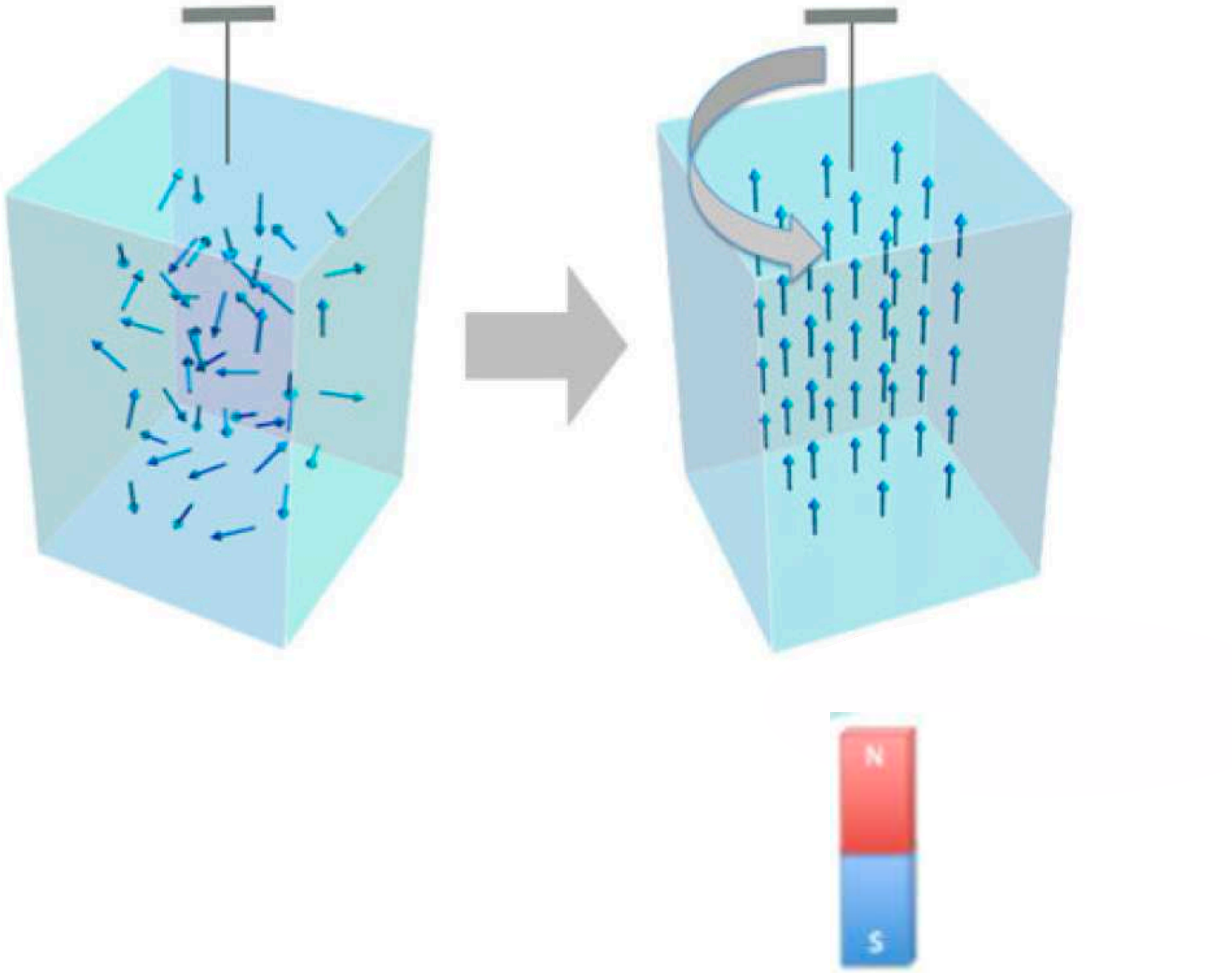


Figure: Matsuo, Ieda, Maekawa Front. Phys. 3:54 (2015)

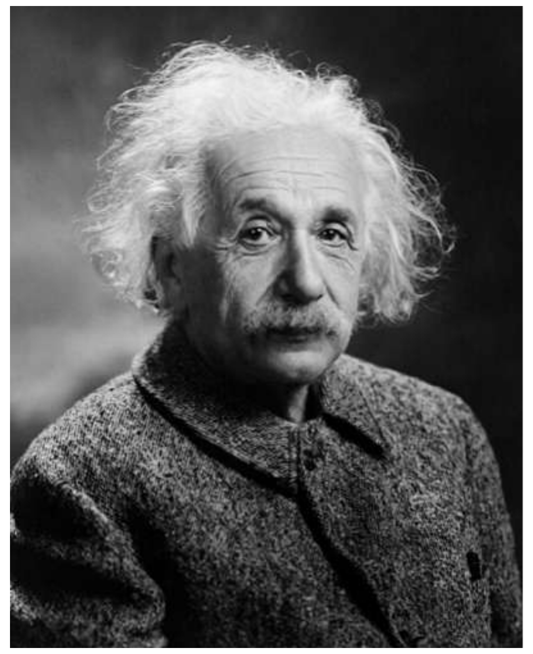
ANGULAR MOMENTUM TRANSFER IN SOLID STATE PHYSICS

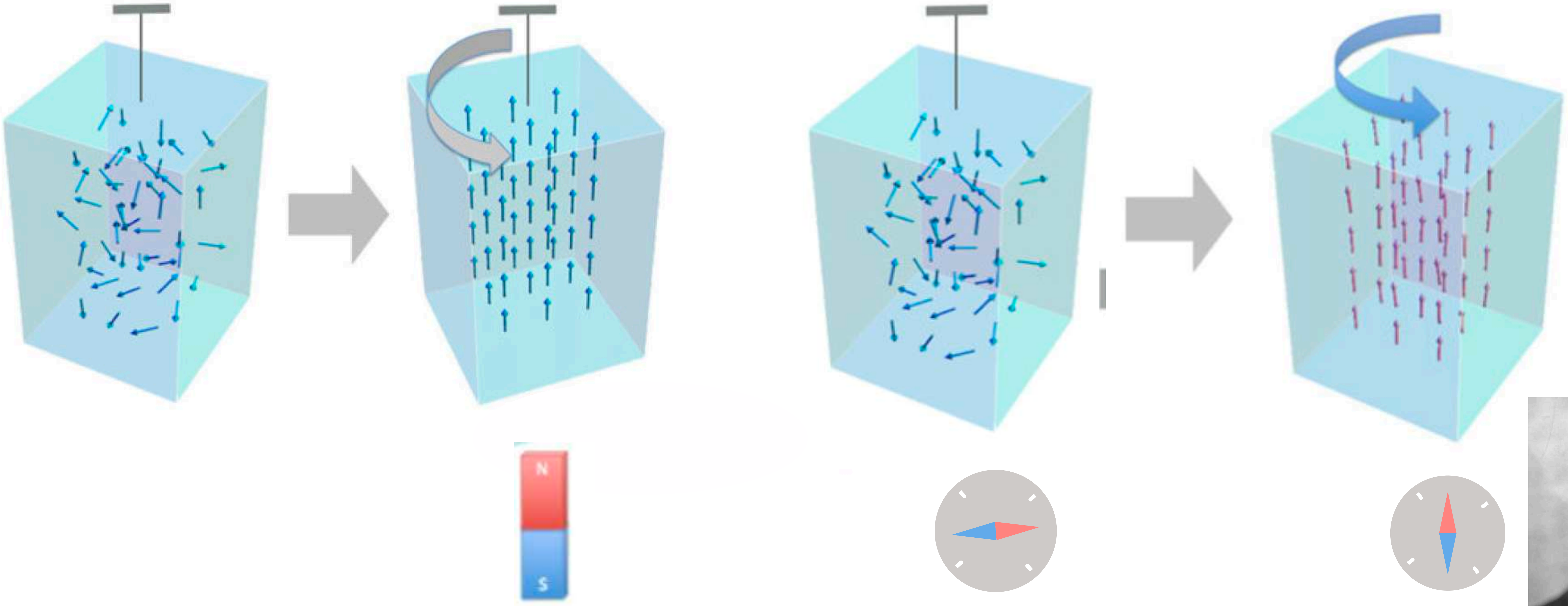
Einstein-de-Haas effect: magnetization → rotation

Einstein, de Haas, Ned. Akad. Wet. Proc. Ser. B Phys. Sci. 18:696 (1915)

Barnett effect: rotation → magnetization

Barnett, Phys. Rev. 6:239 (1915); Rev. Mod. Phys. 7, 129 (1935)





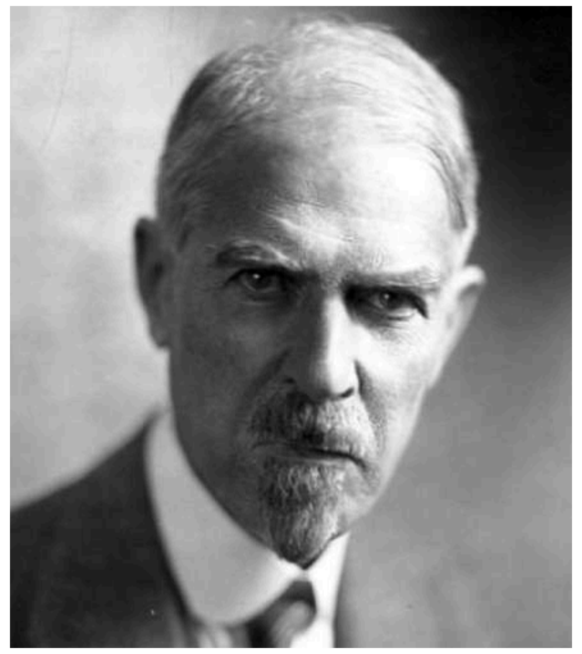
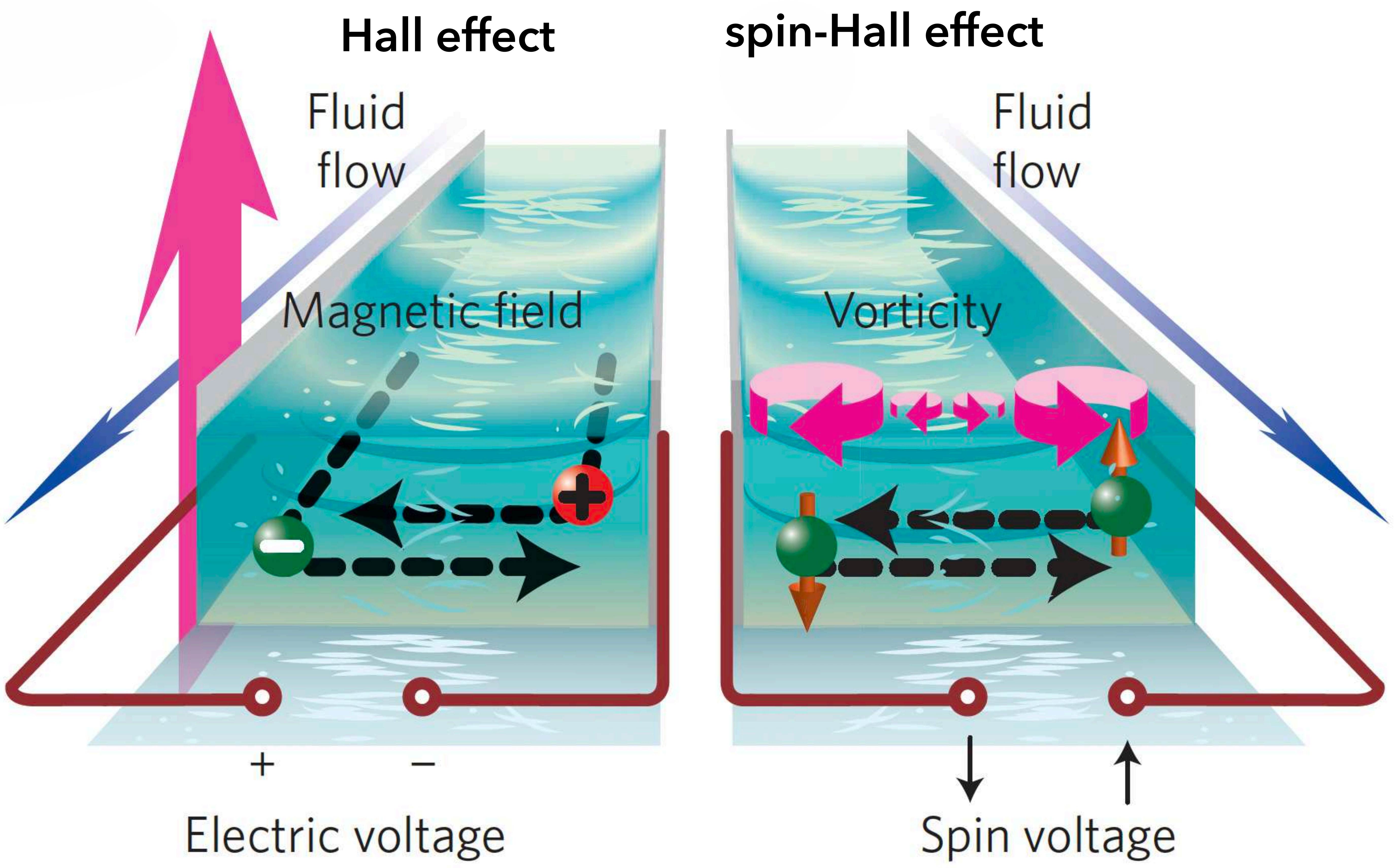


Figure: Matsuo, Ieda, Maekawa Front. Phys. 3:54 (2015)

SPIN POLARIZATION GENERATION FROM VORTICITY IN FLUID'S

Vorticity creates spin imbalance

Takahashi, Matsuo, Ono, et al. Nature Phys ***12****, 52–56 (2016)*



Valet-Fert / Takahashi-Maekawa theory

Maekawa, Valenzuela, Saitoh, Kimura (eds.), Spin Current

$$\mu_S \equiv \mu_\uparrow - \mu_\downarrow$$

$$\frac{\partial \mu_S}{\partial t} = D_s \nabla^2 \mu_S - \frac{\mu_S}{\tau_s} + C\,\boldsymbol{\omega}$$

spin diffusion

spin relaxation

spin–vorticity coupling

MEASUREMENT OF GLOBAL SPIN POLARIZATION IN HIC

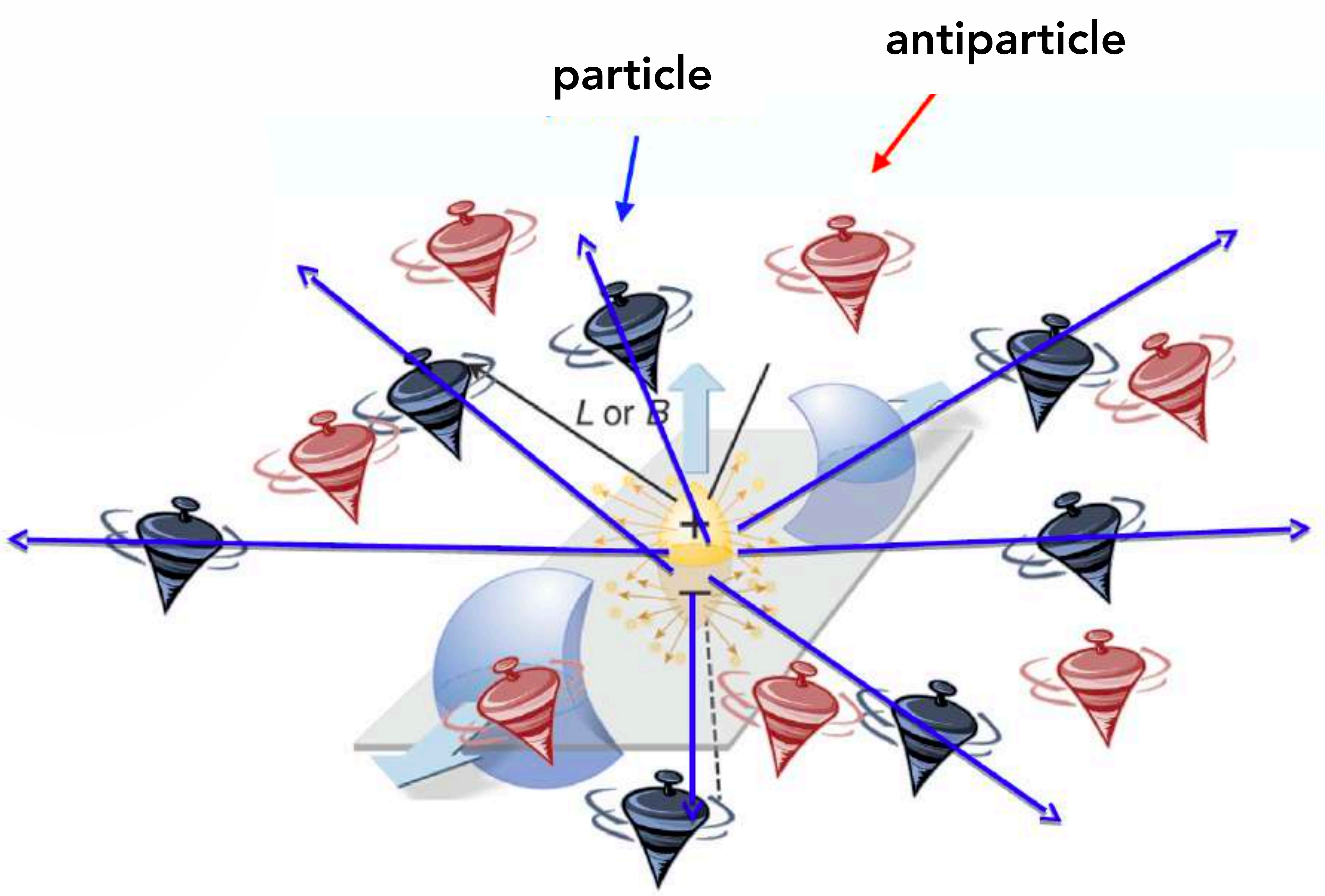
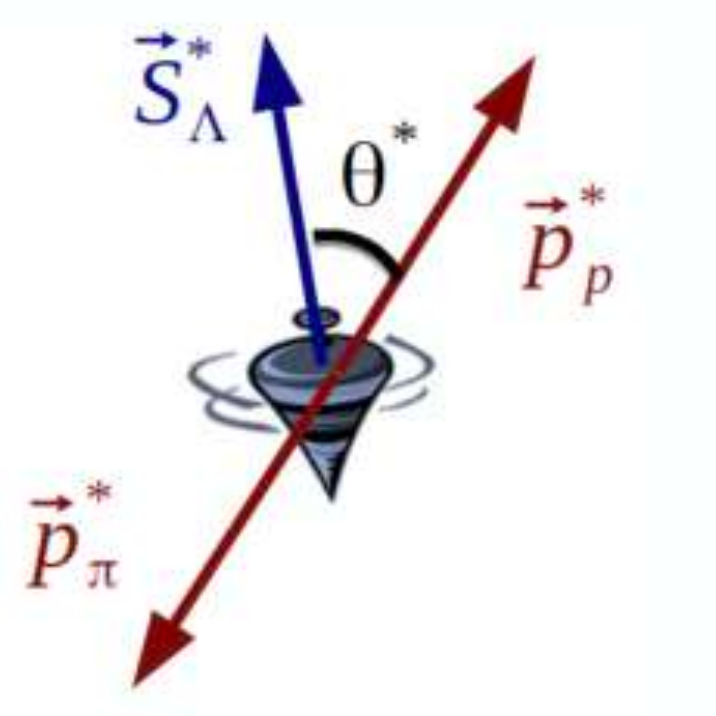


Figure: T. Niida

Self-analysing parity-violating weak decay allows to measure polarization of Λ hyperon



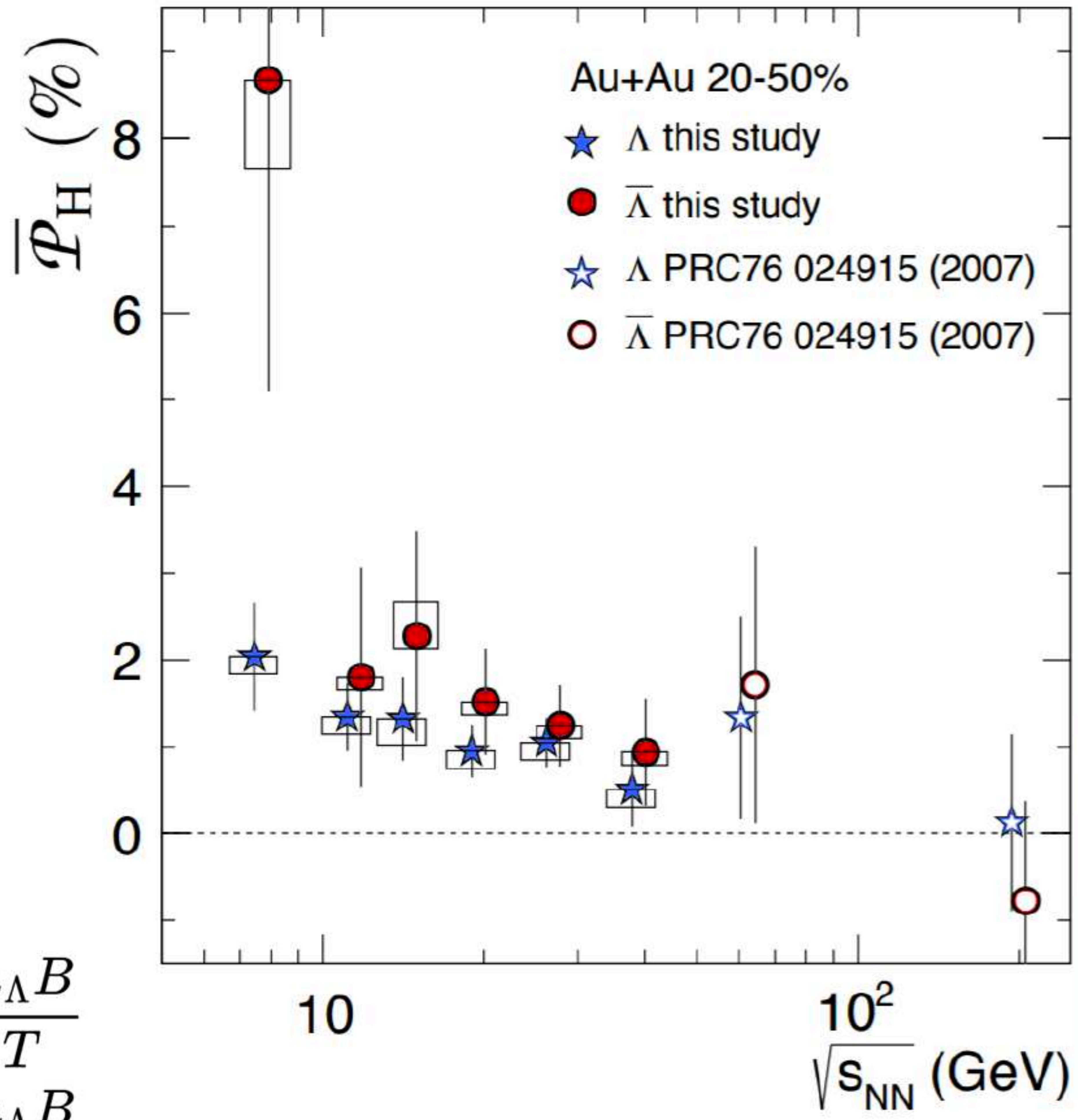
$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha_H |\vec{\mathcal{P}}_H| \cos\theta^*\right)$$

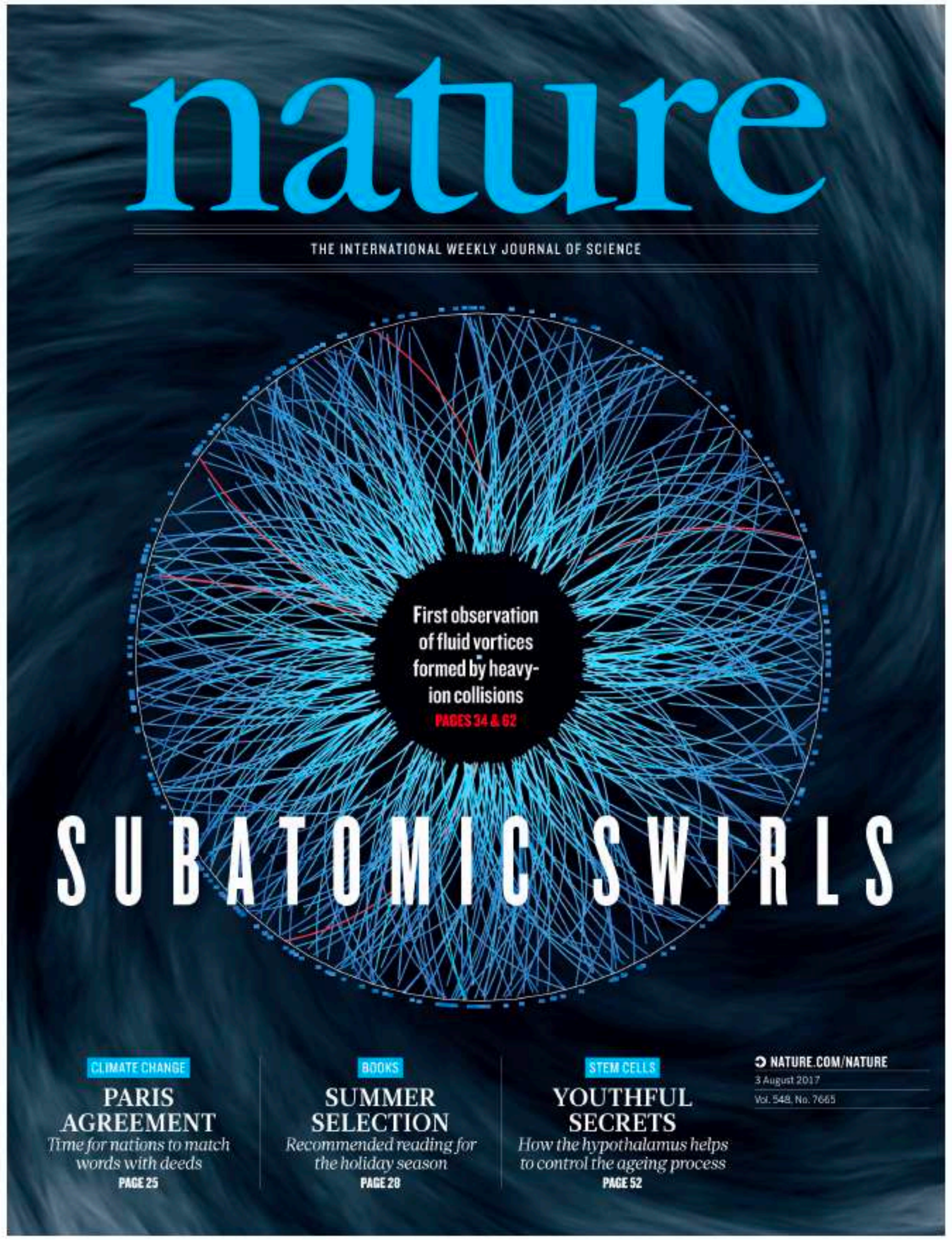
$$(\alpha_\Lambda = 0.732)$$

$$\bar{\mathcal{P}}_H \equiv \left\langle \vec{\mathcal{P}}_H \cdot \hat{J}_{\rm sys} \right\rangle = \frac{8}{\pi\alpha_H} \frac{\left\langle \cos\left(\phi^*_p - \phi_{\hat{J}_{\rm sys}}\right)\right\rangle}{R^{(1)}_{\rm EP}}$$

MEASUREMENT OF GLOBAL SPIN POLARIZATION

Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65





$$\mathcal{P}_{\Lambda} \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_{\Lambda} B}{T}$$

$$\mathcal{P}_{\bar{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_{\Lambda} B}{T}$$

$P_{\Lambda} \approx P_{\bar{\Lambda}}$ ➡ **first direct observation of spin in HIC**

THEORETICAL INTERPRETARTION: *SPIN-THERMAL* APPROACH

Using quantum statistical field theory in **thermodynamic equilibrium,** one can establish a link between **a particle's spin** and fluid's **thermal vorticity**

F. Becattini, F. Piccinini, AP 323, 2452 (2008)
F. Becattini, F. Piccinini, J. Rizzo, PRC 77, 024906 (2008)
Becattini, Chandra, Del Zanna, Grossi, AP 338:32 (2013)
Fang, Pang, Wang, Wang, PRC 94:024904 (2016)
Becattini, Karpenko, Lisa, Upsal, and Voloshin PRC 95, 054902 (2017)

Spin vector of a single massive particle in RQM is given through by **Pauli-Lubański pseudovector**

$$\hat{S}^{\mu} = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\hat{J}_{\nu\rho}\hat{P}_{\sigma} \qquad \hat{S}^{\mu}\hat{P}_{\mu} = 0$$

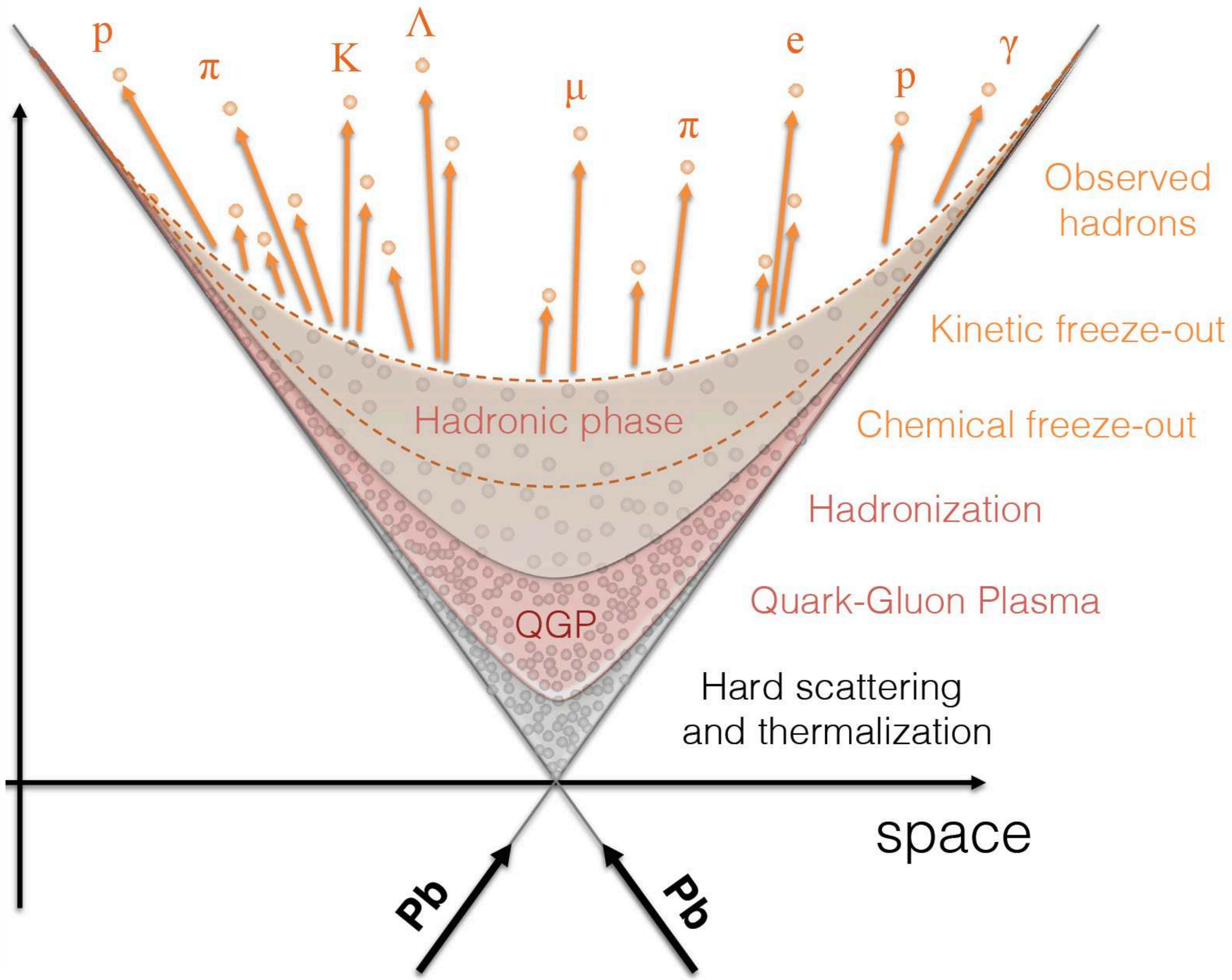


figure: D.D. Chinellato

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The **mean spin vector** of emitted particles is

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m_{\Lambda}}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int d\Sigma \cdot p\, n_F(1-n_F)\varpi_{\nu\rho}}{\int d\Sigma \cdot p\, n_F}$$

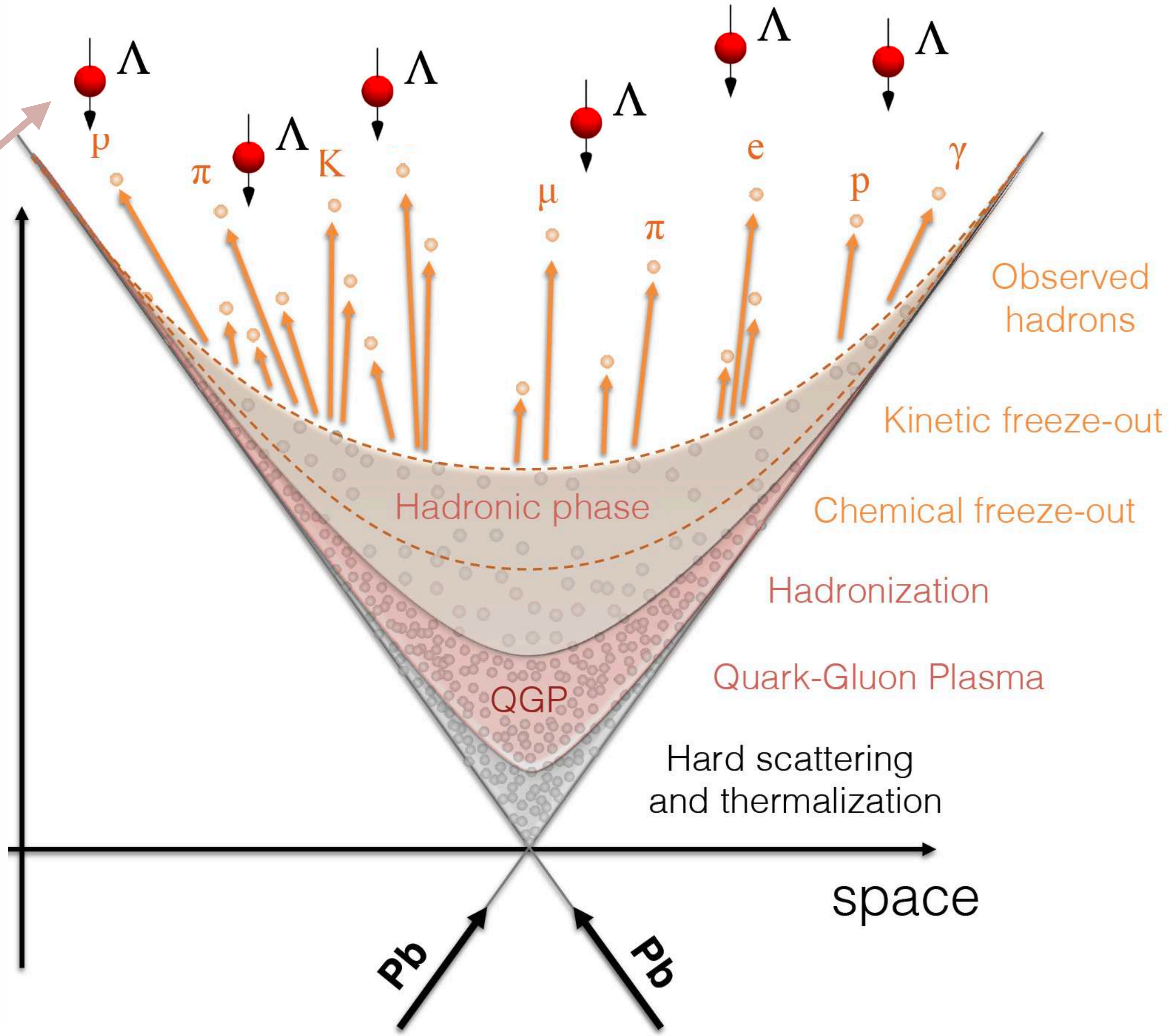

figure: D.D. Chinellato

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Fluid's thermal vorticity is

$$\varpi_{\mu\nu} = \partial_{[\nu}\beta_{\mu]} = -\frac{1}{2}(\partial_{\mu}\beta_{\nu} - \partial_{\nu}\beta_{\mu})$$

$$\beta^{\mu} \equiv u^{\mu}/T$$

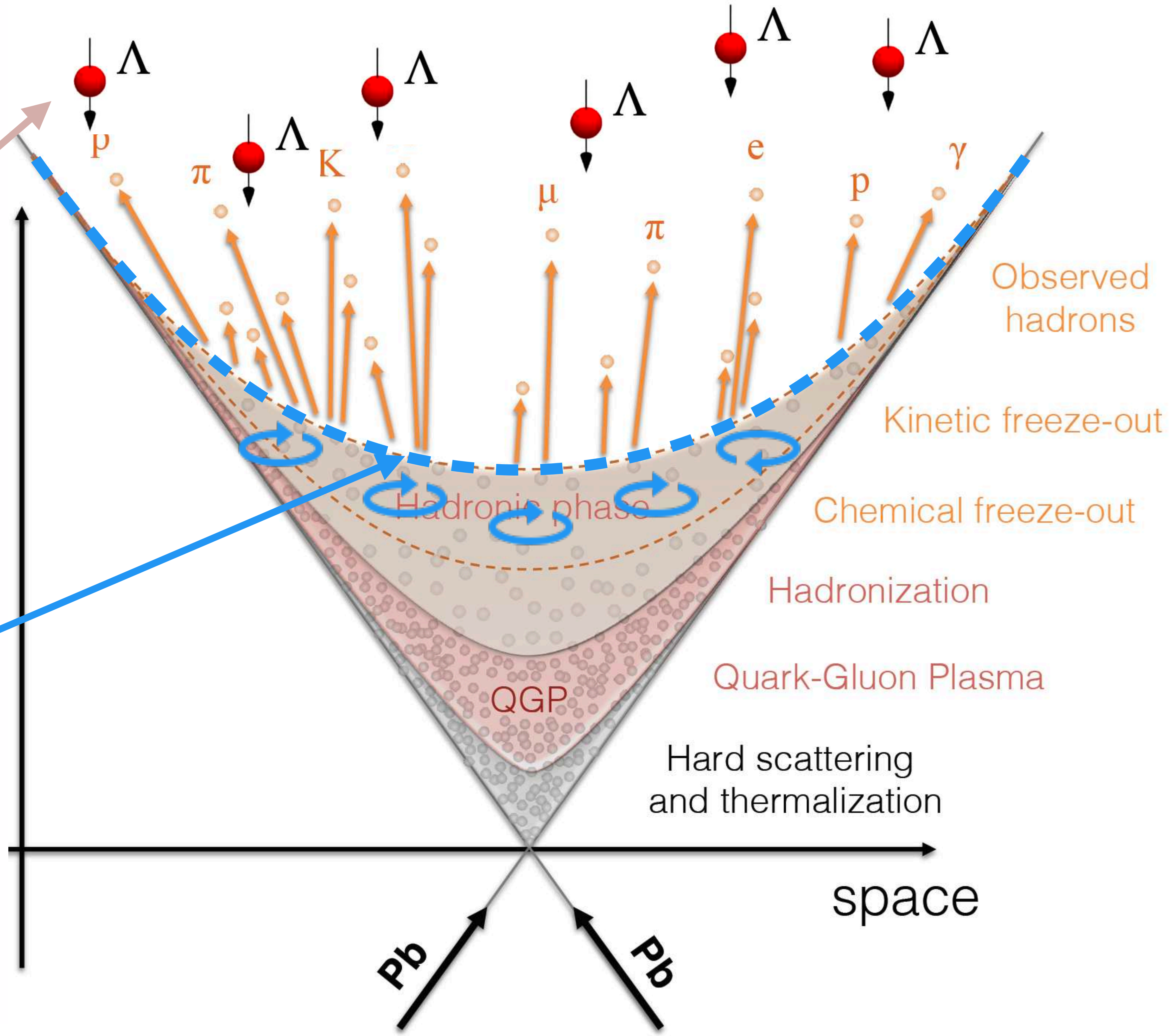


figure: D.D. Chinellato

GLOBAL POLARIZATION: DATA VS *SPIN-THERMAL* APPROACH

Thermal vorticity determines global polarization!

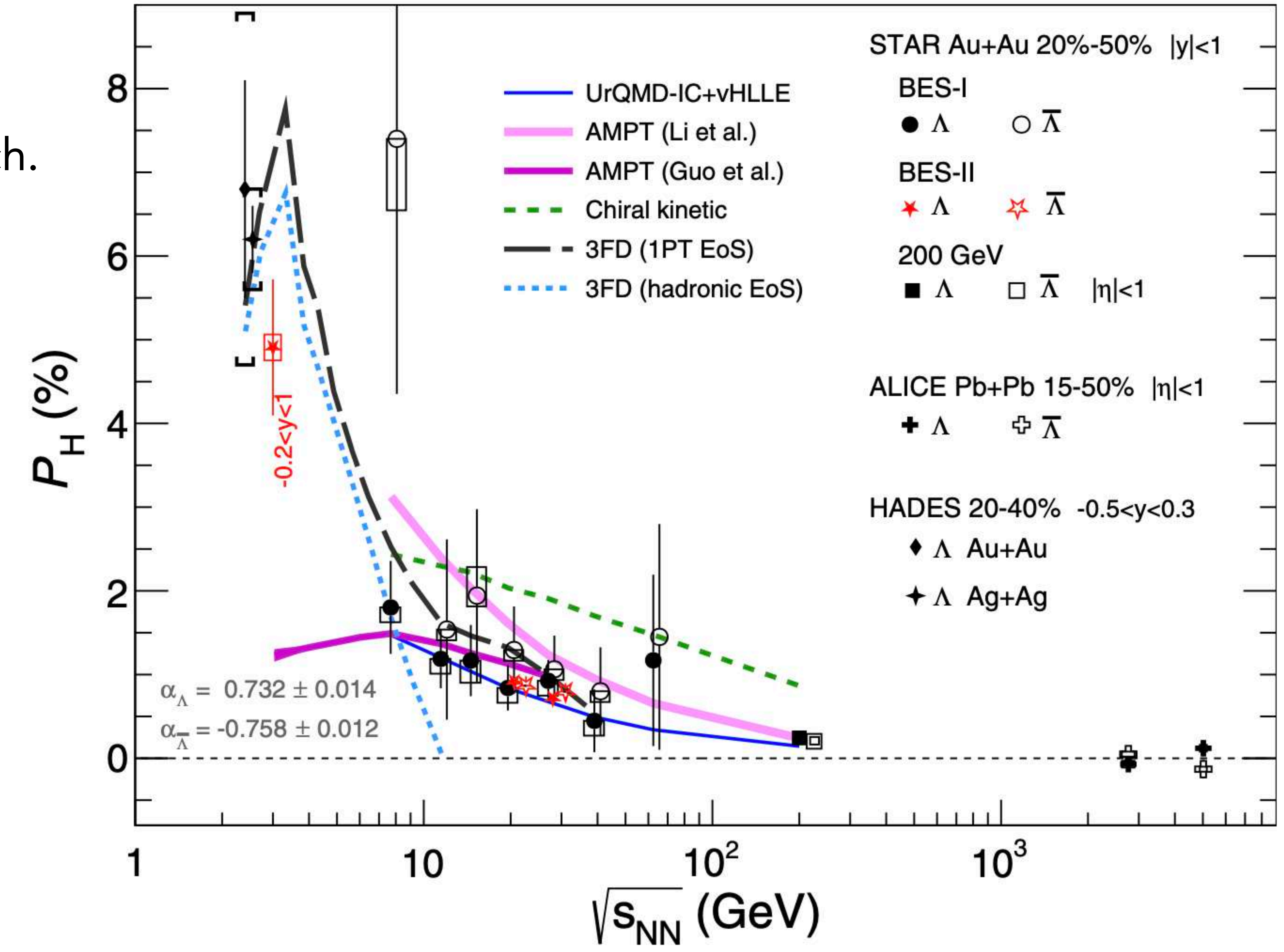
Becattini et al, IJMP EVol. 33, No. 6 (2024) 2430006

Global polarization data agree well with **transport, hydrodynamics, and many other models** that employ the spin-thermal approach.

STAR:
L. Adamczyk et al., Nature 548 (2017) 62,
M. S. Abdallah et al., Phys. Rev. C 104 (2021) L061901
J. Adam et al., Phys. Rev. C 98 (2018) 014910,
M. I. Abdulhamid et al., Phys. Rev. C 108 (2023) 014910

ALICE:
S. Acharya et al., Phys. Rev. C 101 (2020) 044611,

HADES:
R. Abou Yassine et al., Phys. Lett. B 835 (2022) 137506



MOST VORTICAL FLUID PRODUCED IN THE LABORATORY !?

$$\mathcal{P}_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T} \quad \text{and} \quad \mathcal{P}_{\bar{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$$

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ \mathrm{s}^{-1}$$ ← large vorticity

short time → $\Delta t = 1\ \mathrm{fm}/c = 3 \times 10^{-24}\ \mathrm{s}, \quad \Delta\phi = \Delta t\, \omega = 27 \times 10^{-24} \times 10^{21} = 2.7 \times 10^{-2}$ ← small rotation angle

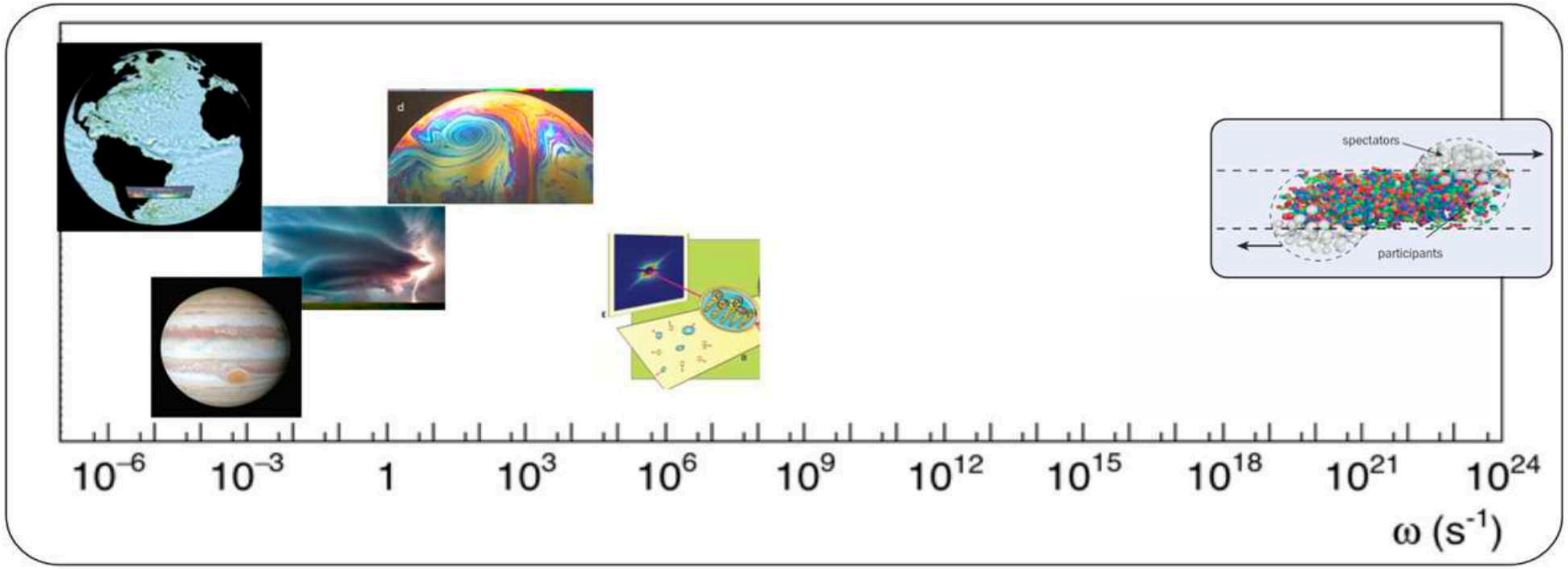
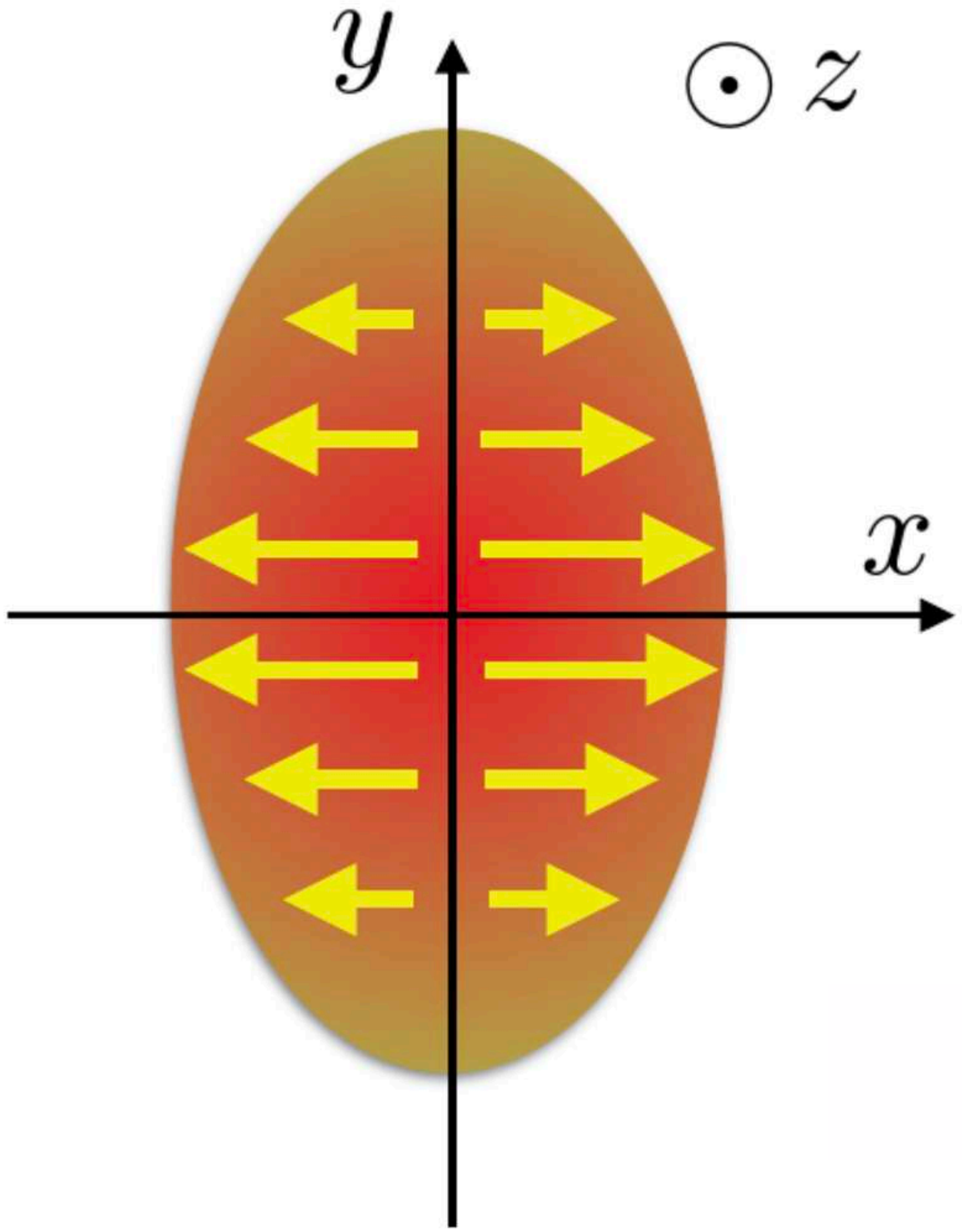
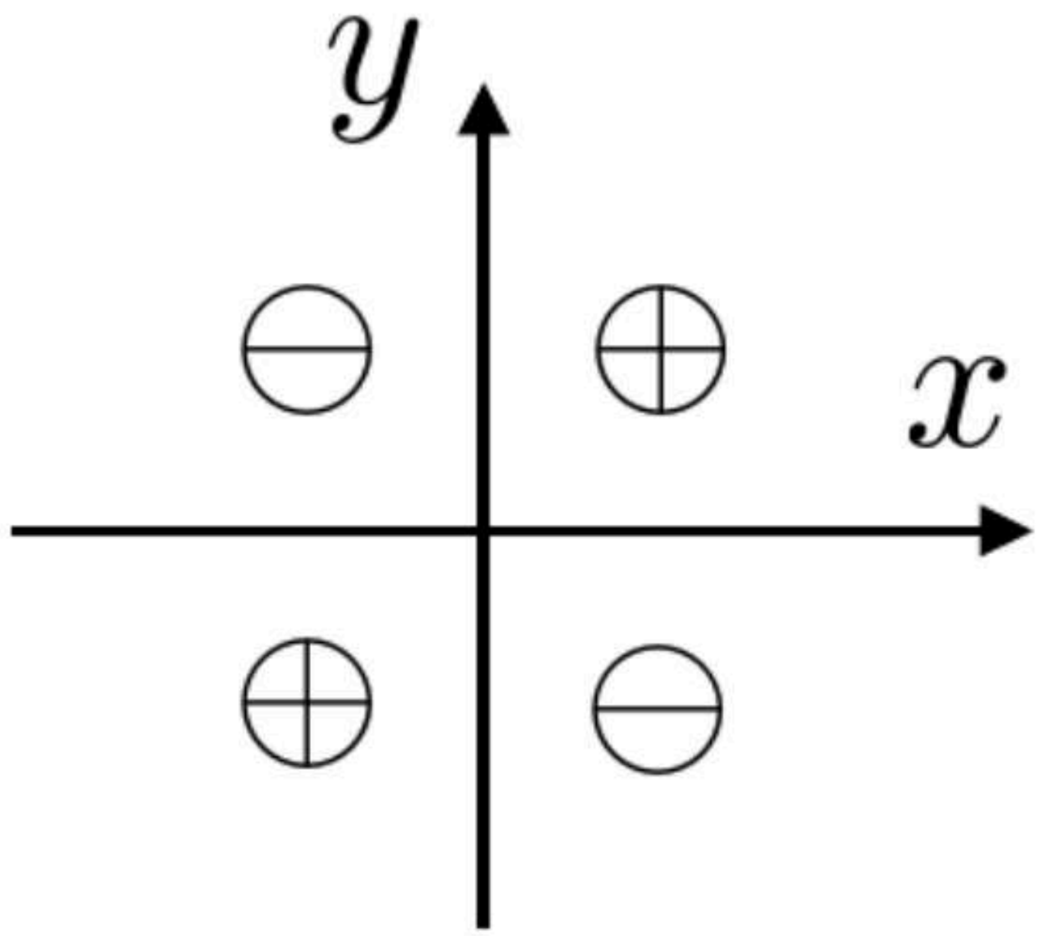


Figure: M. Lisa

LONGITUDINAL POLARIZATION

Local flow structures in the plane transverse to the beam lead to longitudinal (beam-direction) polarization





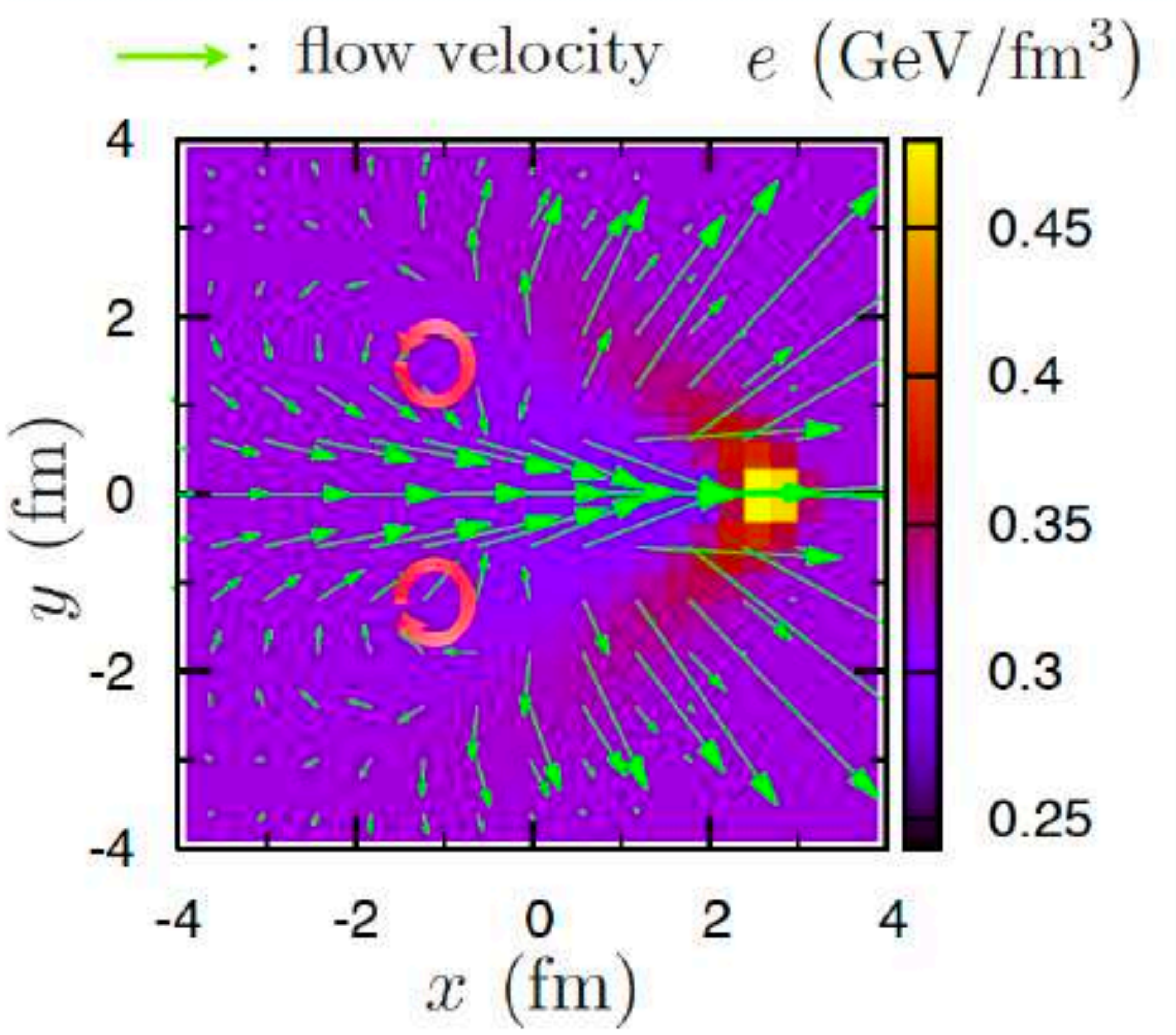
non-relativistic + uniform-T limit:

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{v}$$

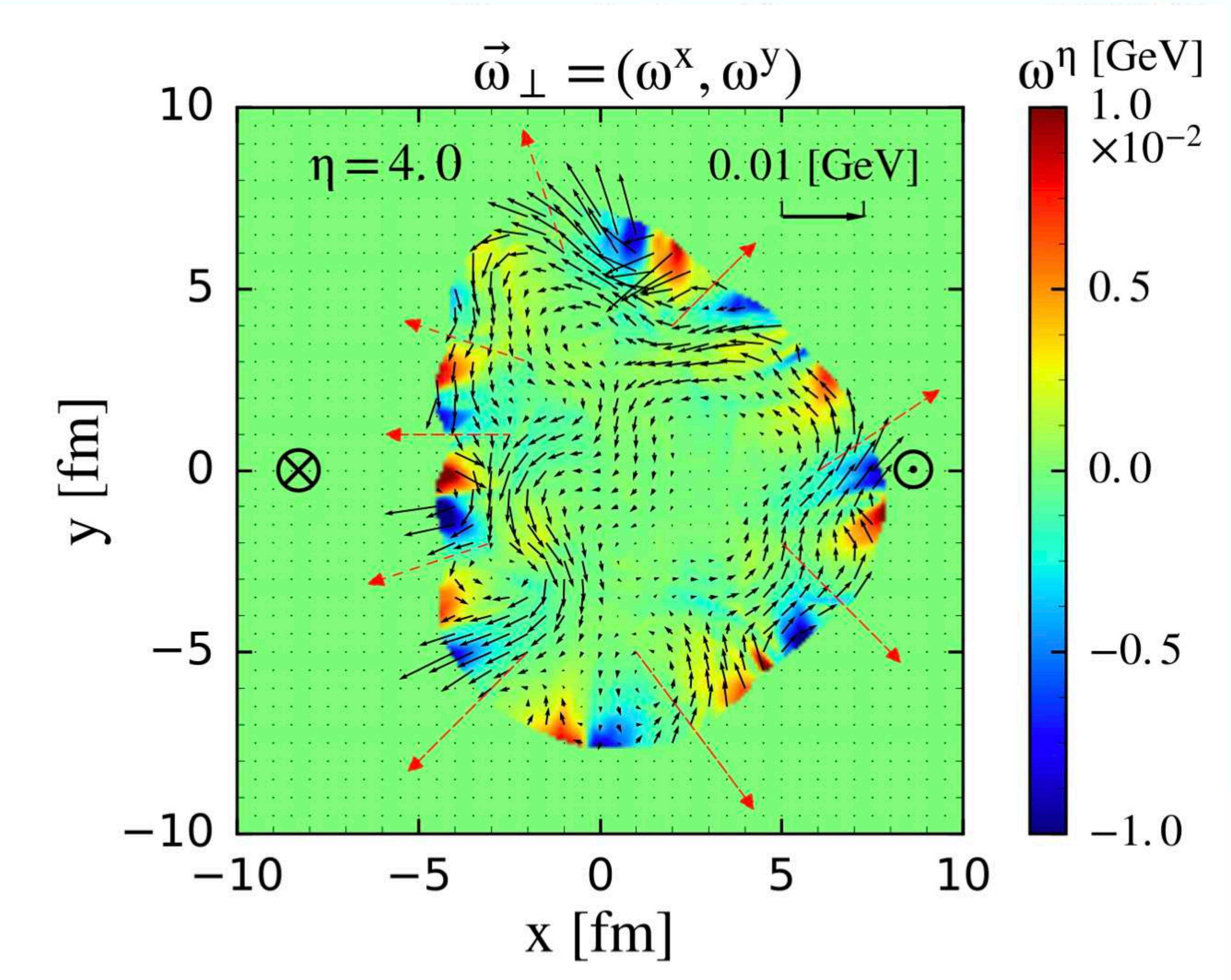
LONGITUDINAL POLARIZATION

Local flow structures in the plane transverse to the beam lead to longitudinal (beam-direction) polarization

Tachibana, Hirano, Nucl.Phys.A 904-905 (2013) 1023c-1026c



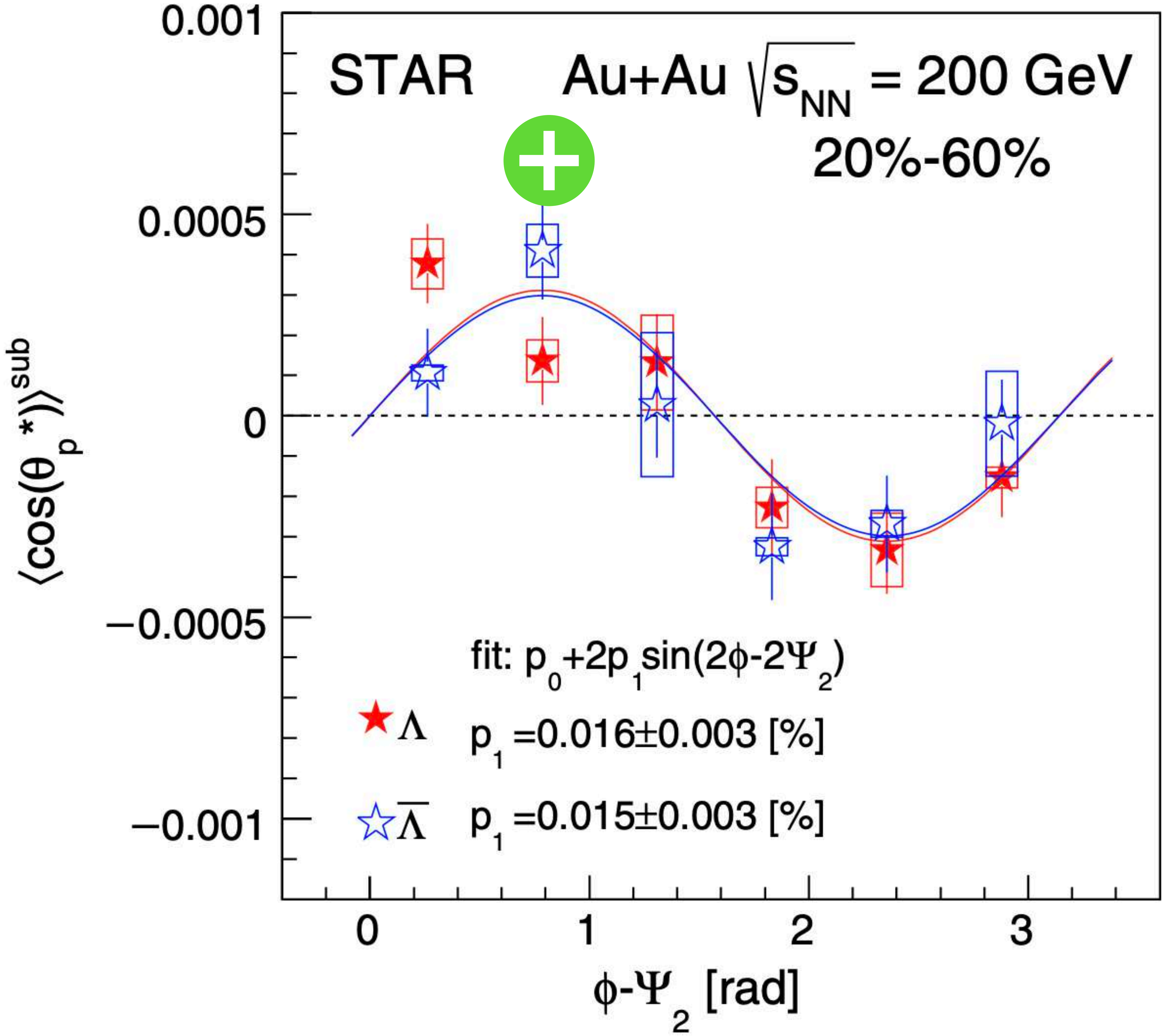
Pang, Petersen, Wang, Wang, Phys.Rev.Lett. 117 (2016) 19, 192301



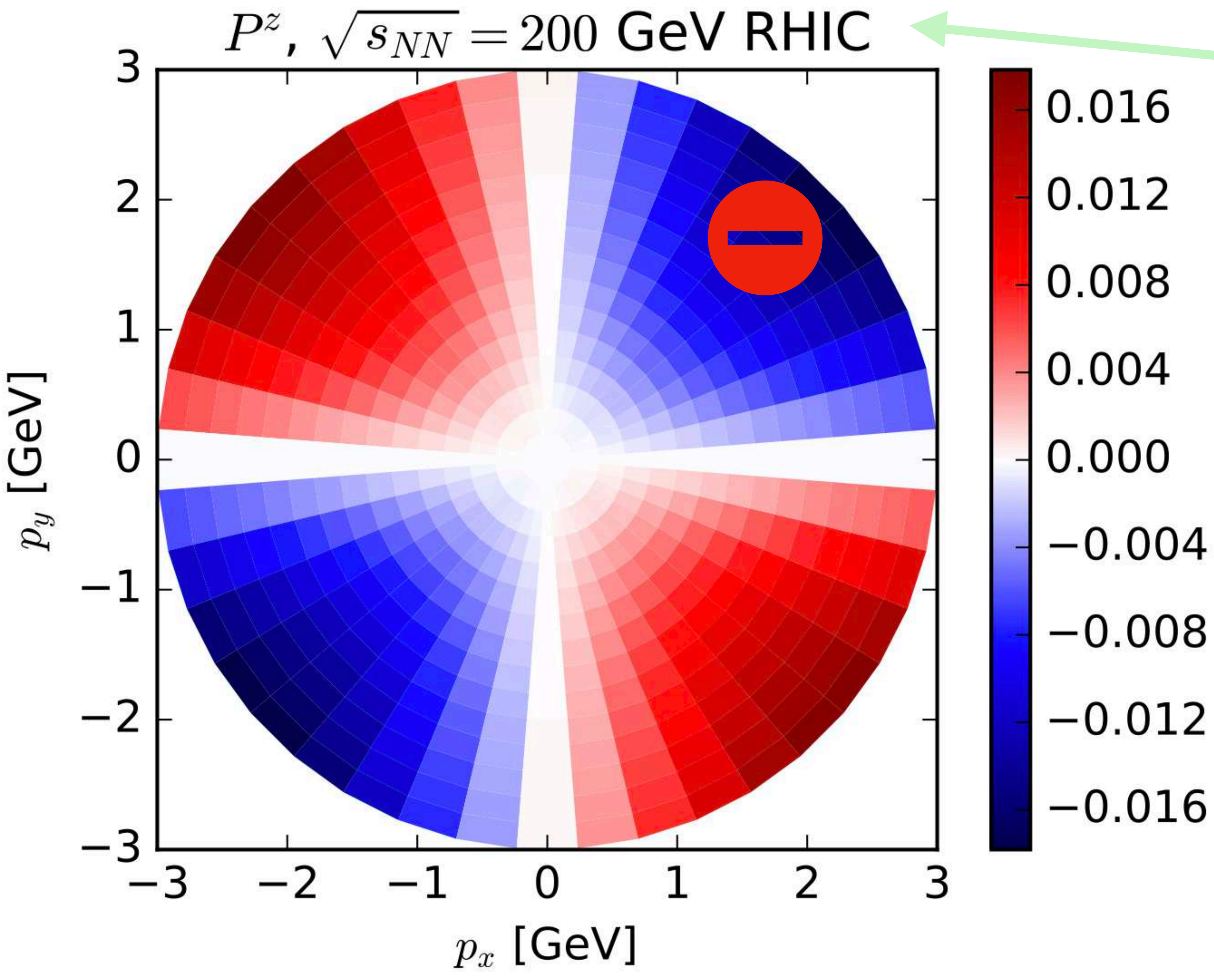
LONGITUDINAL POLARIZATION: DATA VS *SPIN-THERMAL* APPROACH

Spin-thermal approach fails in reproducing local polarization observables

Adam et al (STAR Collaboration) Phys. Rev. Lett. 123, 132301

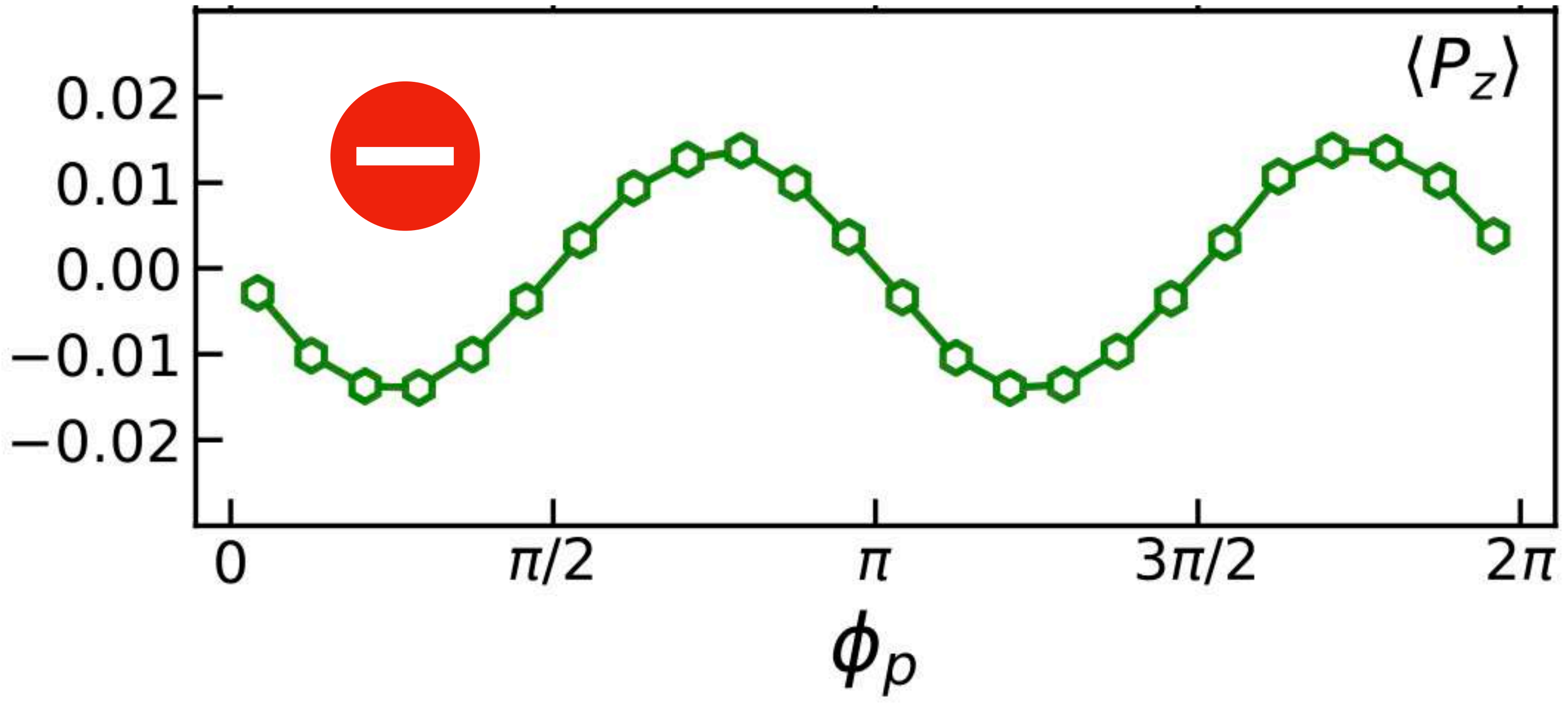


$P^z = S^z_\varpi$



Becattini, Karpenko, Phys.Rev.Lett. 120 (2018) 1, 012302

Hydrodynamics



Transport

Xia, Li, Tang, Wang, Phys.Rev.C 98 (2018) 024905

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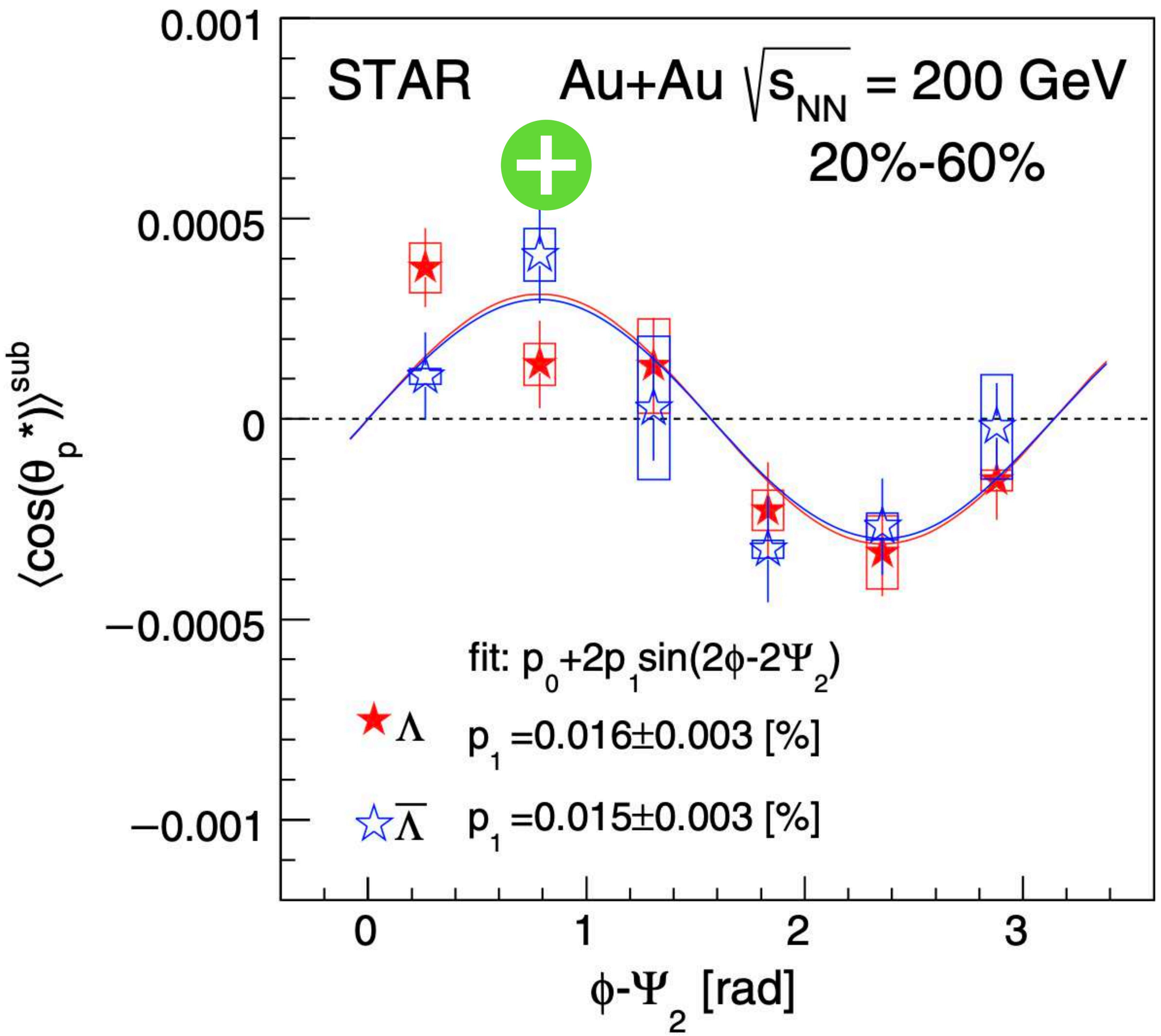
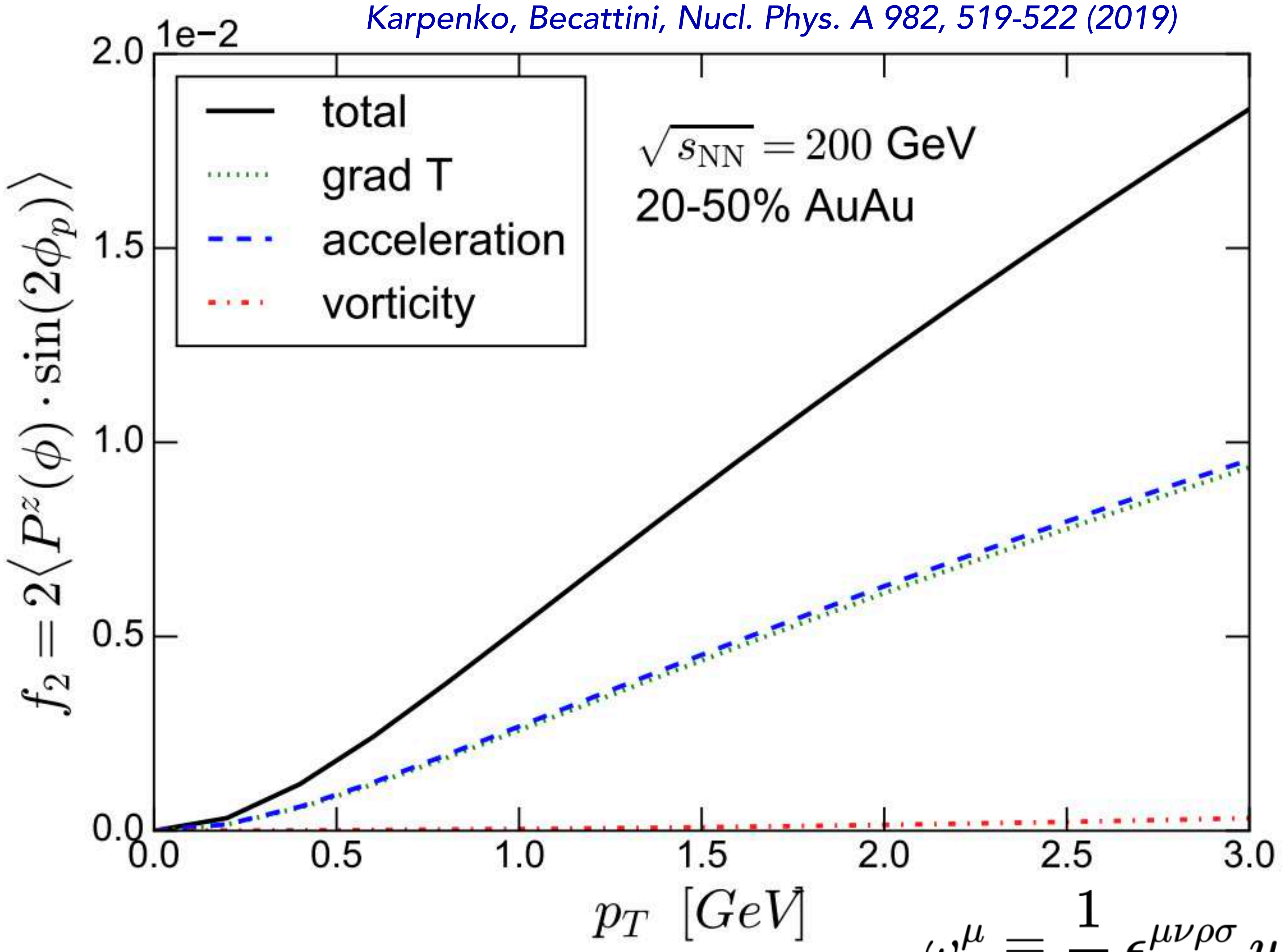


longitudinal polarization is dominated by the *mixed* components

$$S^\mu \propto= \epsilon^{\mu\rho\sigma\tau}\,(\partial_\rho\beta_\sigma)\,p_\tau$$

$$= \underbrace{\epsilon^{\mu\rho\sigma\tau}\,p_\tau\,\partial_\rho\left(\frac{1}{T}\right)u_\sigma}_{\mathrm{grad}T} + \underbrace{\frac{2}{T^2}\left[\omega^\mu(u\cdot p) - u^\mu(\omega\cdot p)\right]}_{\text{``NR vorticity''}} + \underbrace{\epsilon^{\mu\rho\sigma\tau}\,p_\tau\,A_\sigma\,u_\rho}_{\text{acceleration}}$$

Karpenko, Becattini, Nucl. Phys. A 982, 519-522 (2019)



$$\omega^\mu \equiv \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}\,u_\nu\,\partial_\rho u_\sigma.$$

SPIN HYDRODYNAMICS

NEED FOR DYNAMICAL MODELS: SPIN HYDRODYNAMICS

Spin-thermal approach does not describe the local polarization data properly

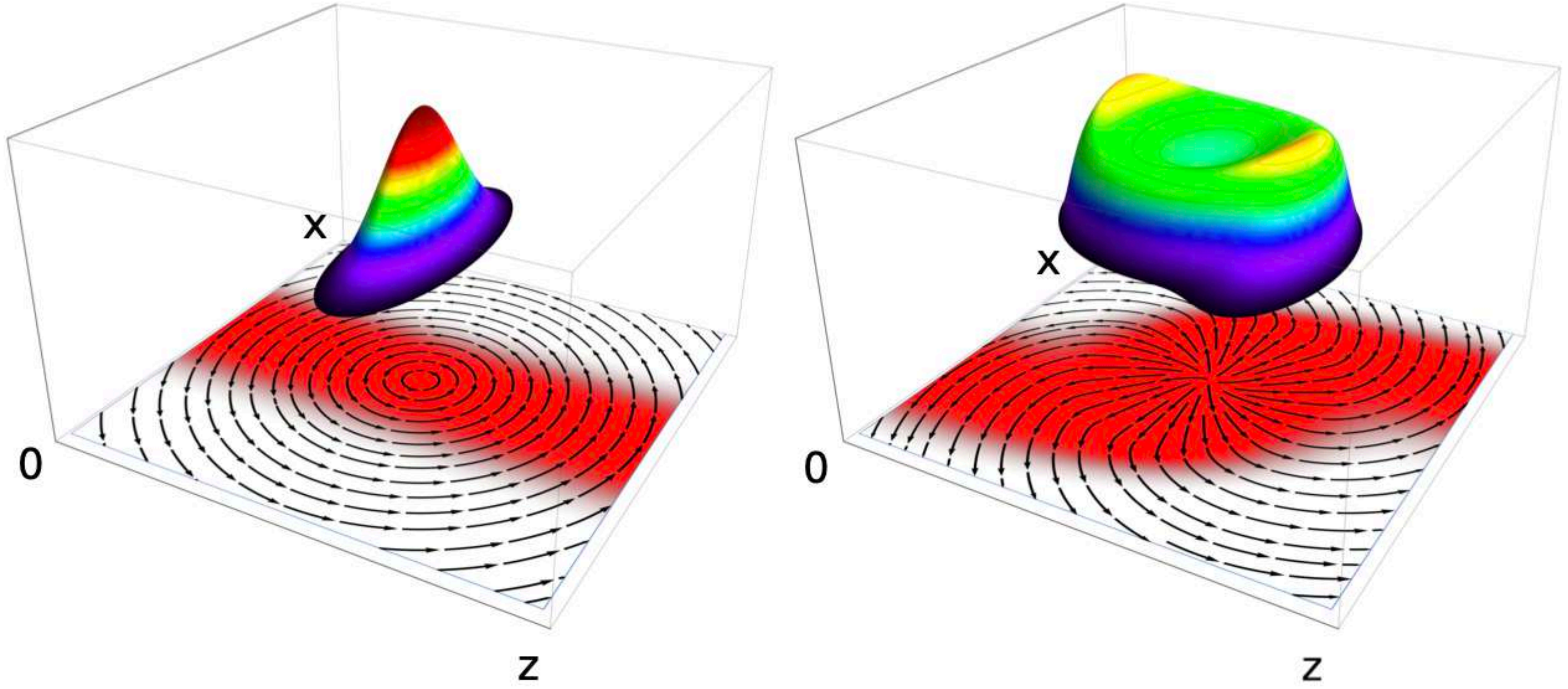
Xia, Li, Tang, Wang, Phys.Rev.C 98 (2018) 024905
Becattini, Karpenko, Phys.Rev.Lett. 120 (2018) 1, 012302

Perfect spin hydrodynamics was proposed phenomenologically

Florkowski, Friman, Jaiswal, Speranza, Phys. Rev. C97 (4) (2018) 041901
Florkowski, Friman, Jaiswal, RR, Speranza, Phys. Rev. D97 (2018) 116017

Spin hydrodynamics is being actively developed

Becattini and Tinti, Annals Phys. 325, 1566 (2010)
Montenegro and Torrieri, PRD 100, 056011 (2019)
Bhadury, Florkowski, Jaiswal, Kumar, and R. R, PRL 129, 192301 (2022)
Weickgenannt, Speranza, Sheng, Wang, and Rischke, PRL 127, 052301 (2021)
Li, Stephanov, and Yee, PRL 127, 082302 (2021)
Gallegos, Gursoy, and Yarom, JHEP 05, 139
Hongo, Huang, Kaminski, Stephanov, Yee JHEP 11, 150
Drogosz, Florkowski, Hontarenko, PRD 110 (2024) 9, 096018



Florkowski, Friman, Jaiswal, RR, Speranza, Nuclear Physics A 00 (2018) 1–4

Future measurements are planned (NA61/SHINE)

Bondar and Florkowski, Acta Phys. Polon. B 55, 9 (2024)

Spin hydrodynamics was studied in simple systems

Florkowski, Kumar, RR, Singh, Phys.Rev.C 99 (2019) 4, 044910
Drogosz, Florkowski, Łygan, RR Phys.Rev.C 111 (2025) 2, 024909
Florkowski, RR, Singh, Sophys, Phys.Rev.D 105 (2022) 5, 054007

Recently realistic modelling in (3+1)D was performed

S. K. Singh, RR, Florkowski, Phys.Rev.C 111 (2025) 2, 024907

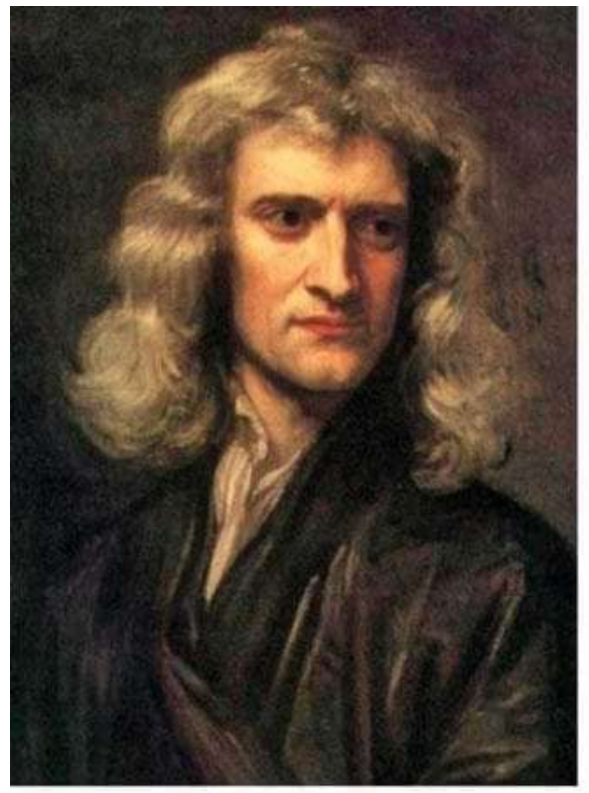
NON-RELATIVISTIC NAVIER-STOKES HYDRODYNAMICS W/O SPIN

A dissipative, uncharged, one-component fluid is described by Navier-Stokes equations

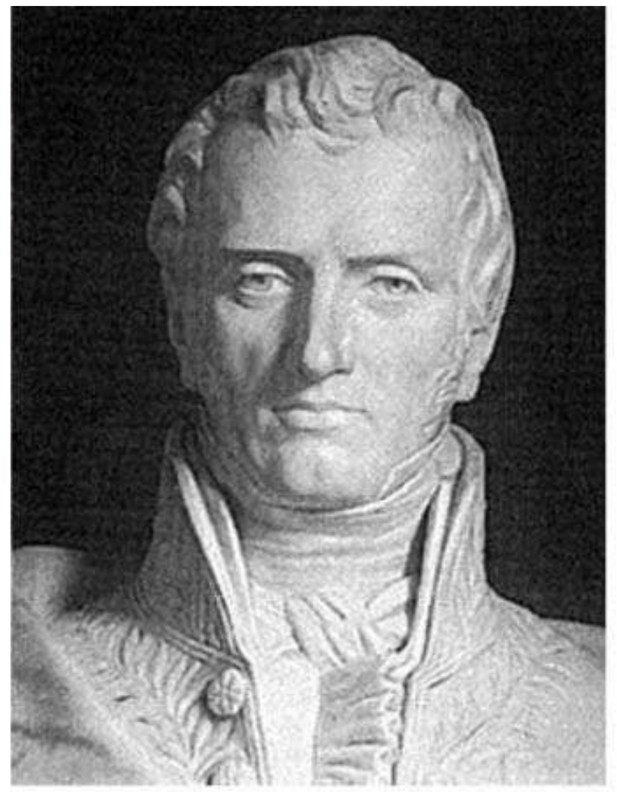
C. L. M. H. Navier, M´em. Acad. Sci. Inst. France 6 (1822) 389.
G. G. Stokes, Trans. Camb. Philos. Soc. 8 (1845) 287

$$\partial_t \rho + \rho \vec{\partial} \cdot \vec{v} + \vec{v} \cdot \vec{\partial} \rho = 0$$

$$\frac{\partial v^i}{\partial t} + v^k \frac{\partial v^i}{\partial x^k} = -\frac{1}{\rho}\frac{\partial p}{\partial x^i} - \frac{1}{\rho}\frac{\partial \Pi^{ki}}{\partial x^k},$$



Isaac Newton
1642-1727



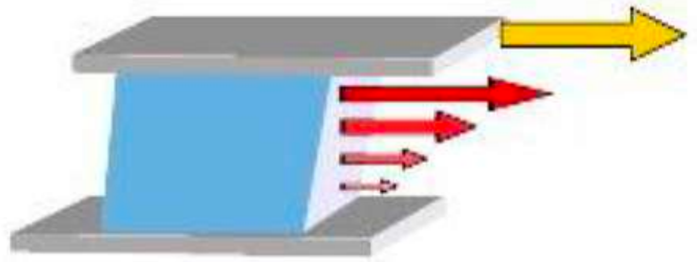
Claude-Louis Navier
1785-1836

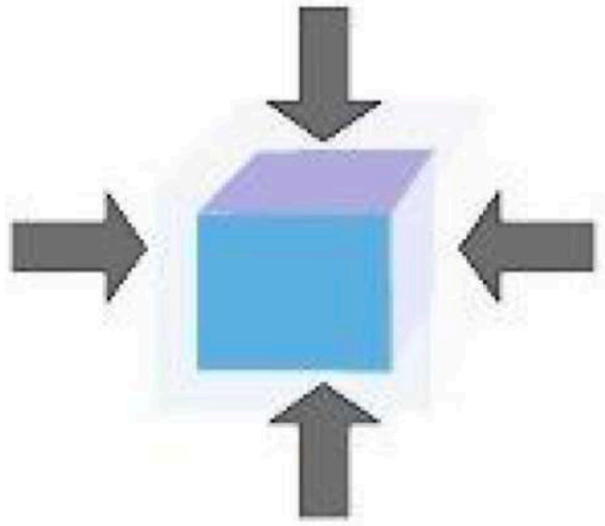
Sir George Stokes
1819-1903

$$\Pi^{ki} = -\eta \left(\frac{\partial v^i}{\partial x^k} + \frac{\partial v^k}{\partial x^i} - \frac{2}{3}\delta^{ki}\frac{\partial v^l}{\partial x^l} \right) - \zeta\, \delta^{ik}\frac{\partial v^l}{\partial x^l}$$

Equation of state is needed

$$p = p(\rho)$$





RELATIVISTIC NAVIER-STOKES HYDRODYNAMICS W/O SPIN

For relativistic fluids one needs to make some generalizations

L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Course of Theoretical Physics, Vol. 6

$$\begin{matrix} \boldsymbol{v}(t,\boldsymbol{x}) \\ \rho(t,\boldsymbol{x}) \end{matrix} \quad \Longrightarrow \quad \begin{matrix} u^\mu = \gamma(\boldsymbol{v})(1,\boldsymbol{v}) \\ \varepsilon(t,\boldsymbol{x}) \end{matrix}$$

EOM are given by energy-momentum conservation

P. Romatschke Int.J.Mod.Phys.E 19 (2010) 1-53

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \underbrace{\varepsilon u^\mu u^\nu - p g^{\mu\nu}}_{\text{perfect fluid}} + \Pi^{\mu\nu}$$

$$u_\nu\, \partial_\mu T^{\mu\nu} = D\varepsilon + (\varepsilon+p)\,\partial_\mu u^\mu + u_\nu\,\partial_\mu \Pi^{\mu\nu} = 0,$$

$$\Delta^\alpha{}_\nu\, \partial_\mu T^{\mu\nu} = (\varepsilon+p)\, Du^\alpha - \nabla^\alpha p + \Delta^\alpha{}_\nu\, \partial_\mu \Pi^{\mu\nu} = 0.$$

energy density   *energy flux*

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$$

momentum density   *momentum flux*   *isotropic pressure*

Eckart-Landau theory is acausal!

$$\xrightarrow{|\boldsymbol{v}| \ll 1}$$

$$\partial_t \rho + \rho\, \vec{\partial}\cdot\vec{v} + \vec{v}\cdot\vec{\partial}\rho = 0$$

$$\frac{\partial v^i}{\partial t} + v^k \frac{\partial v^i}{\partial x^k} = -\frac{1}{\rho}\frac{\partial p}{\partial x^i} - \frac{1}{\rho}\frac{\partial \Pi^{ki}}{\partial x^k}$$

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$$\xrightarrow{|\boldsymbol{v}| \ll 1}$$

$$\begin{aligned} \partial_t \rho + \rho\, \vec{\partial} \cdot \vec{v} + \vec{v} \cdot \vec{\partial} \rho &= 0 \\ \frac{\partial v^i}{\partial t} + v^k \frac{\partial v^i}{\partial x^k} &= -\frac{1}{\rho}\frac{\partial p}{\partial x^i} - \frac{1}{\rho}\frac{\partial \Pi^{ki}}{\partial x^k} \end{aligned}$$

Eckart-Landau theory is acausal!

CONSERVED *CANONICAL* CURRENTS IN QUANTUM FIELD THEORY

Noether's theorem:
For each continuous global symmetry of the action, there exists a (not unique) Noether current whose divergence vanishes on-shell
Weinberg, The Quantum Theory Of Fields Vol. 1 Cambridge University Press (1995)

- **invariance under space-time translation** **conservation of energy and linear momentum**

$$\hat{T}_{\mathrm{C}}^{\mu\nu} = \sum_a \frac{\partial \mathcal{L}}{\partial(\partial_\mu \hat{\psi}^a)} \partial^\nu \hat{\psi}^a - g^{\mu\nu}\mathcal{L} \quad \Rightarrow \quad \partial_\mu \hat{T}_{\mathrm{C}}^{\mu\nu} = 0$$

- **invariance under rotations and boosts** **conservation of total angular momentum**

$$\hat{J}_{\mathrm{C}}^{\mu,\lambda\nu} = x^\lambda \hat{T}_{\mathrm{C}}^{\mu\nu} - x^\nu \hat{T}_{\mathrm{C}}^{\mu\lambda} + \hat{S}_{\mathrm{C}}^{\mu,\lambda\nu} \quad \Rightarrow \quad \partial_\mu \hat{J}_{\mathrm{C}}^{\mu,\lambda\nu} = \hat{T}_{\mathrm{C}}^{\lambda\nu} - \hat{T}_{\mathrm{C}}^{\nu\lambda} + \partial_\mu \hat{S}_{\mathrm{C}}^{\mu,\lambda\nu} = 0$$

$$\hat{S}_{\mathrm{C}}^{\mu,\lambda\nu} = -i \sum_{a,b} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \hat{\psi}^a)} \left[D(J^{\lambda\nu})\right]^a{}_b \, \hat{\psi}^b$$

← non-vanishing for fields with spin

ORBITAL AND SPIN ANGULAR MOMENTUM

$$J^{\mu,\lambda\nu} = \underbrace{x^\lambda T^{\mu\nu} - x^\nu T^{\mu\lambda}}_{\text{orbital part}} + \underbrace{S^{\mu,\lambda\nu}}_{\text{spin part}}$$

orbital angular momentum of a point particle

$$\vec{L} = \vec{x} \times \vec{p} \quad \Rightarrow \quad L_i = \varepsilon_{ijk}\, x_i\, p_j$$

its dual is

$$L_{ij} \equiv \varepsilon_{ijk}\, L_k \quad \Rightarrow \quad L_{ij} = x_i\, p_j - x_j\, p_i$$

relativistic generalization is

$$L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu$$

for relativistic fluid one has

$$L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$$

CONSERVATION LAWS AND LAGRANGE MULTIPLIERS

conservation laws + (near) local equilibrium $\Rightarrow$ hydrodynamics

- **conservation of charge (baryon number, electric charge, …)**

$$\partial_\mu N^\mu(x) = 0 \quad \Rightarrow \quad \mu \equiv \xi T$$

(1 eq / charge)

- **conservation of energy and linear momentum**

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \Rightarrow \quad T, u^\nu$$

(4 eqs)

CONSERVATION LAWS AND LAGRANGE MULTIPLIERS

conservation laws + (near) local equilibrium → hydrodynamics

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$$\partial_\mu N^\mu(x) = 0 \quad \rightarrow \quad \mu \equiv \xi T$$ (1 eq / charge)

- **conservation of energy and linear momentum**

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \rightarrow \quad T, u^\nu$$ (4 eqs)

- **conservation of angular momentum**

$$\partial_\lambda J^{\lambda\mu\nu}(x) = 0 \quad \rightarrow \quad \Omega_{\mu\nu} \equiv T\omega_{\mu\nu}$$ (6 eqs)

↑ spin chemical potential ↑ spin polarization tensor

Florkowski, Friman, Jaiswal, Speranza, Phys. Rev. C97 (4) (2018) 041901
Florkowski, Friman, Jaiswal, R. R., Speranza, Phys. Rev. D97 (2018) 116017
Becattini, Florkowski, Speranza, Phys. Lett. B 789 (2019) 419-425

PERFECT SPIN HYDRODYNAMICS

The conservation law for total angular momentum is

$$D_\alpha J^{\alpha,\beta\gamma} = 0$$

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The total angular momentum is decomposed into orbital angular momentum and intrinsic spin tensor

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu} = \left(x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}\right) + S^{\lambda\mu\nu}$$

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$$\boxed{D_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu}}$$

PERFECT SPIN HYDRODYNAMICS

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$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu} = \left(x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}\right) + S^{\lambda\mu\nu}$$

$$D_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \xrightarrow{T^{\nu\mu} = -T^{\mu\nu}} \boxed{D_\alpha S^{\alpha,\beta\gamma}(x) = 0}$$

For conserved symmetric EMT implies the conservation of the spin tensor (if the lattter is nonzero)

Florkowski, Friman, Jaiswal, Speranza, PRC 97 (4) (2018) 041901
Florkowski, Friman, Jaiswal, R. R., Speranza, PRD 97 (2018) 116017

PERFECT SPIN HYDRODYNAMICS

$$\partial_\mu N^\mu(x) = 0$$
$$\partial_\mu T^{\mu\nu}(x) = 0$$
$$\partial_\mu S^{\mu\nu\lambda}(x) = 0$$

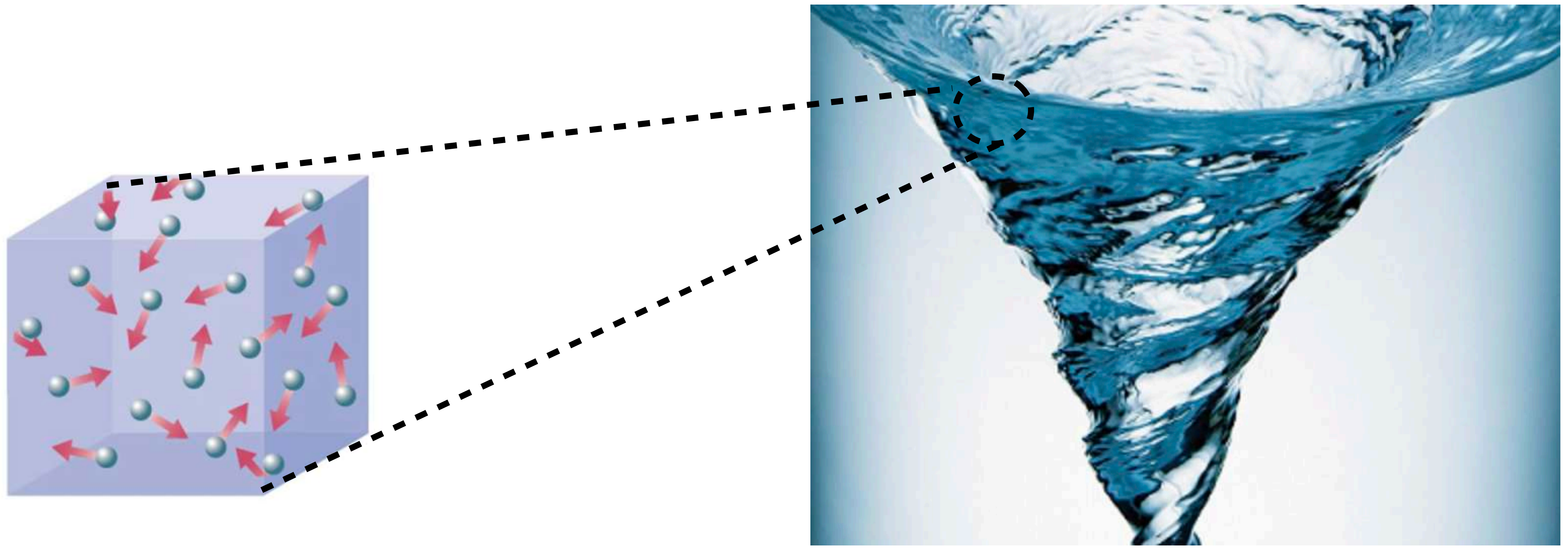
What are the constitutive relations which enter equations of motion?

$$T^{\mu\nu} = T^{\mu\nu}[\beta, \omega, \xi], \quad S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta, \omega, \xi], \quad N^\mu = N^\mu[\beta, \omega, \xi]$$

Coarse-graining of the underlying microscopic theory may provide this information.
Relativistic kinetic theory is commonly used.

RELATIVISTIC KINETIC THEORY DERIVATION OF SPIN HYDRODYNAMICS

For dilute systems, the derivation of fluid dynamics can be done starting from the underlying kinetic theory



Classical RKT

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)] \xrightarrow{\text{moments method}} \begin{aligned} \partial_\mu N^\mu(x) &= 0 \\ \partial_\mu T^{\mu\nu}(x) &= 0 \end{aligned}$$

$$\int dP p^\mu \partial_\mu f = \int dP C[f]$$

$$\int dP p^\mu p^\nu \partial_\mu f = \int dP p^\nu C[f]$$

1 and p^ν are collision invariants

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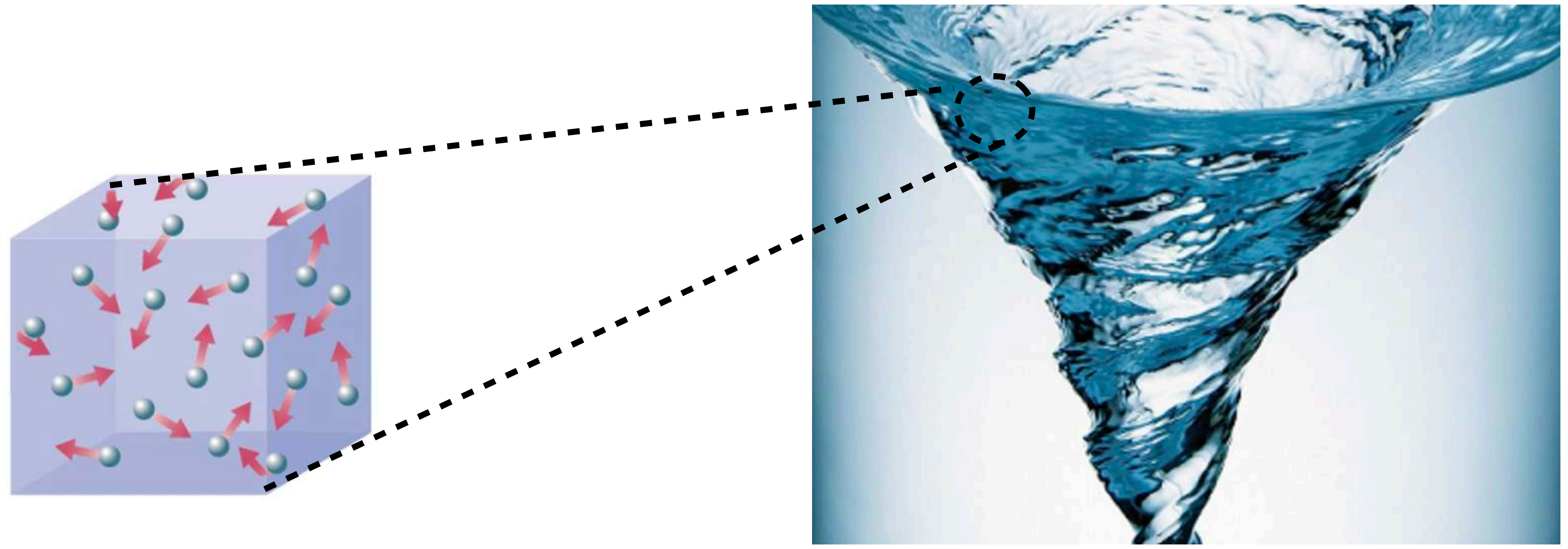

Classical RKT (constitutive relations)

$$f(x,p) \approx f^{\pm}_{\rm eq}(p\cdot u(x), T(x), \mu(x)) = \exp[\pm\xi - \beta^\mu p_\mu]$$

$$N^\mu(x) \equiv \int dP p^\mu f(x,p)$$

$$T^{\mu\nu}(x) \equiv \int dP p^\mu p^\nu f(x,p)$$

$$\longrightarrow$$

$$N^\mu = n u^\mu,$$

$$T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu}$$

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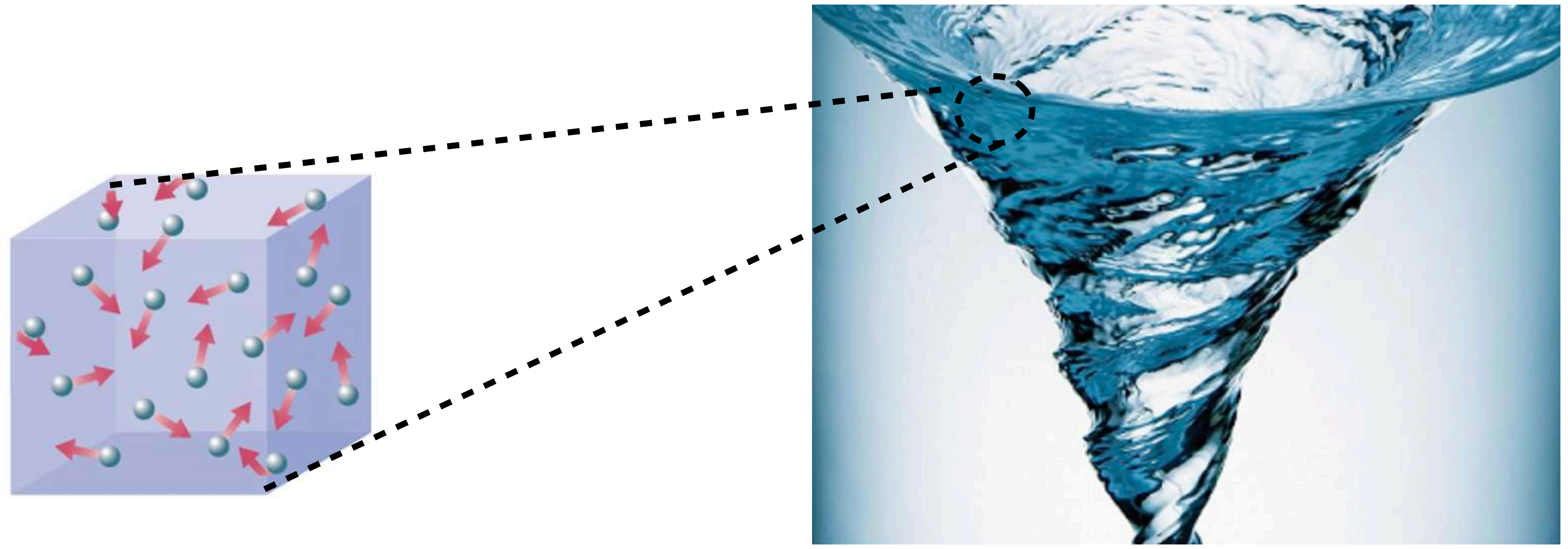
For dilute systems, the derivation of fluid dynamics can be done starting from the underlying kinetic theory



Quantum RKT

$$(\gamma_\mu K^\mu - m)\,\mathcal{W}(x,k) = C[\mathcal{W}(x,k)]$$

semi-classical expansion

$$k^\mu \partial_\mu \mathcal{F}_{\rm eq}(x,k) = 0$$
$$k^\mu \partial_\mu \mathcal{A}^\nu_{\rm eq}(x,k) = 0$$

moments method

$$\partial_\mu N^\mu(x) = 0$$
$$\partial_\mu T^{\mu\nu}(x) = 0$$
$$\partial_\mu S^{\mu\nu\lambda}(x) = 0$$

De Groot, van Leeuwen, van Weert: Relativistic Kinetic Theory. Principles and Applications, 1980.
Florkowski, Kumar, R. R., PRC 98 (2018) 044906

RELATIVISTIC KINETIC THEORY DERIVATION OF SPIN HYDRODYNAMICS

Quantum RKT (constitutive relations)

Becattini, Chandra, Del Zanna, Grossi , Annals Phys. 338 (2013) 32
Florkowski, Friman, Jaiswal, Speranza, PRC 97 (4) (2018) 041901
Florkowski, Friman, Jaiswal, R. R., Speranza, PRD 97 (11) (2018) 116017

$$f^{+}_{rs}(x,p) = \frac{1}{2m}\,\bar{u}_r(p)\,X^{+}\,u_s(p)\,.$$

$$f^{-}_{rs}(x,p) = -\frac{1}{2m}\,\bar{v}_s(p)\,X^{-}\,v_r(p)\,.$$

$$X^{\pm} = \exp\left[\pm\xi(x) \;-\; \beta_\mu(x)\,p^\mu \;\pm\; \frac{1}{2}\,\omega_{\mu\nu}(x)\,\Sigma^{\mu\nu}\right].$$

$$\mathcal{W}^{+}_{\rm eq}(x,k) = \tfrac{1}{2}\sum_{r,s=1}^{2}\int dP\delta^{(4)}(k-p)u^r(p)\bar{u}^s(p)f^{+}_{rs}(x,p)$$

$$W^{-}_{\rm eq}(x,k) = -\tfrac{1}{2}\sum_{r,s=1}^{2}\int dP\delta^{(4)}(k+p)v^s(p)\bar{v}^r(p)f^{-}_{rs}(x,p)$$

$$\mathcal{W}_{\rm eq}(x,k) = \mathcal{W}^{+}_{\rm eq}(x,k) + \mathcal{W}^{-}_{\rm eq}(x,k)$$

$$\Longrightarrow \quad T^{\beta\alpha}_{\rm eq}(x) = T^{\alpha\beta}_{\rm eq}(x).$$

Spin is conserved

De Groot, van Leeuwen, van Weert: Relativistic Kinetic Theory. Principles and Applications, 1980.
Florkowski, Kumar, R. R., PRC 98 (2018) 044906

$$\hat{\Sigma}^{\mu\nu} = (i/4)\left[\gamma^\mu, \gamma^\nu\right]$$

RELATIVISTIC KINETIC THEORY DERIVATION OF SPIN HYDRODYNAMICS

Linear order in spin polarization tensor (small polarization limit)

Florkowski, Kumar, and RR, Phys. Rev. C98, 044906 (2018)
Florkowski, RR, and Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$T^{\mu\nu} = T^{\mu\nu}[\beta, \cancel{\omega}, \xi], \quad \boxed{S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta, \omega, \xi],} \quad N^{\mu} = N^{\mu}[\beta, \cancel{\omega}, \xi]$$

Background hydrodynamics decouples from spin hydrodynamics!

PERFECT SPIN HYDRODYNAMICS

Background hydrodynamics

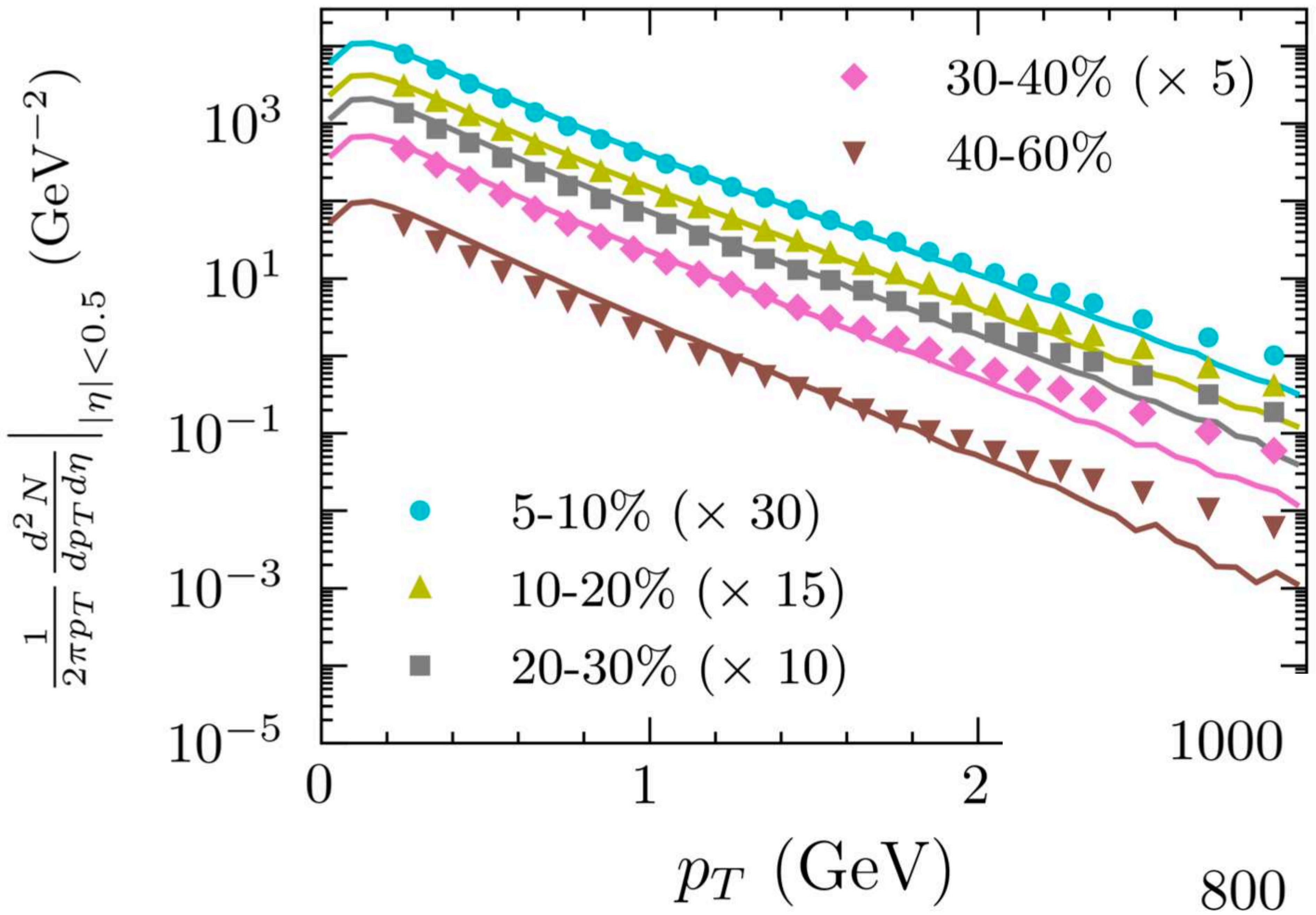
$$T^{\mu\nu} = T^{\mu\nu}[\beta, \xi]$$
$$N^{\mu} = N^{\mu}[\beta, \xi]$$

$$D_\alpha T^{\alpha\beta}(x) = 0$$
$$D_\alpha N^{\alpha}(x) = 0$$

$\Rightarrow$

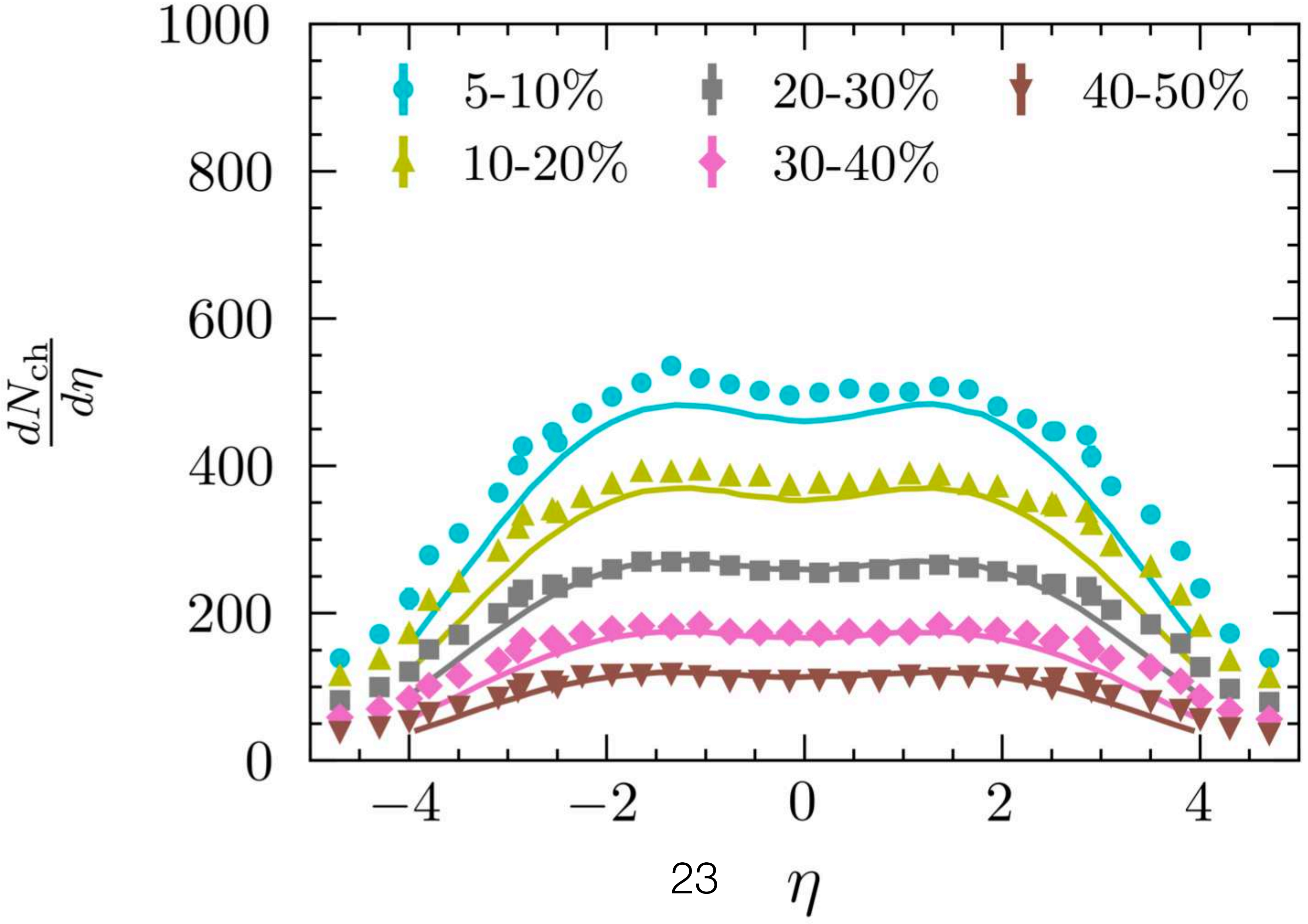
$$T, \mu_B \; u^\rho$$

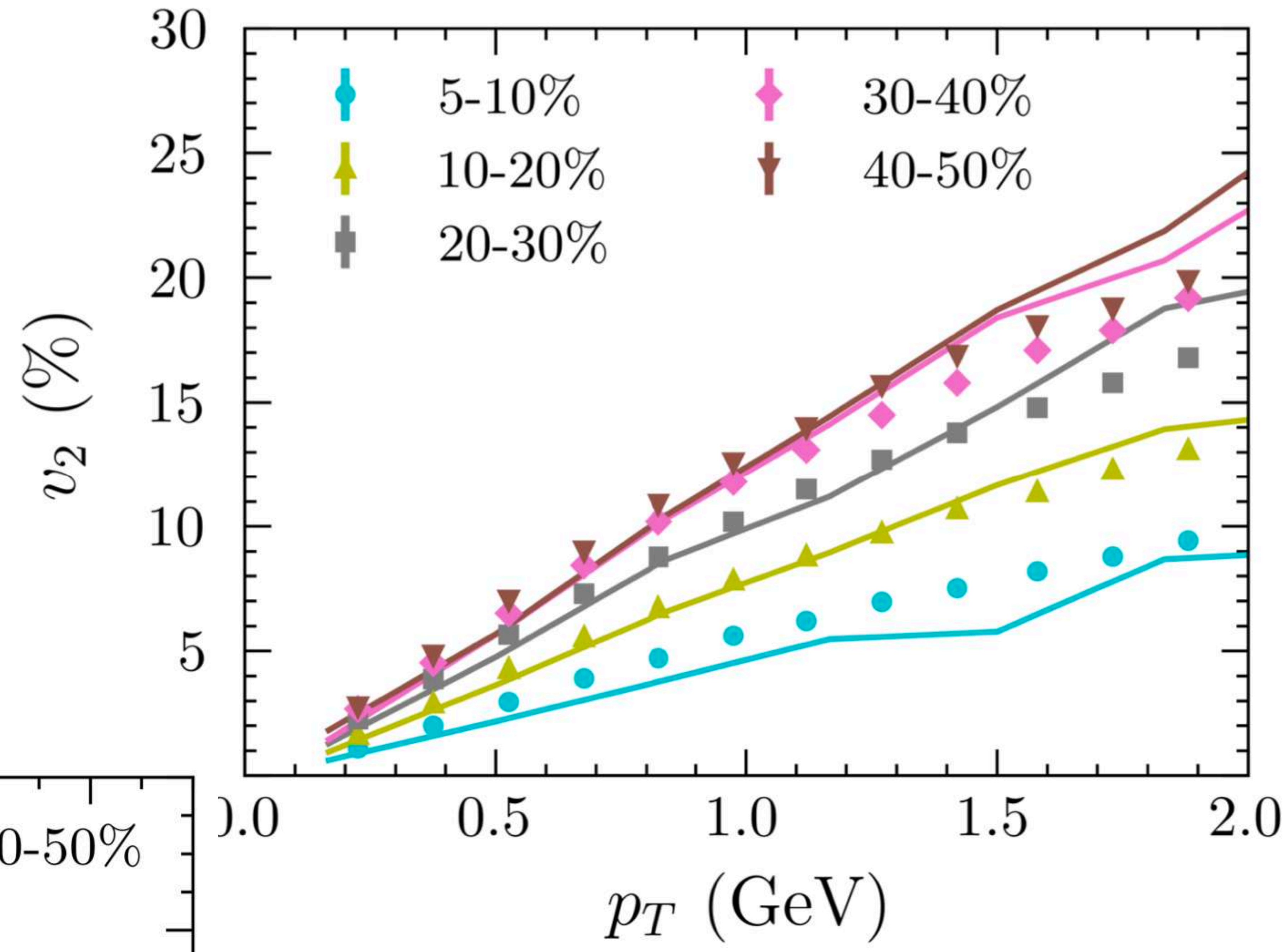
Results for background hydrodynamics



Adams et al. (STAR Collaboration), Phys. Rev. Lett. 91, 172302 (2003)

Bearden et al. (BRAHMS Collaboration), Phys. Rev. Lett. 88, 202301 (2002).





Adams et al. (STAR and STAR-RICH Collaboration), Phys. Rev. C 72, 014904 (2005)

PERFECT SPIN HYDRODYNAMICS

Background hydrodynamics

$$T^{\mu\nu} = T^{\mu\nu}[\beta, \xi]$$
$$N^{\mu} = N^{\mu}[\beta, \xi]$$

$$D_\alpha T^{\alpha\beta}(x) = 0$$
$$D_\alpha N^{\alpha}(x) = 0$$

$$T, \mu_B \; u^\rho$$

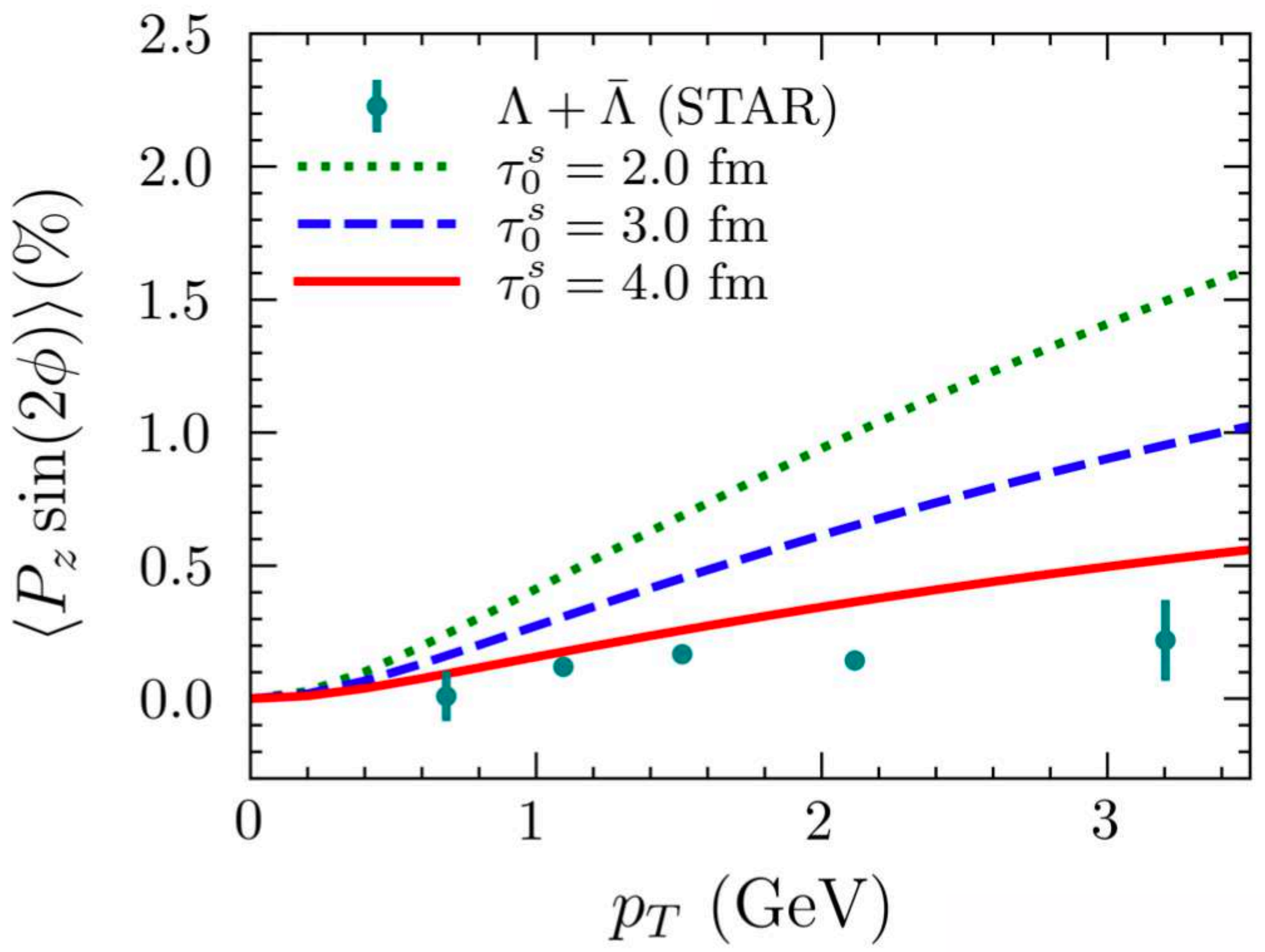
Spin hydrodynamics

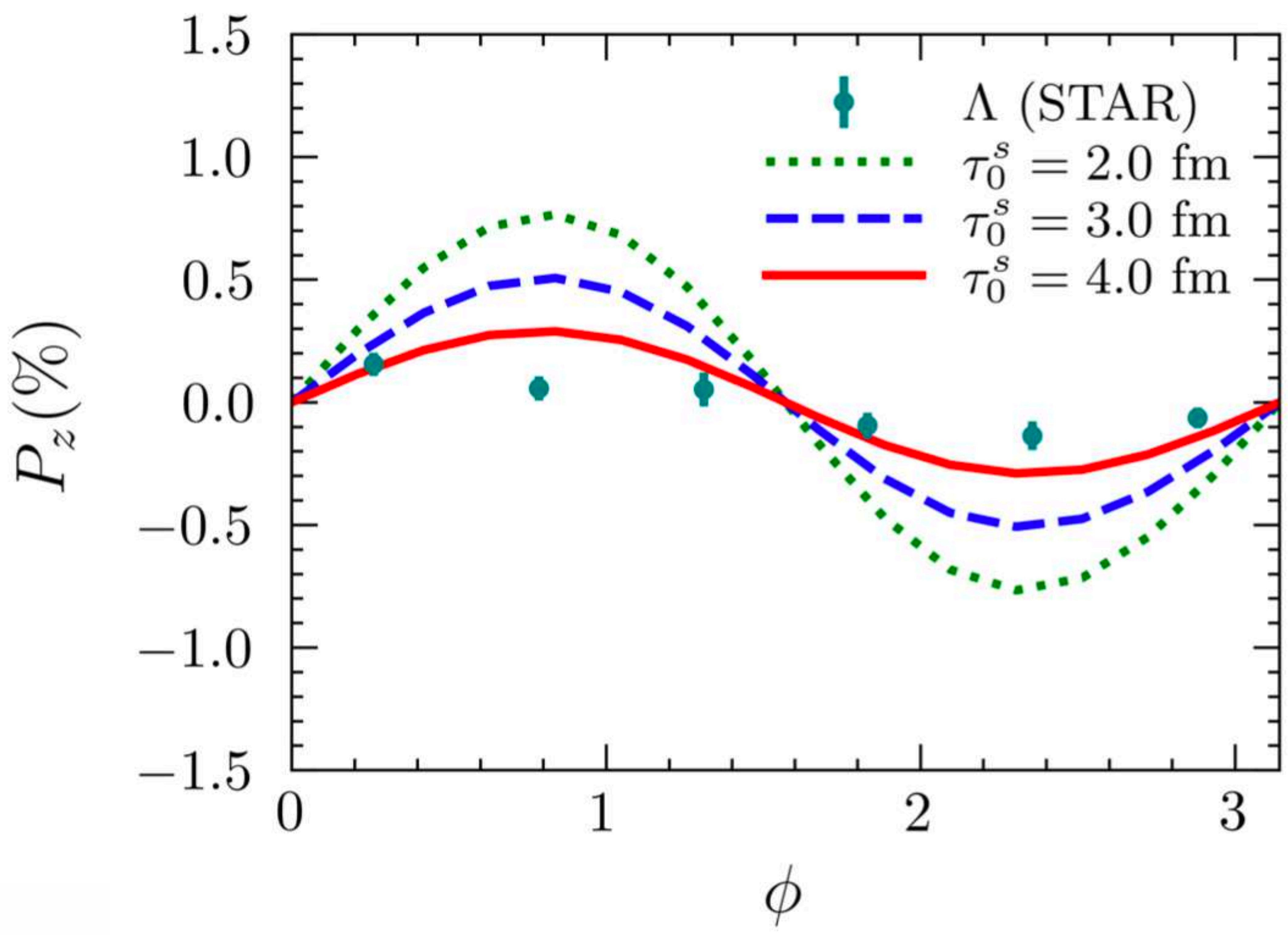
$$S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta, \omega, \xi]$$

$$D_\alpha S^{\alpha,\beta\gamma}(x) = 0$$

$$\omega_{\mu\nu}$$

RESULTS FOR SPIN HYDRODYNAMICS





$$m = m_\Lambda$$

$$\varpi_{\mu\nu} = \varpi^{\text{iso}}_{\mu\nu} + \cancel{\varpi^{\text{T}}_{\mu\nu}} \qquad \xi_{\mu\nu} = \xi^{\text{iso}}_{\mu\nu} + \cancel{\xi^{\text{T}}_{\mu\nu}}$$

OTHER DEVELOPMENTS OF HYDRODYNAMICS WITH SPIN

- **Lagrangian effective field theory approach**
  D. Montenegro, G. Torrieri, Phys.Rev. D94 (2016) no.6, 065042
  D. Montenegro, L. Tinti, G. Torrieri, Phys. Rev. D 96(5) (2017) 056012; Phys. Rev. D 96(7) (2017) 076016
  D. Montenegro, G. Torrieri, Phys. Rev. D 100, 056011 (2019)

- **Hydrodynamics with spin based on entropy-current analysis**
  K. Hattori, M. Hongo, X-G Huang, M. Matsuo, H. Taya, PLB 795 (2019) 100-106

- **Hydrodynamics of spin currents using presence of torsion**
  D. Gallegos, U. Gursoy, A. Yarom arXiv:2101.04759

- **Relativistic viscous hydrodynamics with spin using Navier-Stokes type gradient expansion analy**
  D. She, A. Huang, D. Hou, J. Liao, arXiv:2105.04060

- **Relativistic viscous spin hydrodynamics from chiral kinetic theory**
  S. Shi, C. Gale, and S. Jeon, Phys. Rev. C 103, 044906 (2021)

- **Spin polarization generation from vorticity through nonlocal collisions**
  N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, arXiv:2005.01506, arXiv:2103.04896

- **Spin polarisation due to thermal shear**
  F. Becattini, M. Buzzegoli, and A. Palermo, arXiv:2103.10917
  S. Y. F. Liu and Y. Yin, arXiv:2103.09200

CONCLUSIONS

A complete numerical framework for perfect spin hydrodynamics

We solved perfect spin hydrodynamics in realistic 3+1 dimensional case

We tuned the background to describe basic hadronic observables

We determined polarization vector for Lambda hyperons and compared with data and other frameworks

Acceptable agreement is obtained with delayed initialization time for spin evolution

The first kinetic theory formulation of **relativistic dissipative nonresistive MHD with spin** in the limit of small polarization was provided.

We showed that our framework naturally leads to the **emergence of the relativistic analog of Barnett effect**.

We show that the **coupling between the magnetic field and spin polarization appears at gradient order**.

WEISSENBERG EFFECT

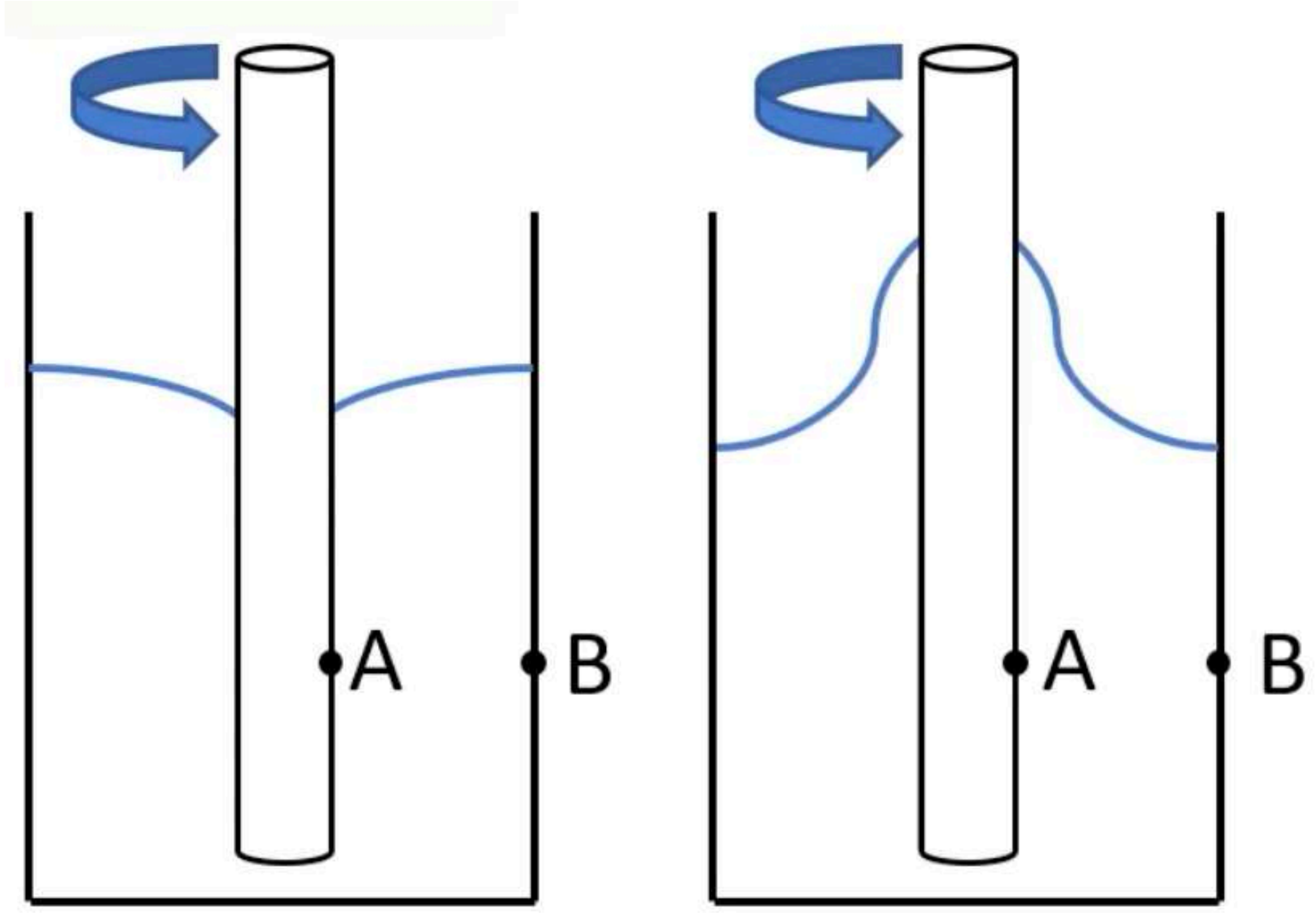
Visco-elastic materials (non-newtonian fluids) subjected to torsional movements by a rotating rod develop normal forces which make them climb up on the rod

Weissenberg, K. "A Continuum Theory of Rhelogical Phenomena". Nature. 159 (4035): 310–311

rheology: ***(rhéō)*** **'flow' and -λογια** ***(-logía)*** **'study of')**

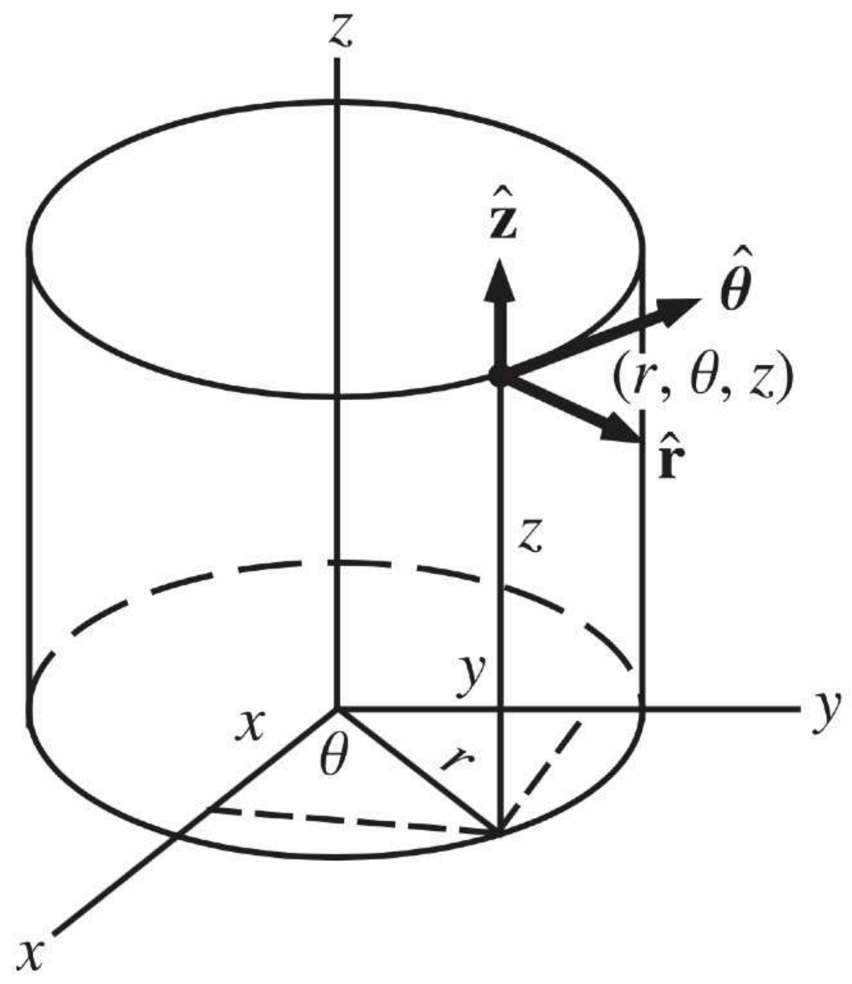
WEISSENBERG EFFECT

$$v_r = 0, \quad v_z = 0, \quad v_\theta = v_\theta(r), \quad \tau_{ij} = \tau_{ij}(r)$$



Newtonian Non-Newtonian

$p_A < p_B$ $p_A > p_B$



For viscoelastic fluids, stress has an additional normal-stress part with its own dynamics

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g} \qquad \boldsymbol{\sigma} = -p\,\mathbf{I} + \boldsymbol{\tau}$$

Balance equation for pressure

$$\frac{d}{dr}(p + \tau_{rr}) = \rho\,\frac{v_\theta^2}{r} + \frac{\tau_{\theta\theta} - \tau_{rr}}{r}.$$

centrifugal force
(drives pressure higher outward)
free surface dips near rod

hoop/streamline tension
(acts like stretched polymers
pulling inward)

THANK YOU FOR YOUR ATTENTION.