Are there spherical vibrational nuclei?

Paul Garrett
University of Guelph

Vibrations in nuclei…

- Liquid drop model was first used to describe static nuclear properties (Gamow, *Proc. Roy. Soc. [A]* 136, *386 (1929)*)
- Dynamic properties of liquid drop (Bohr, *Mat. Fys. Medd. Dan. Vid. Selsk.* 14, no. 10 *(1937)*) – possibility of vibrational oscillations of nucleus discussed, and first estimation of energy, ≈ 1 MeV, given
- Describing the nuclei as a liquid drop leads to the idea of *collective coordinates* – ignore motions of individual nucleons and treat nucleus as a continuous medium
- Surface parameterized as

$$R(\vartheta,\phi,t) = R_0\left(1+\sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha^*_{\lambda\mu}(t)Y_{\lambda\mu}(\vartheta,\varphi)\right)$$

where $\alpha^*_{\lambda\mu}(t)$ are the collective coordinates which are time dependent – this allows for vibrations of the surface

Vibrations of a classical liquid drop

- Assuming a charged liquid drop, E will have 3 terms

$$E = T + E_C + E_S$$

where T is the kinetic term, E_C the Coulomb energy, and E_S the surface energy

- The Coulomb and surface energies leads to the form

$$\frac{1}{2}\sum_{\lambda\mu} C_\lambda \left|\alpha_{\lambda\mu}\right|^2$$

and the kinetic energy $$\frac{1}{2}\sum_{\lambda\mu} B_\lambda \left|\dot{\alpha}_{\lambda\mu}\right|^2 = \sum \frac{1}{2B_\lambda}\left|\pi_{\lambda\mu}\right|^2$$

resulting in the Hamiltonian $$H = \sum_{\lambda\mu}\left\{\frac{1}{2B_\lambda}\left|\pi_{\lambda\mu}\right|^2 + \frac{C_\lambda}{2}\left|\alpha_{\lambda\mu}\right|^2\right\}$$

This is nothing more than a simple harmonic oscillator with frequency

$$\omega_\lambda = \sqrt{\frac{C_\lambda}{B_\lambda}}$$

Vibrations of a QM liquid drop

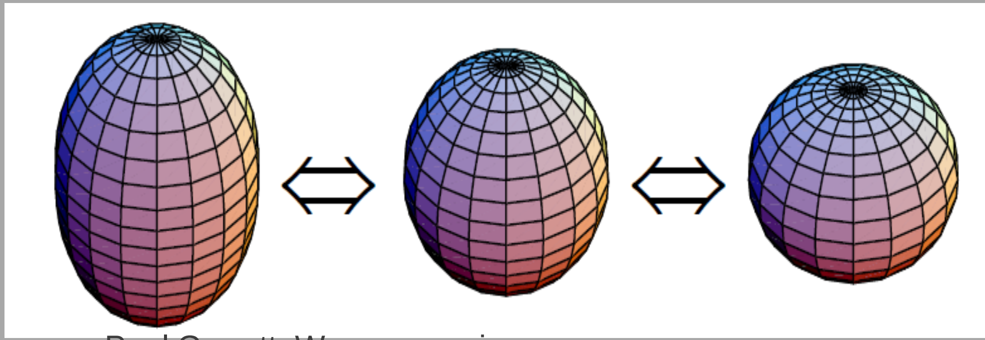
- Hamiltonian can be cast into form

$$\hat{H} = \sum_{\lambda\mu} \hbar\omega \left\{ b^{+}_{\lambda\mu} b_{\lambda\mu} + \tfrac{1}{2} \right\}$$

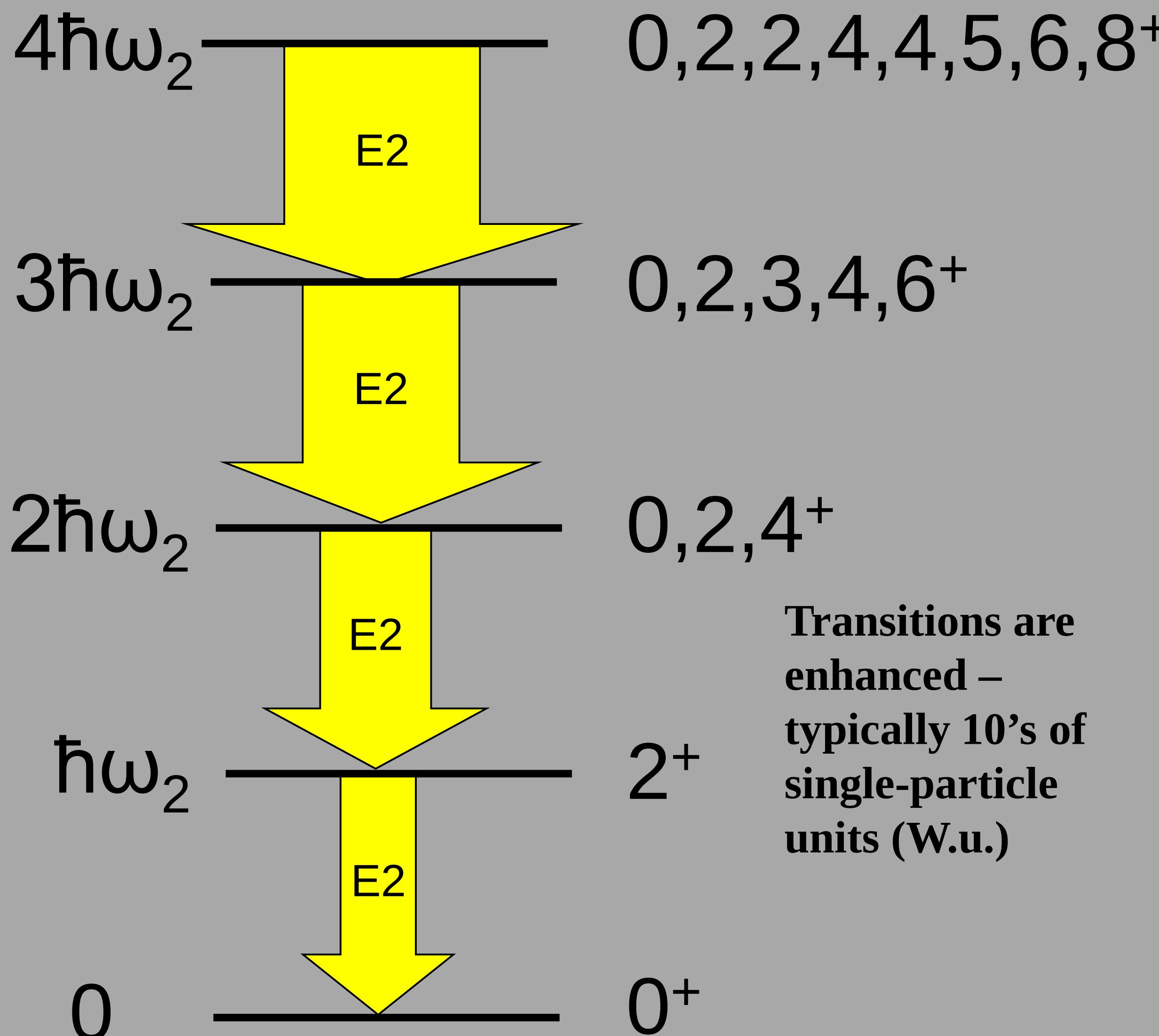
where $b^{+}_{\lambda\mu}$ ($b_{\lambda\mu}$) is a phonon creation (annihilation) operator

- The terms in order of importance (for small amplitude motion and low excitation energies) are λ=2 (quadrupole vibrations), λ=3 (octupole vibrations), etc.

- For quadrupole vibrations $\boldsymbol{E_n = \hbar\omega_2\left(n + \tfrac{5}{2}\right)}$

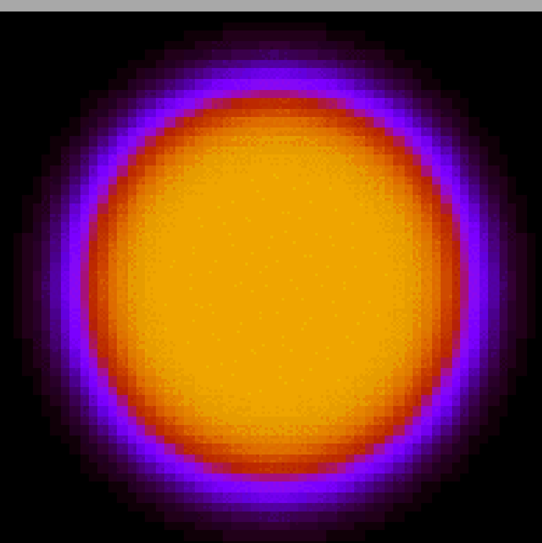


Spherical Quadrupole Vibrations



$\Delta n = \pm 1$

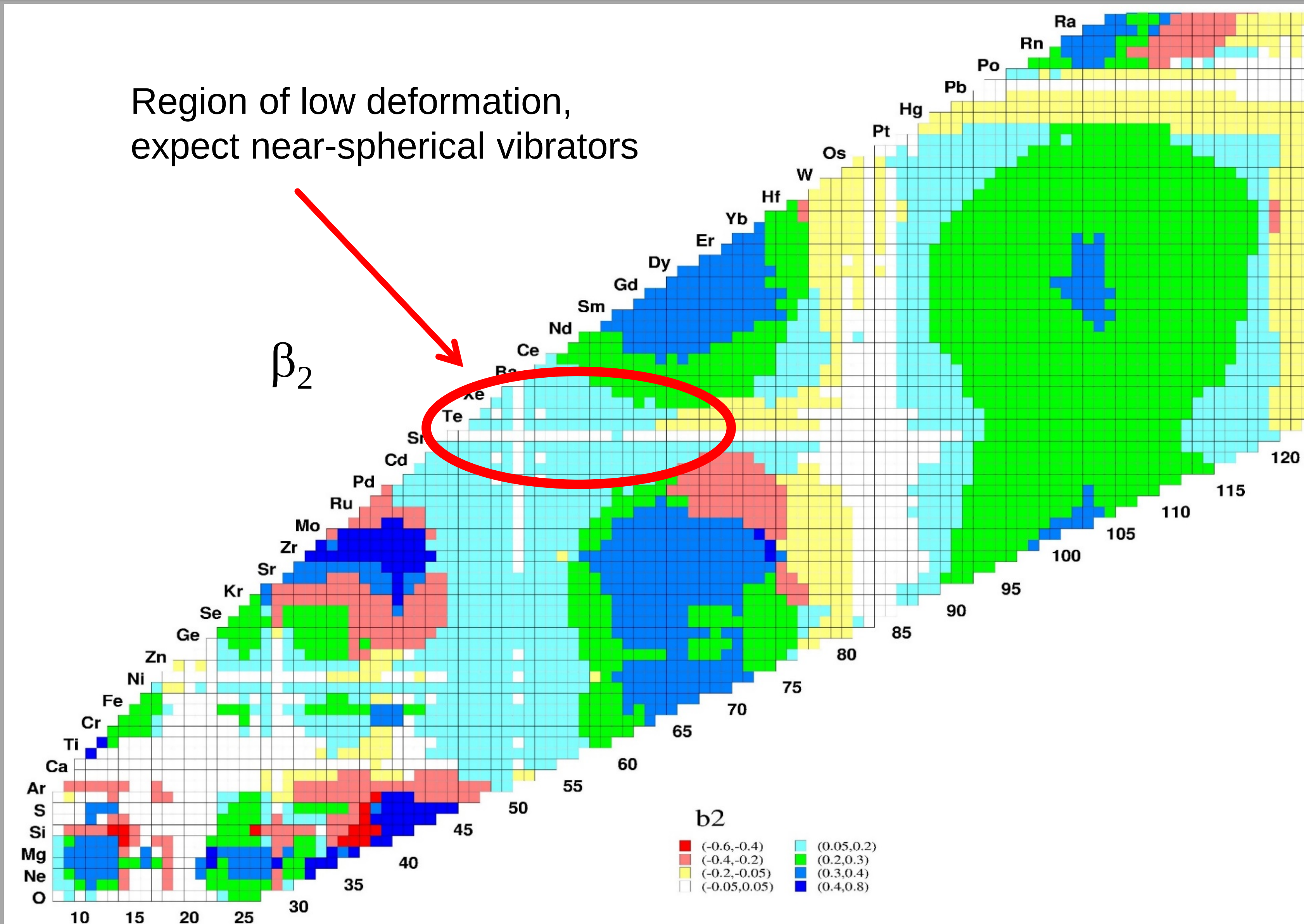
Transitions are enhanced – typically 10's of single-particle units (W.u.)



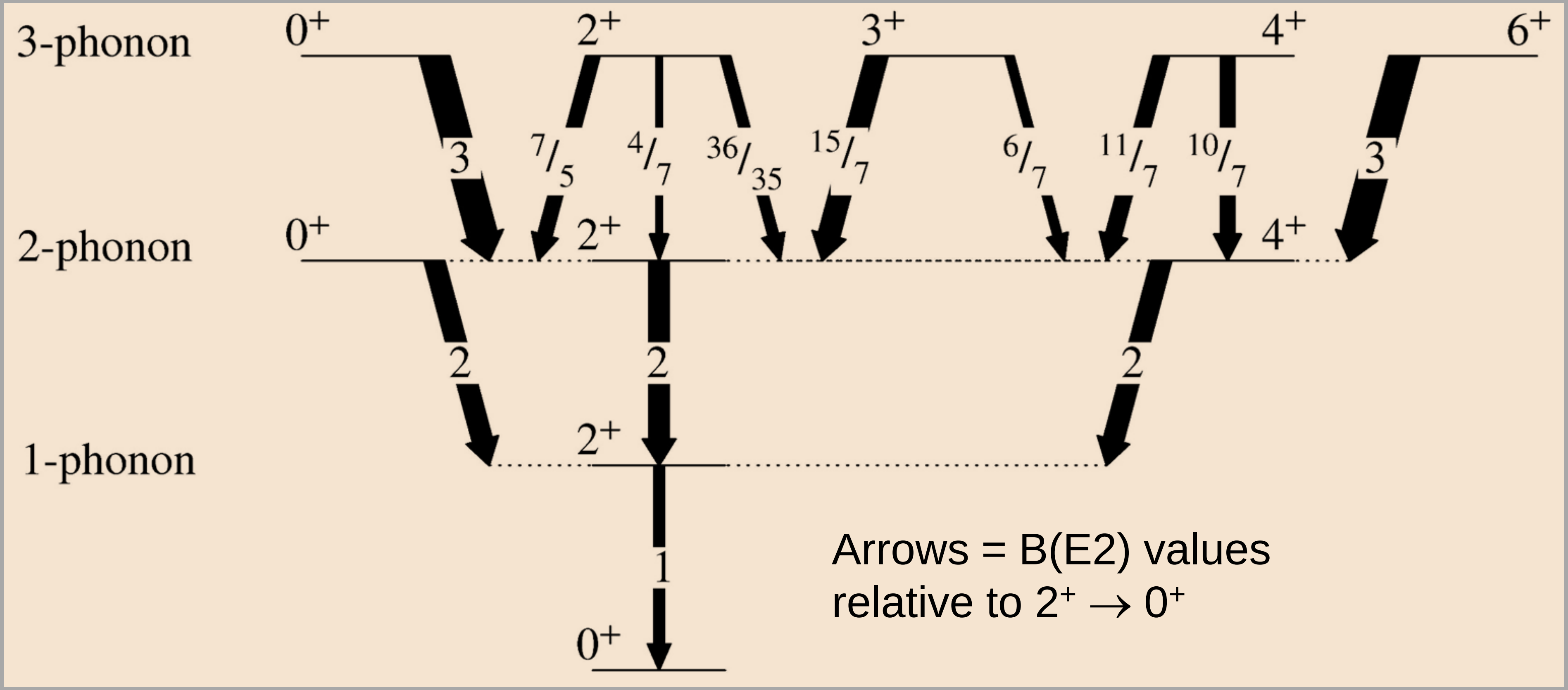
Courtesy of M. Itoh and Y. Fujita

The $Z = 50$ region – rich in spherical vibrators?

Region of low deformation, expect near-spherical vibrators



Quadrupole harmonic oscillator



- **Use expectations of harmonic vibrator as a guide for multiphonon states**

Candidates for near harmonic vibrational motion (or U(5) symmetry) near Z=50

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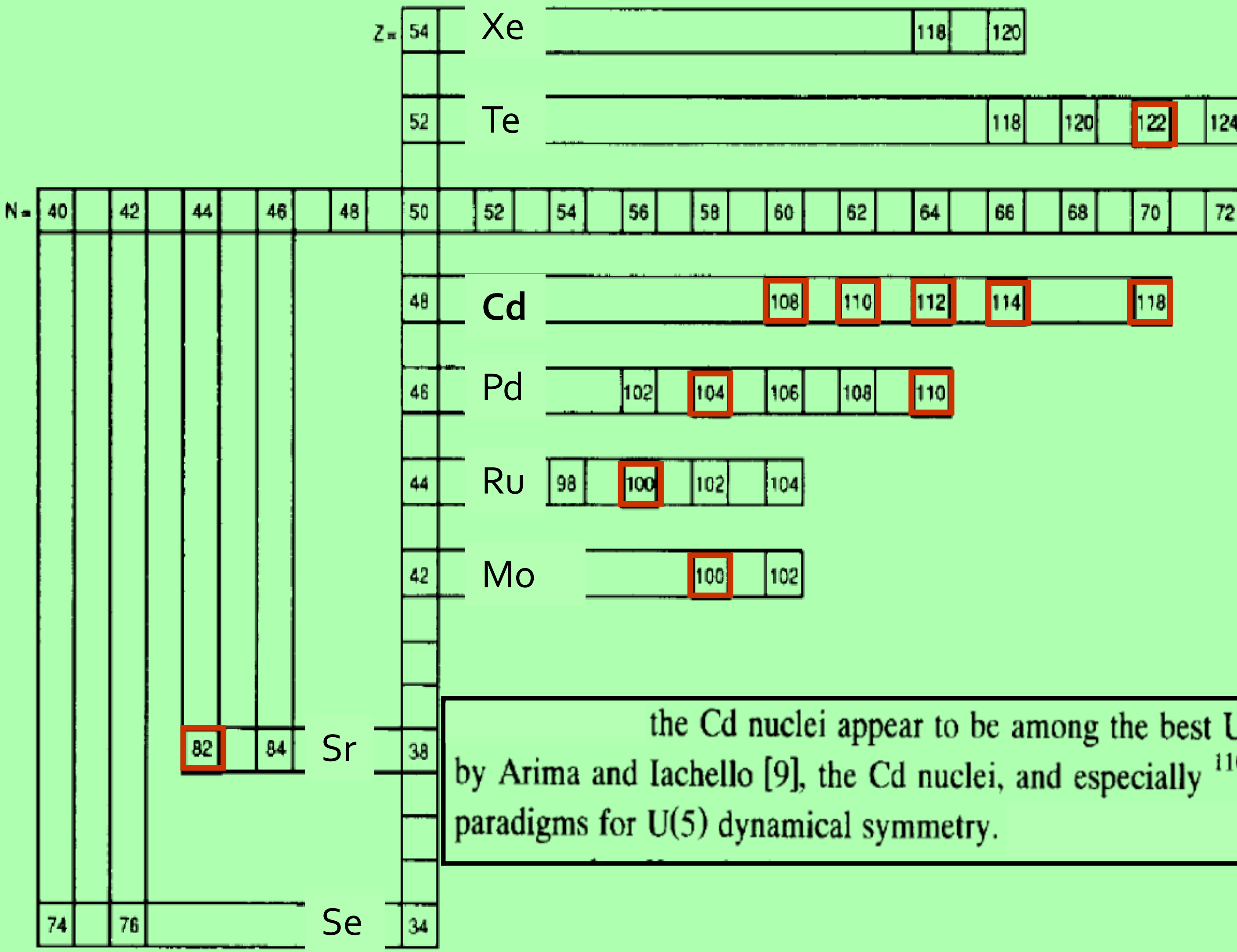
J. Kern et al. / Nuclear Physics A 593 (1995) 21–47 27



the Cd nuclei appear to be among the best U(5) candidates. As proposed by Arima and Iachello [9], the Cd nuclei, and especially ^{110}Cd, can still be regarded as paradigms for U(5) dynamical symmetry.

Criteria for valid vibrational multiphonon state

For good spherical vibrational, or U(5), candidates, Kern *et al*. considered:

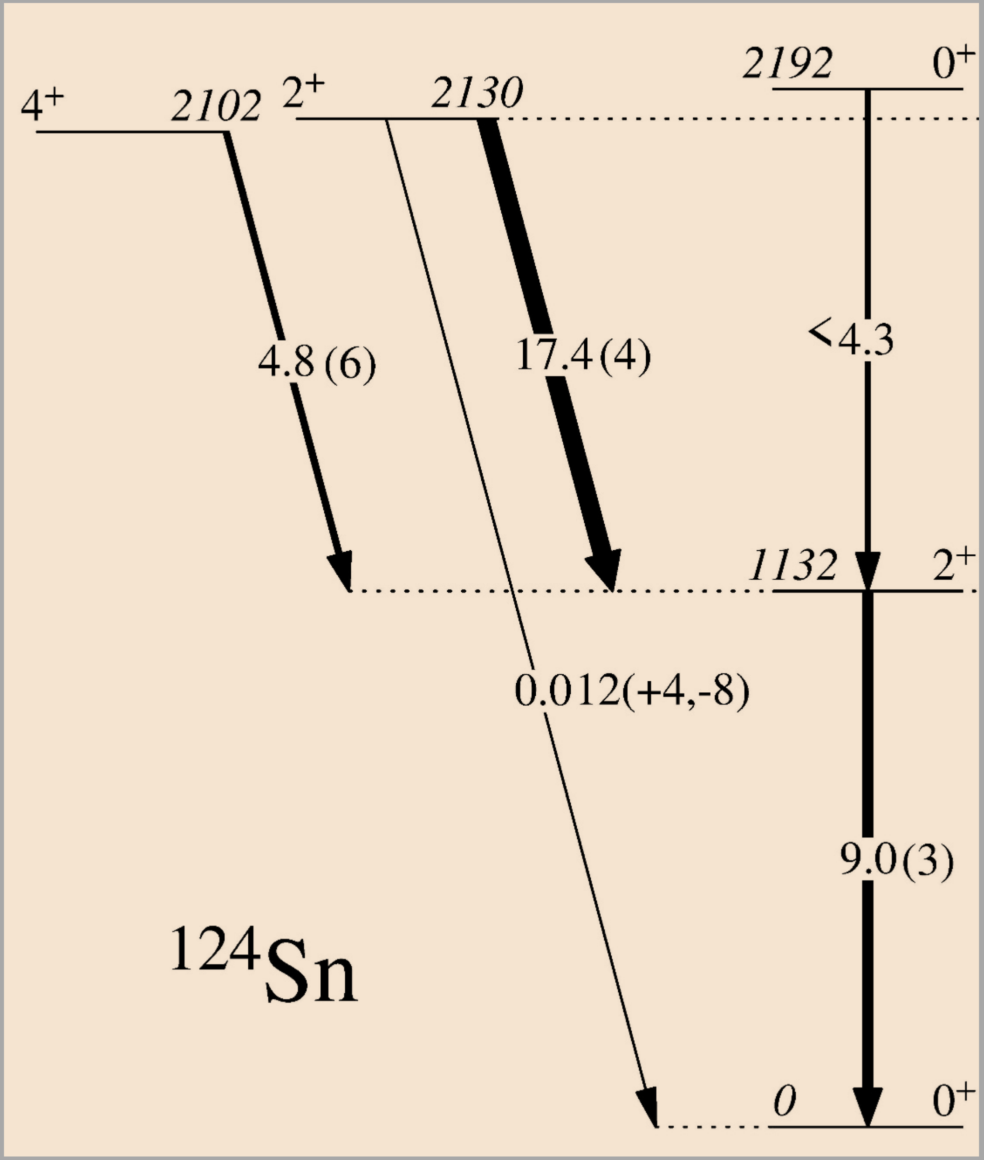
- **Excitation spectrum – existence of a *full* set of two-phonon states, and perhaps even 3-phonon states**
- **E_4/E_2 ratio approximately 2**
- **Energies could be fit with the U(5) energy formula**
- **The $\Delta N = 1$ $E2$ transitions strongly favoured over possible decays**

Now consider expanded criteria:

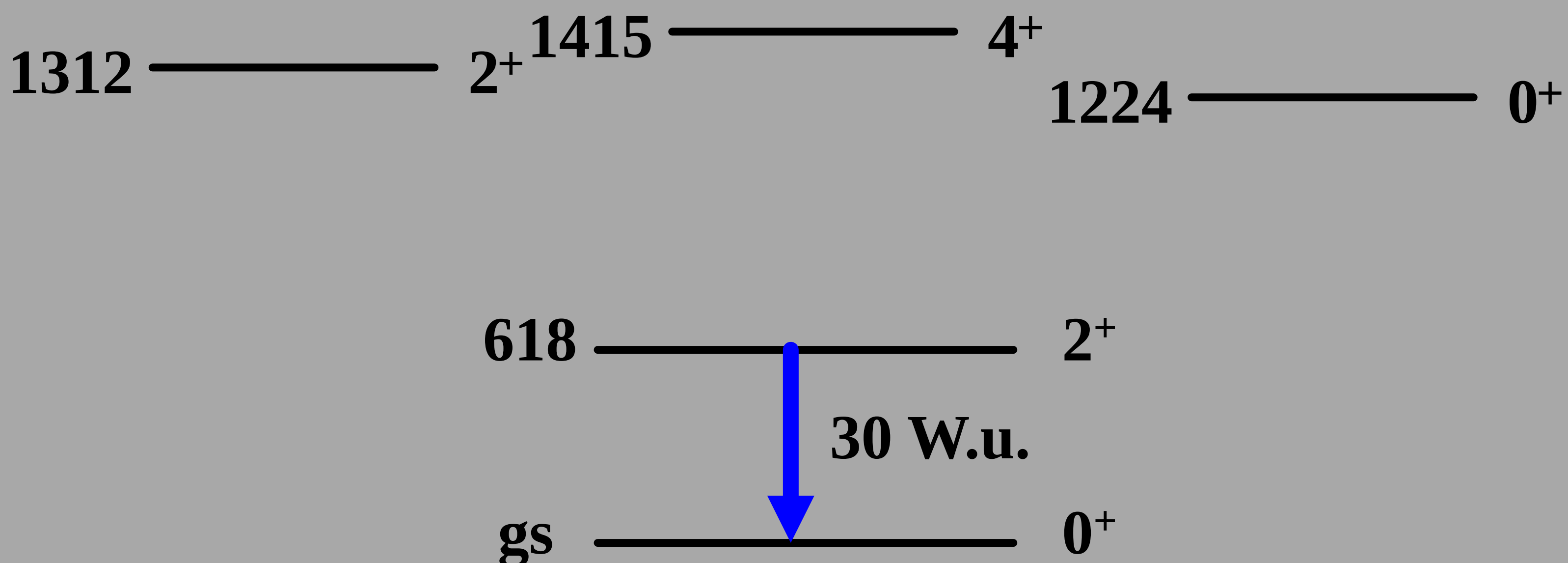
- **Smooth evolution of states as a function of *A***
- **Enhanced set of $B(E2)$ values between phonon states**
- **Deformation parameters extracted from Coulomb excitation or inelastic scattering follow expectations**
- **Consistent transfer results**
  - **One-phonon states may be strongly populated in SNT, but multiphonon should have (ideally) zero spectroscopic strengths**
  - **Weak populations in two-nucleon transfer**

Why are more stringent criteria needed?

- **Considering only energies and branching ratios would lead to conclusion that ^{124}Sn is a good harmonic vibrational nucleus**
  - **E(4^+)/E(2^+) ratio is 1.86**
  - **Energy spread of 2-phonon triplet is only 90 keV**
  - **Relative *B*(*E*2) strongly favour decay to one-phonon 2^+ state**
- **Absolute *B*(*E*2) values immediately rule out harmonic vibrations**



e.g. levels near 1.3 MeV in ^{112}Cd – U(5) candidate



e.g. levels near 1.3 MeV in ^{112}Cd – U(5) candidate

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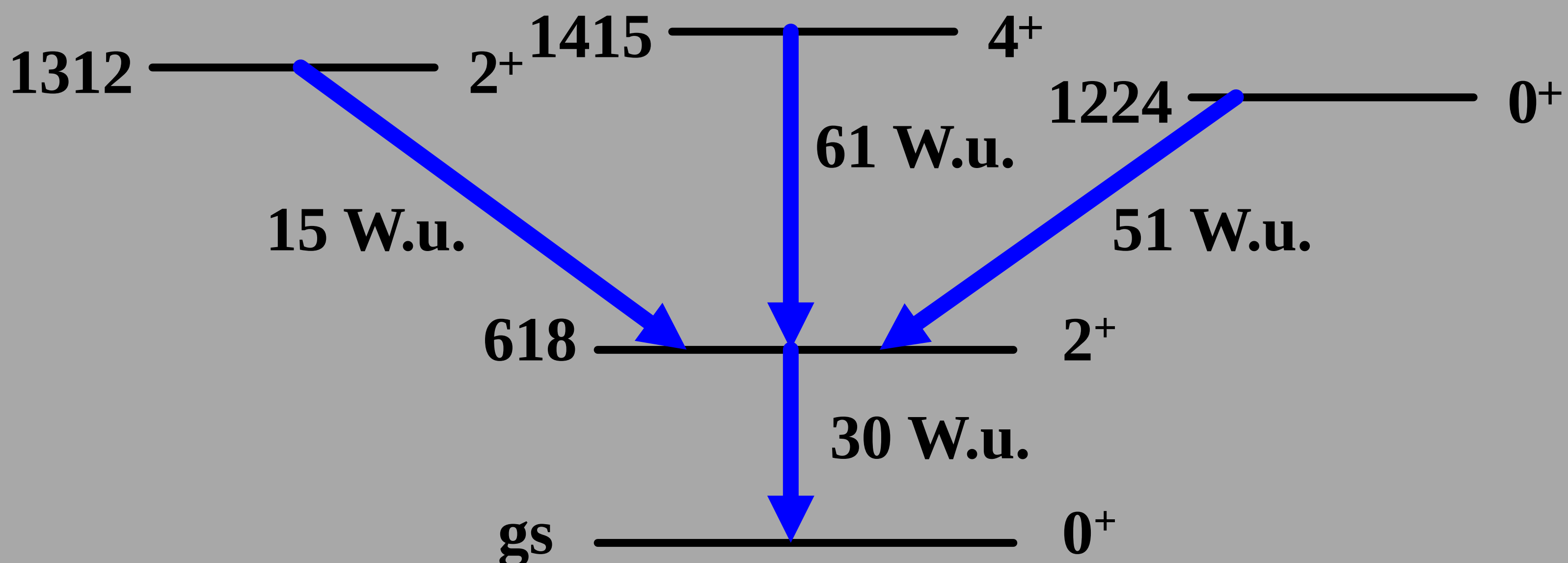
- **Appear to have the right levels and decays**



But there are extra states in the vicinity of 2-phonon states… e.g. ^{112}Cd – U(5) candidate

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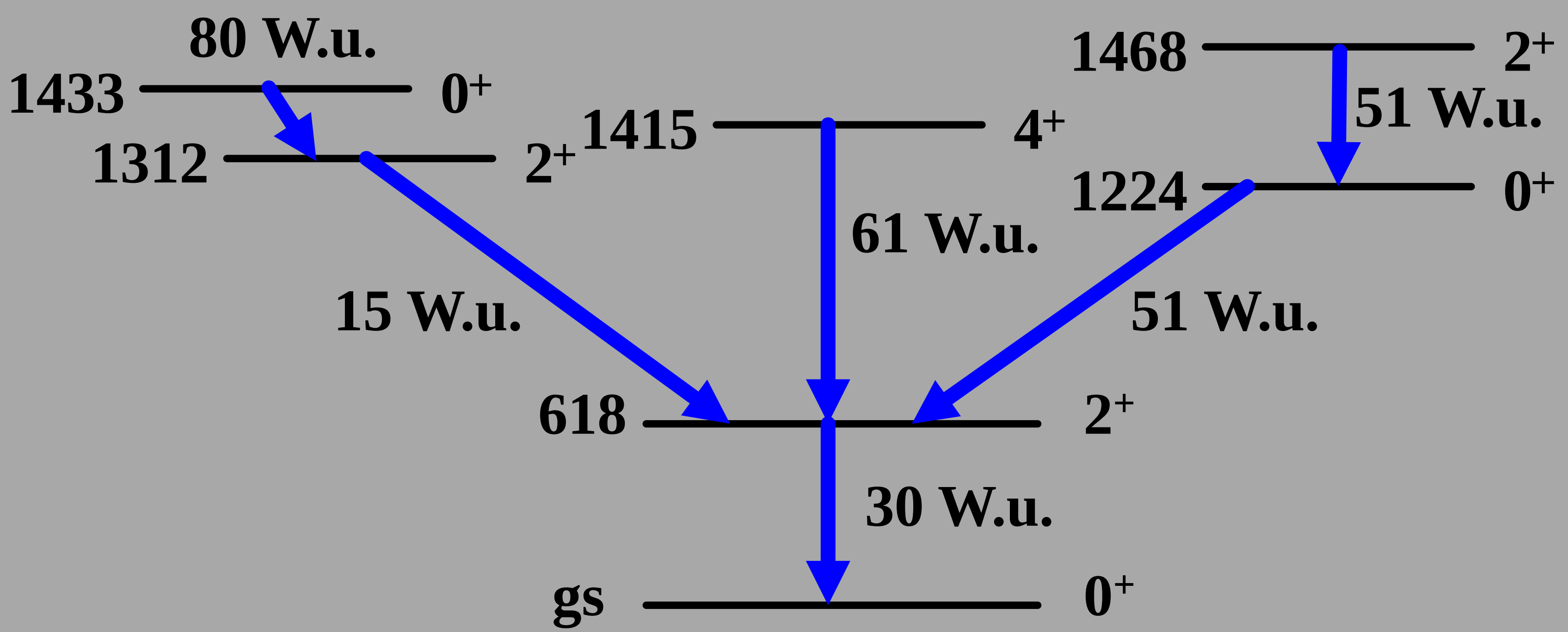
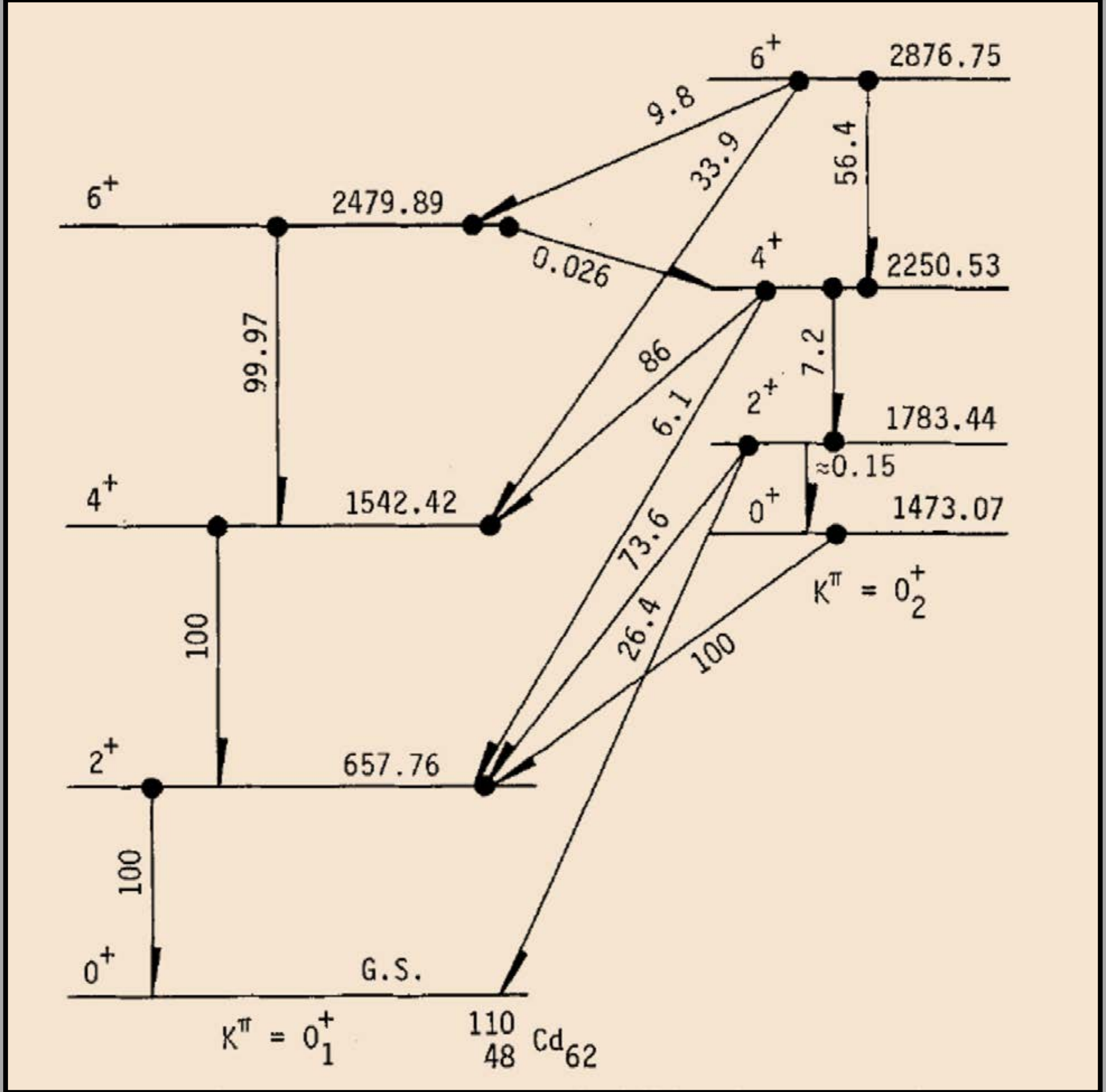
- Appearance of additional 0^+ and 2^+ states with enhanced *E*2 decays systematic in Cd isotopes near midshell



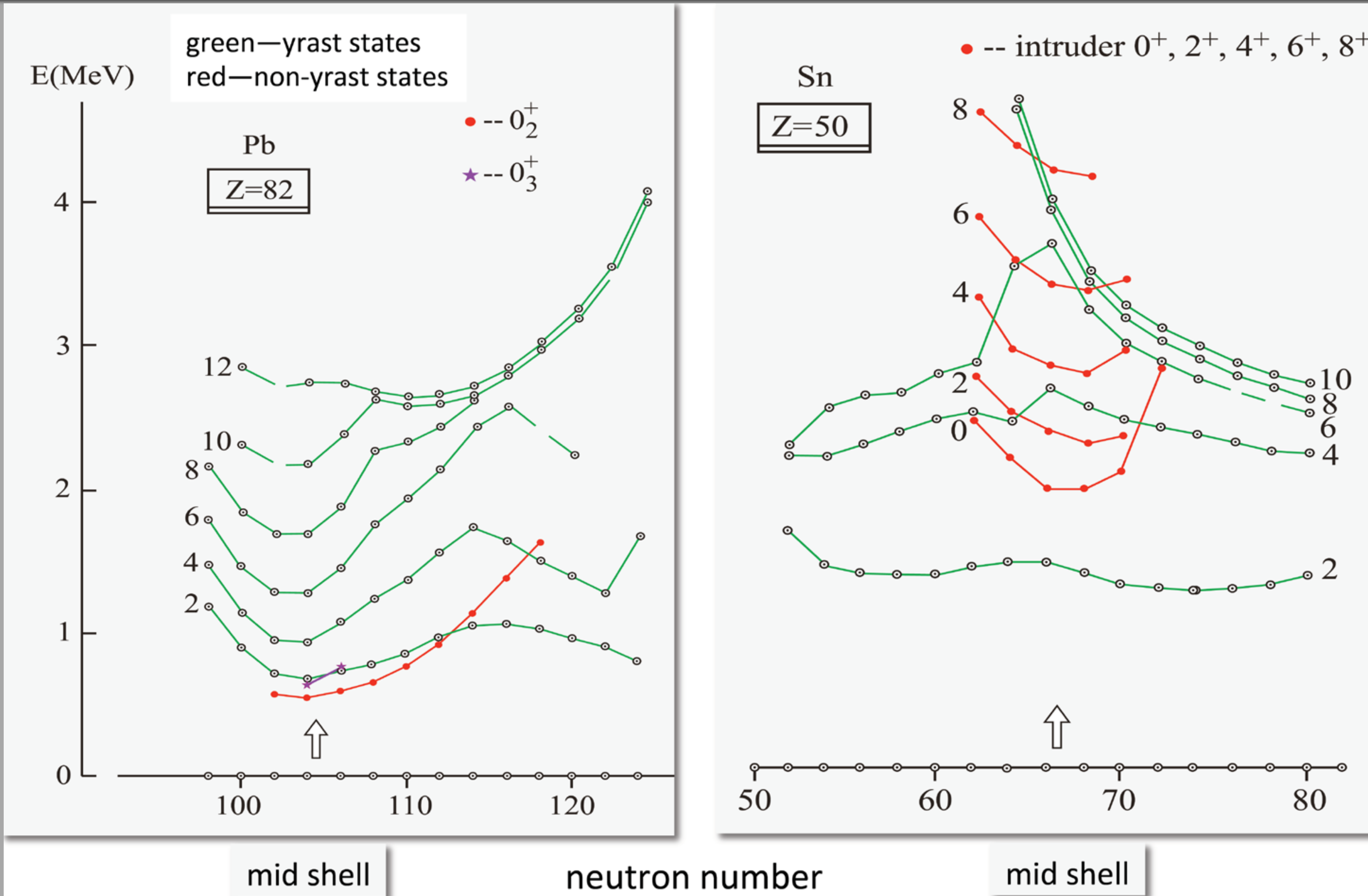
First firm evidence for deformed coexisting band in Cd isotopes – observed with β-decay

- Detailed spectroscopy on ^{110}Cd via β-decay reveals in-band transitions
- "Extra" states in vicinity of 2-phonon triplet explained as part of "intruder" band

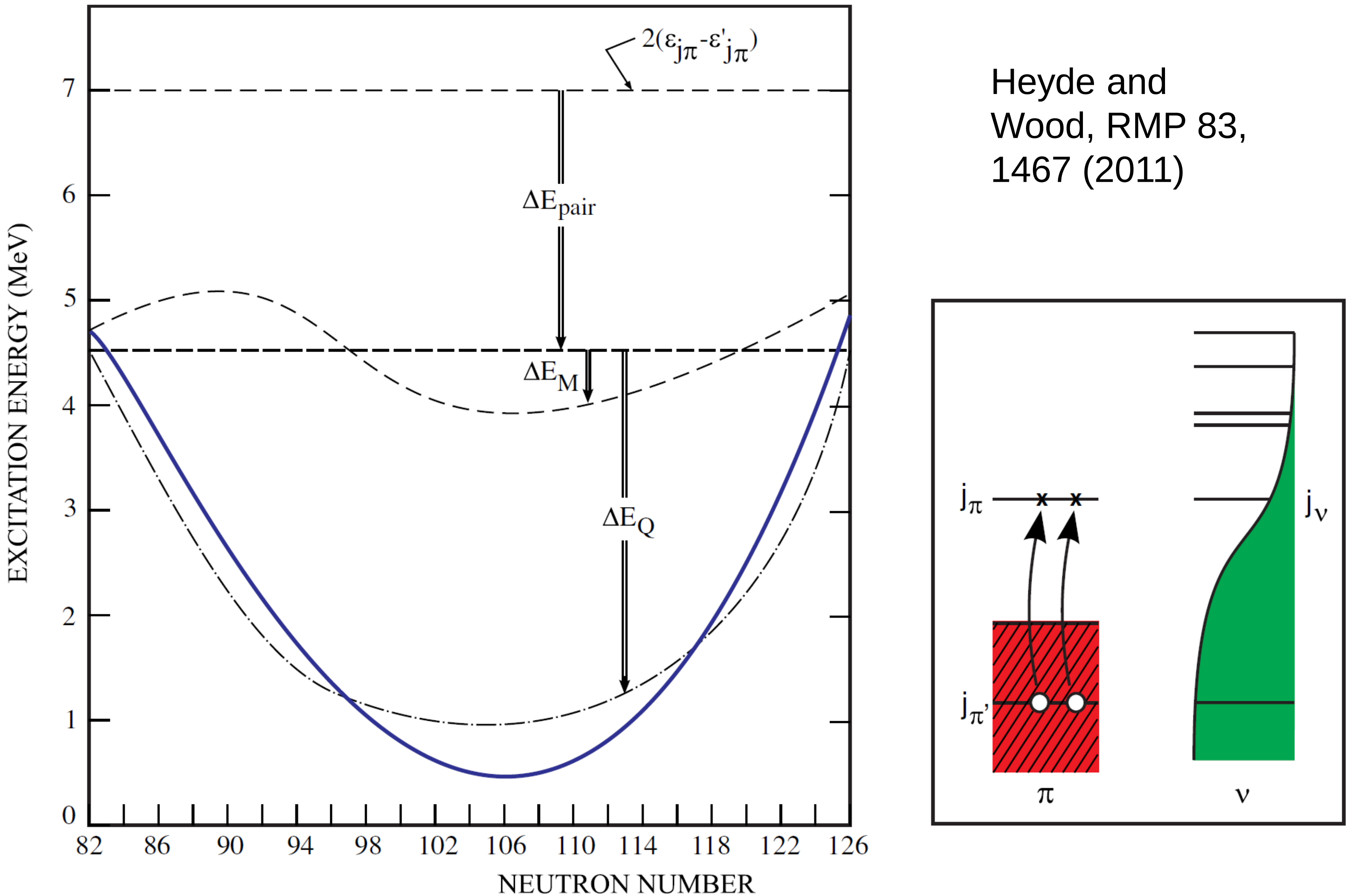
R. Meyer and L. Peker, Z.Phys. **A283**, 379 (1977)



Characteristic pattern of deformed intruder bands at closed shells



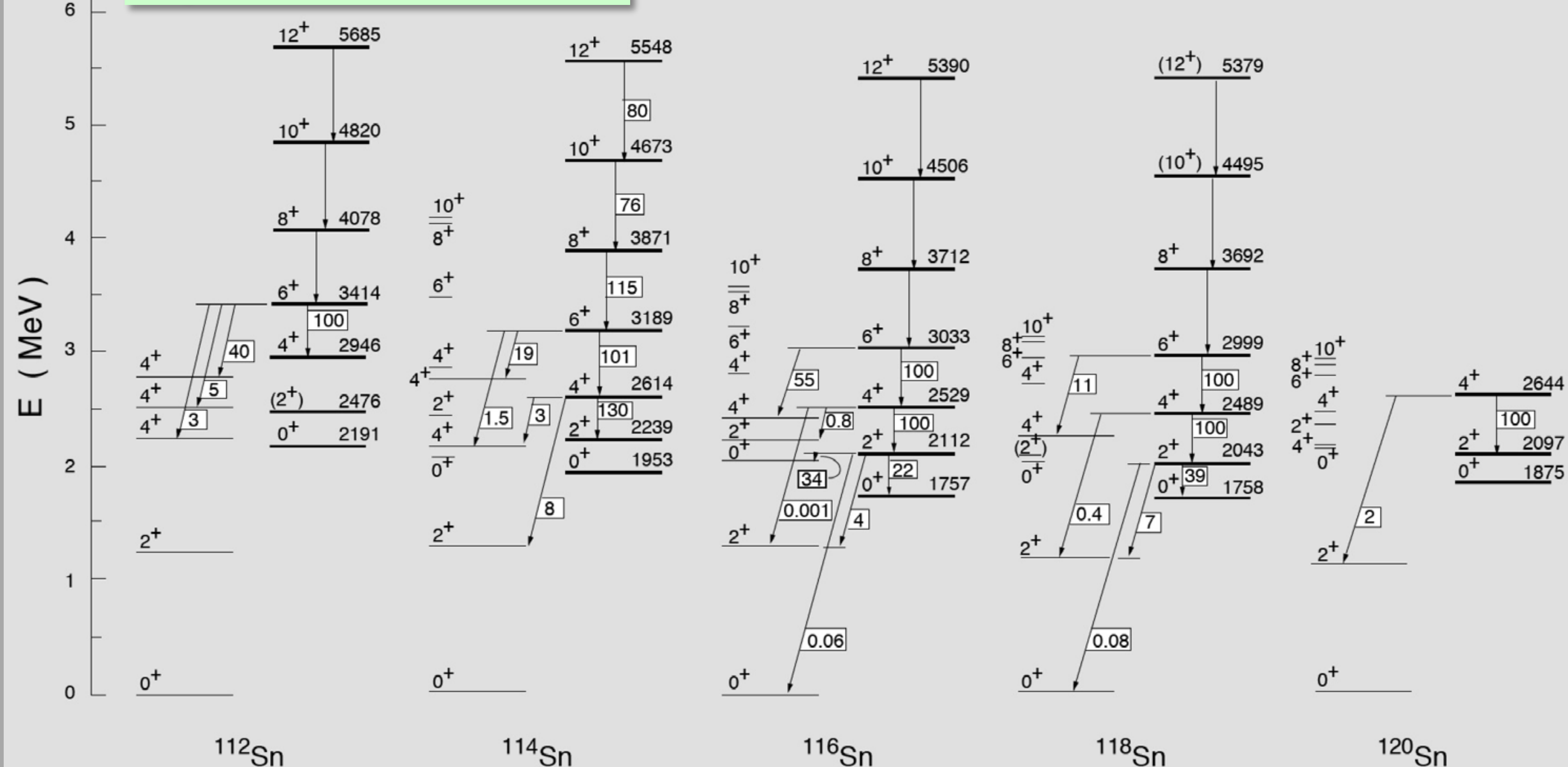
Characteristic "V"-shaped pattern driven by quadrupole interactions



Heyde and Wood, RMP 83, 1467 (2011)

Deformed bands in the even-even Sn isotopes

Figure from Rowe & Wood



Evidence for intruder states – *2p-2h* proton excitation

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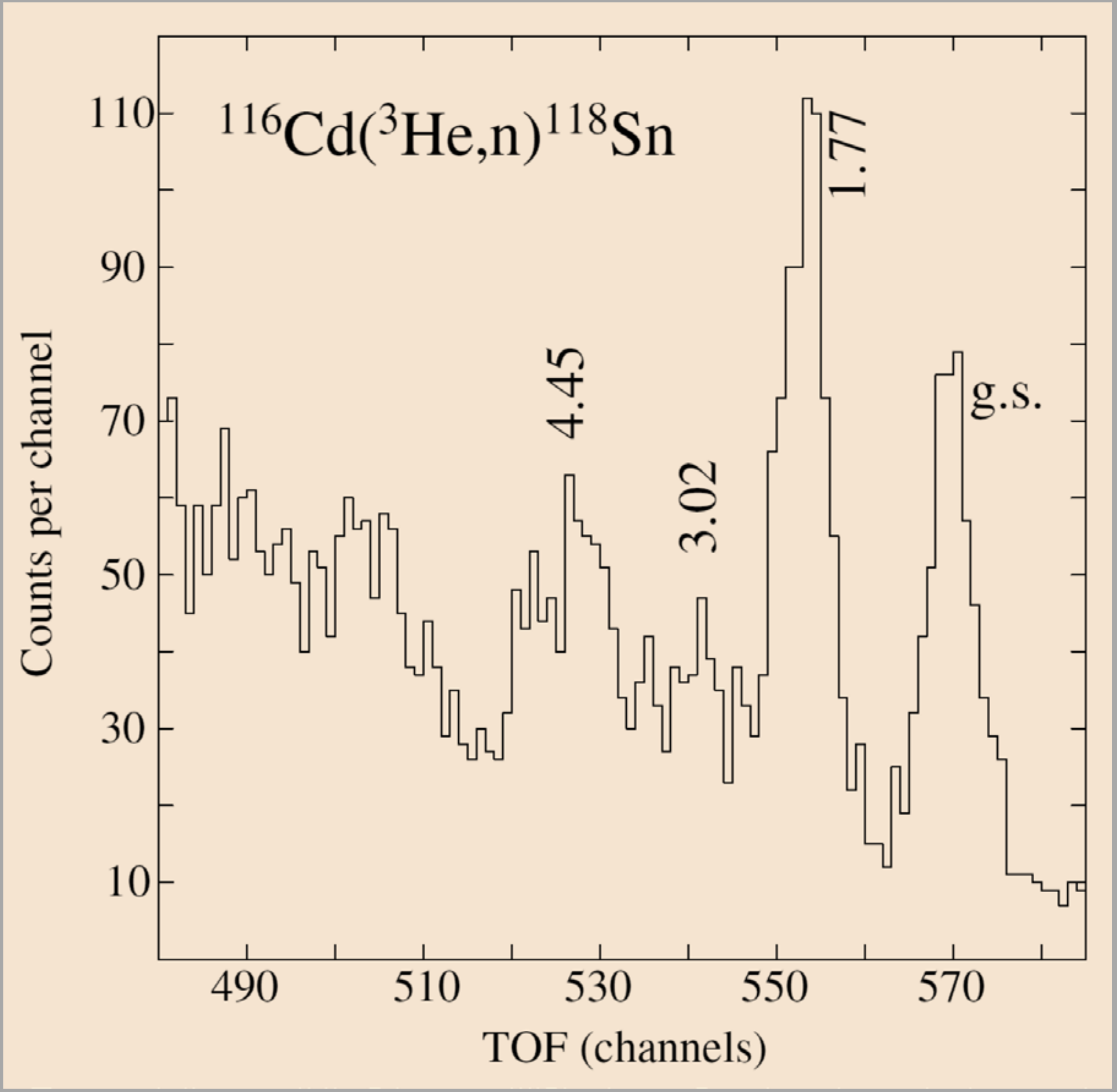
- **In normal or superfluid nuclei, the two-nucleon-transfer should be dominated by ground state–to ground state transitions – typically >95% of *L*=0 strength to the ground state**

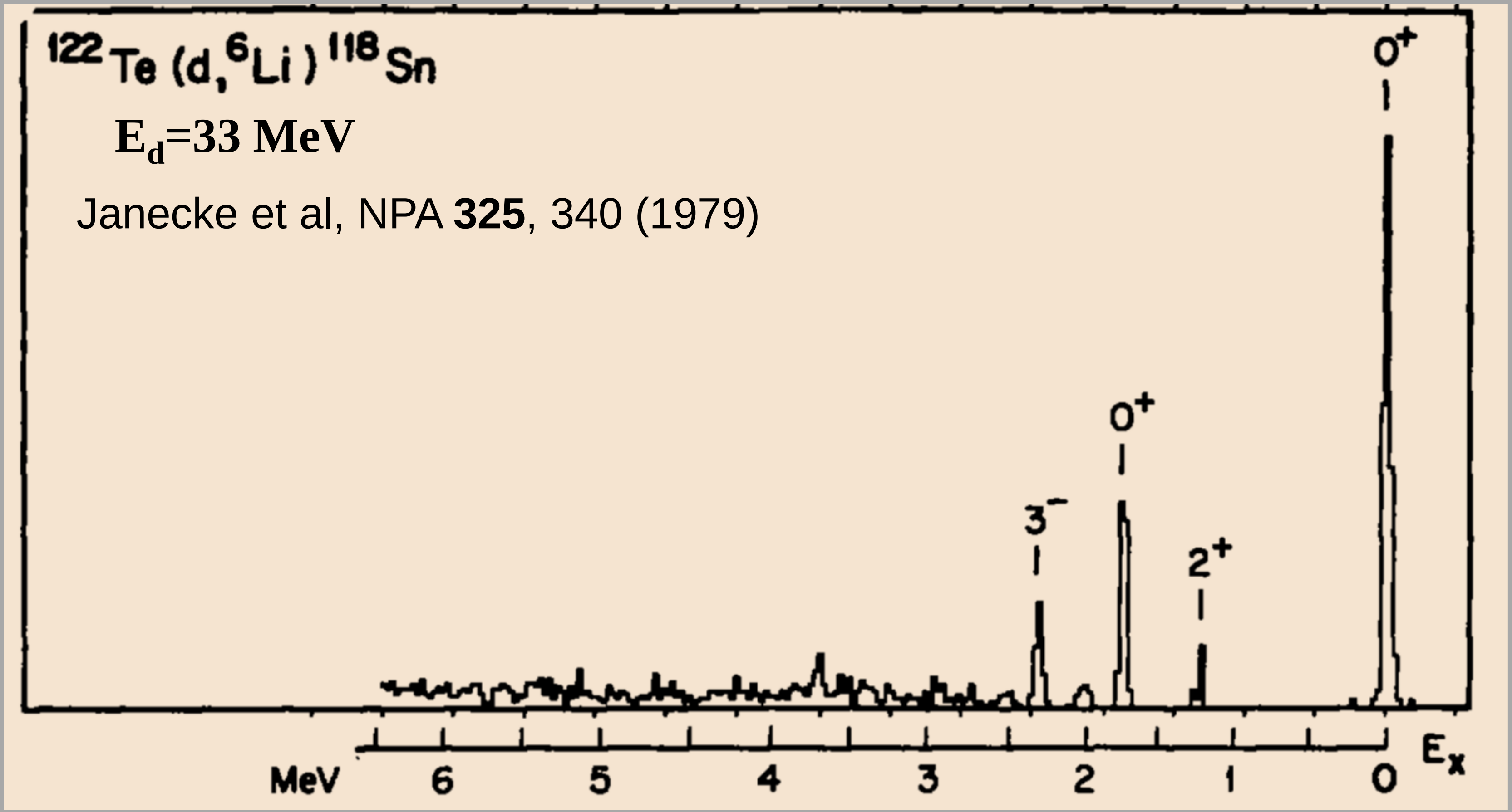
- **Near *Z*=50, two-proton transfer strongly populates excited 0^+ state – reminiscent of proton pairing vibration – 2*p*-2*h* excitation across *Z*=50 closed shell**



Fielding et al., Nucl. Phys. **A281**, 392 (1977)

Further evidence for proton *2p-2h* character from α-particle transfer

- **Te target – protons in 2*p* state**
- **Removal of α particle favours population of 0*p*-0*h* (gs) or 2*p*-2*h* state (intruder)**



Systematic studies of Cd isotopes reveal intruder configuration evolution

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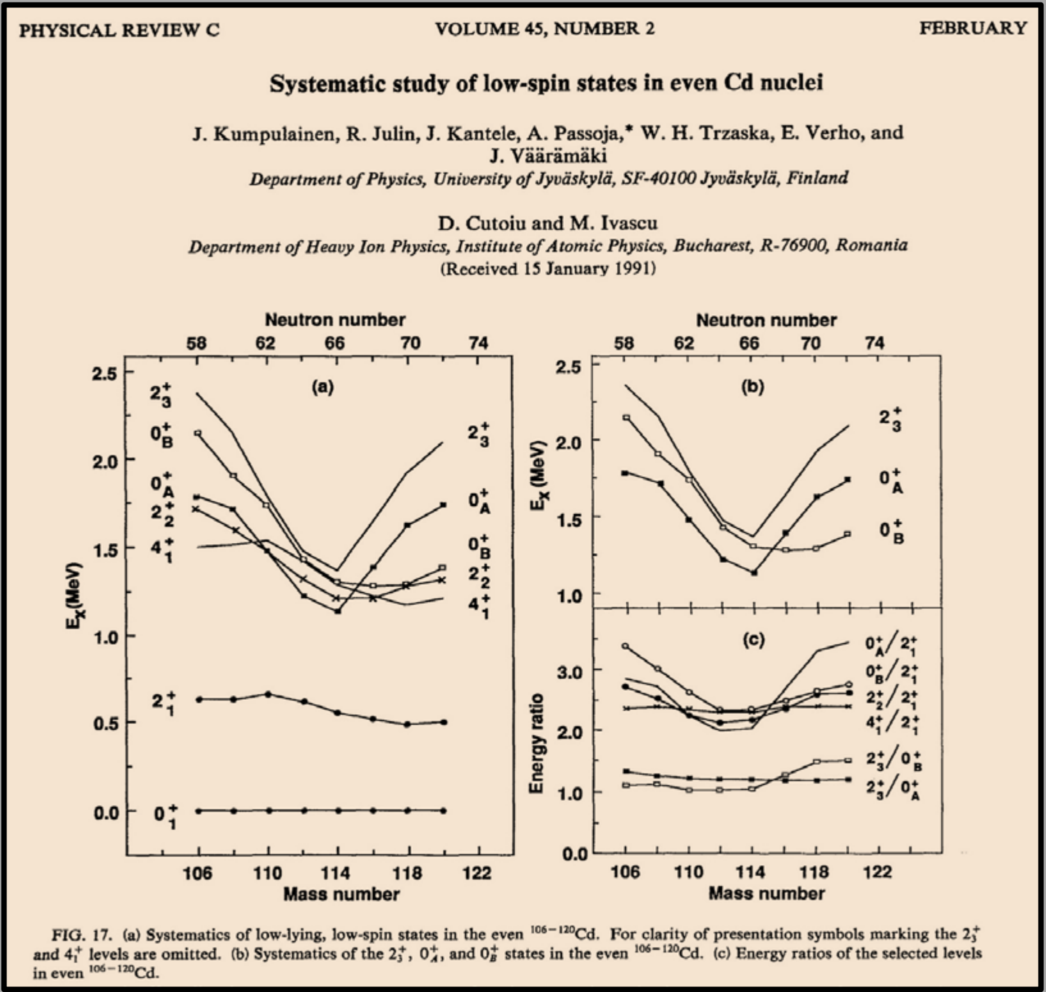

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Systematic study of low-spin states in even Cd nuclei

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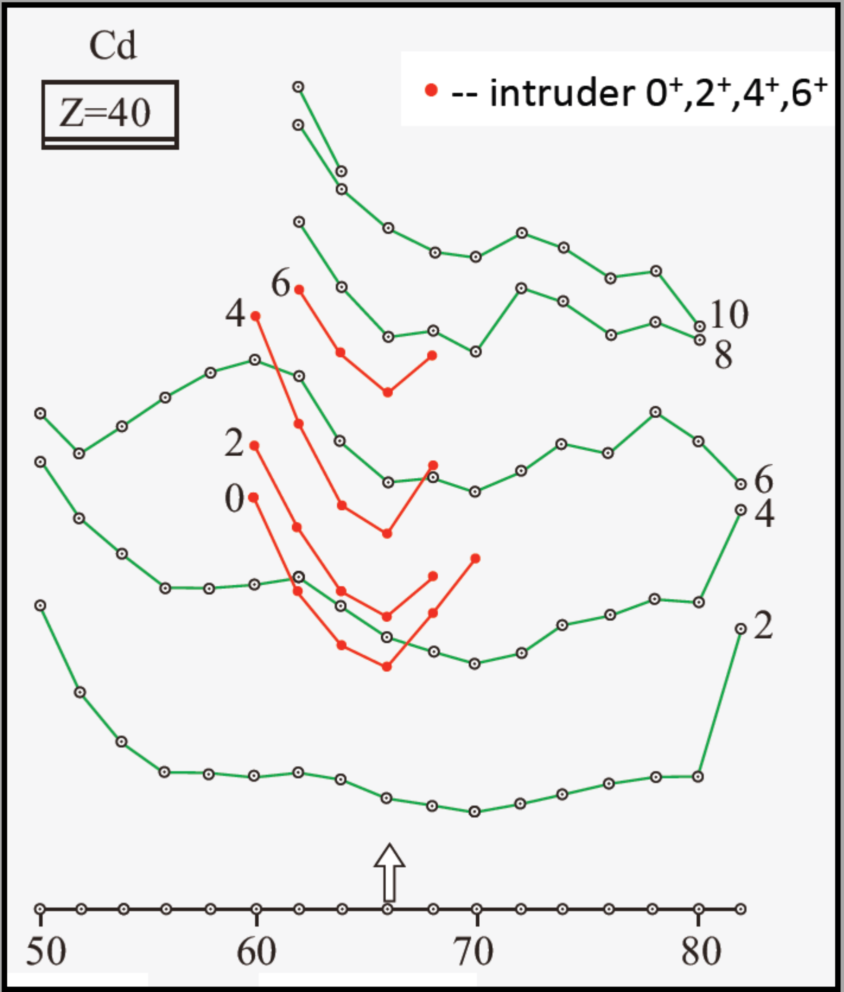
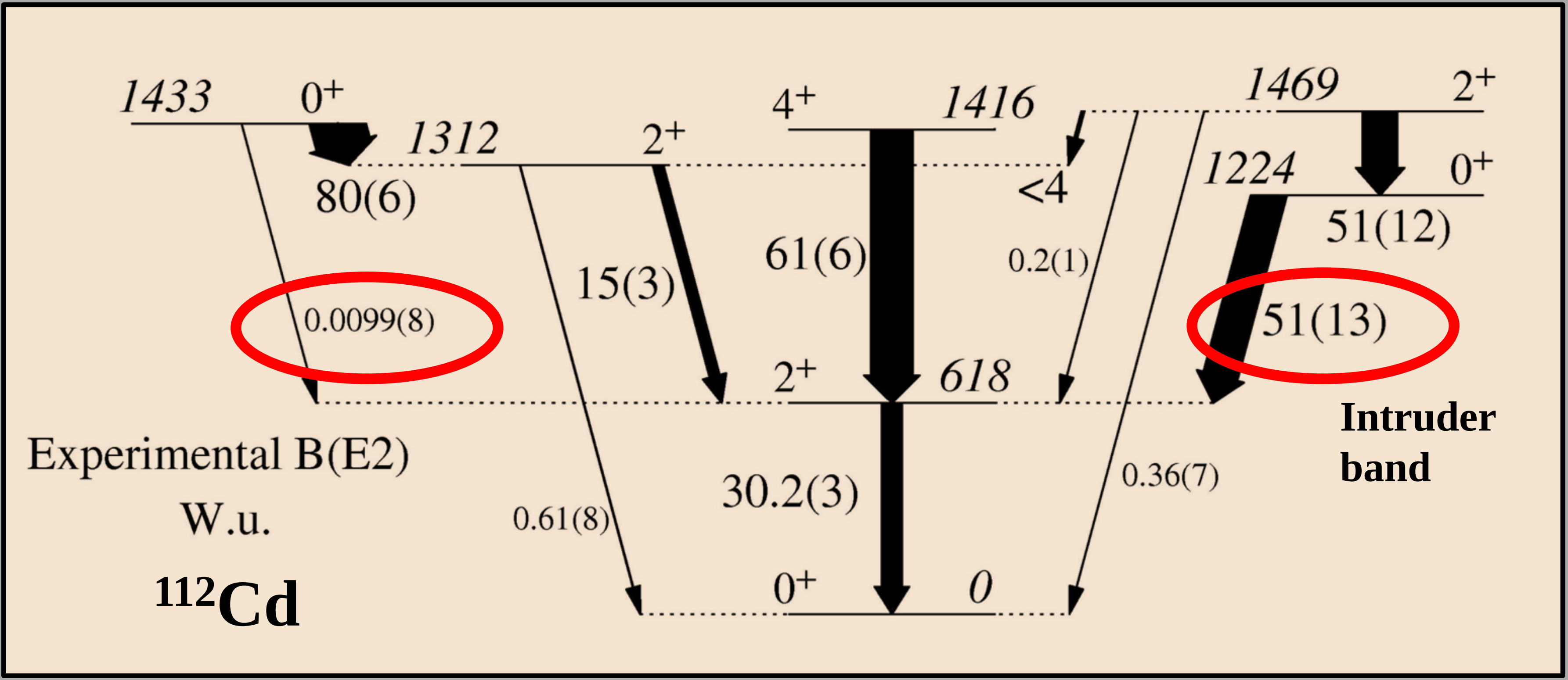
(Received 15 January 1991)

FIG. 17. (a) Systematics of low-lying, low-spin states in the even $^{106-120}$Cd. For clarity of presentation symbols marking the 2_3^+ and 4_1^+ levels are omitted. (b) Systematics of the 2_3^+, 0_A^+, and 0_B^+ states in the even $^{106-120}$Cd. (c) Energy ratios of the selected levels in even $^{106-120}$Cd.



Problem – decays of low-lying 0^+ states

- 0^+ intruder band head has enhanced decay to 1-phonon state, but 0^+ "two-phonon" does not – e.g. ^{112}Cd



Very small of $0_3^+ \rightarrow 2_1^+$ B(E2) and enhancement of 0_2^+ (shape coexisting band head) $\rightarrow 2_1^+$ B(E2) was explained as consequence of strong mixing between intruder configuration and "normal" phonon states

Systematics of $B(E2;0_{2,3}^+ \rightarrow 2_1^+)$ in $^{110-116}$Cd

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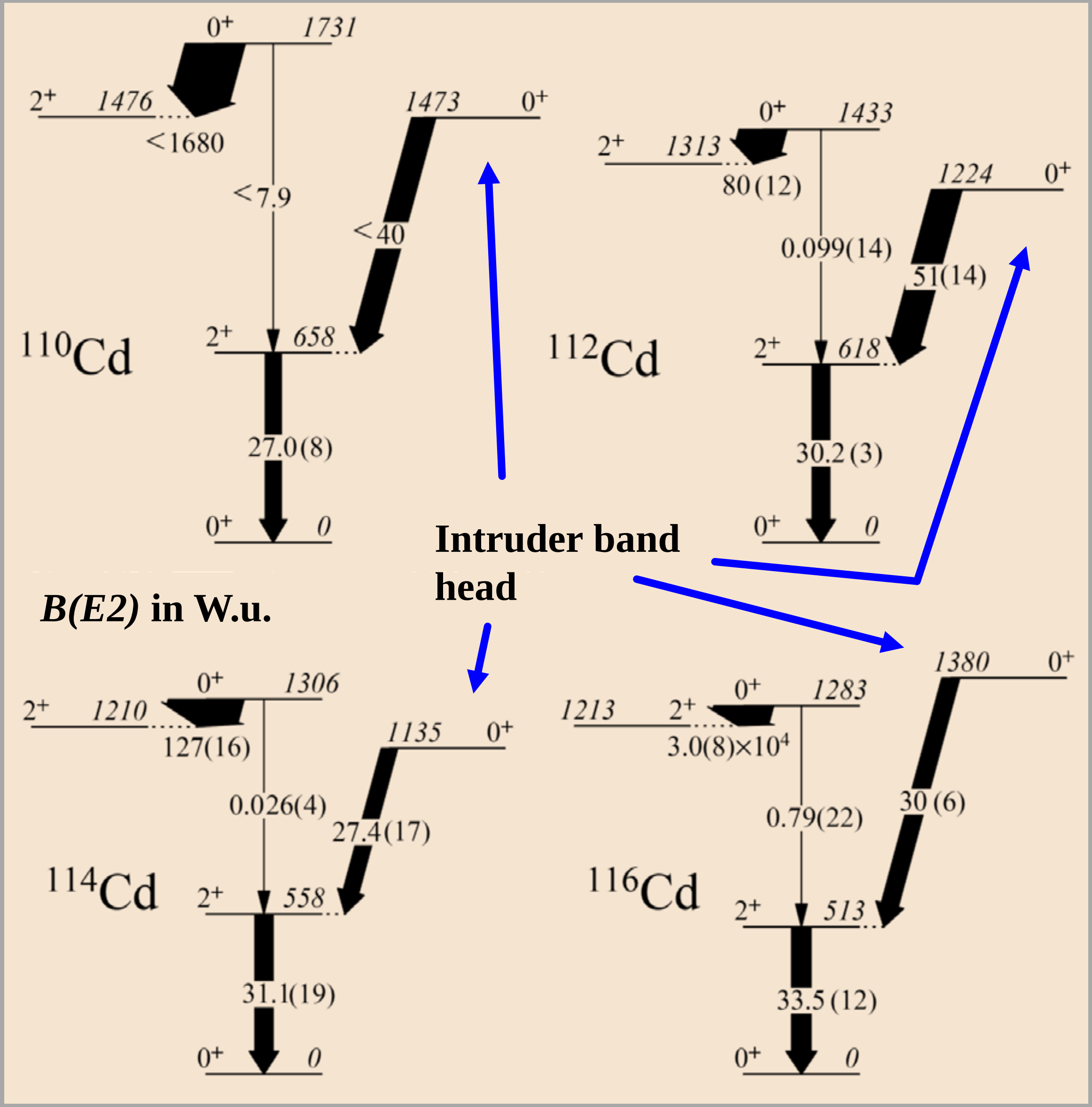

Transitions labelled with *B*(*E2*) values in W.u.

If mixing were the underlying cause of the decay pattern, would expect much larger variation in the B(E2) values

PG et al., Phys. Rev. C 86, 044304 (2012)

^{114}Cd deformation parameters from rotational invariants

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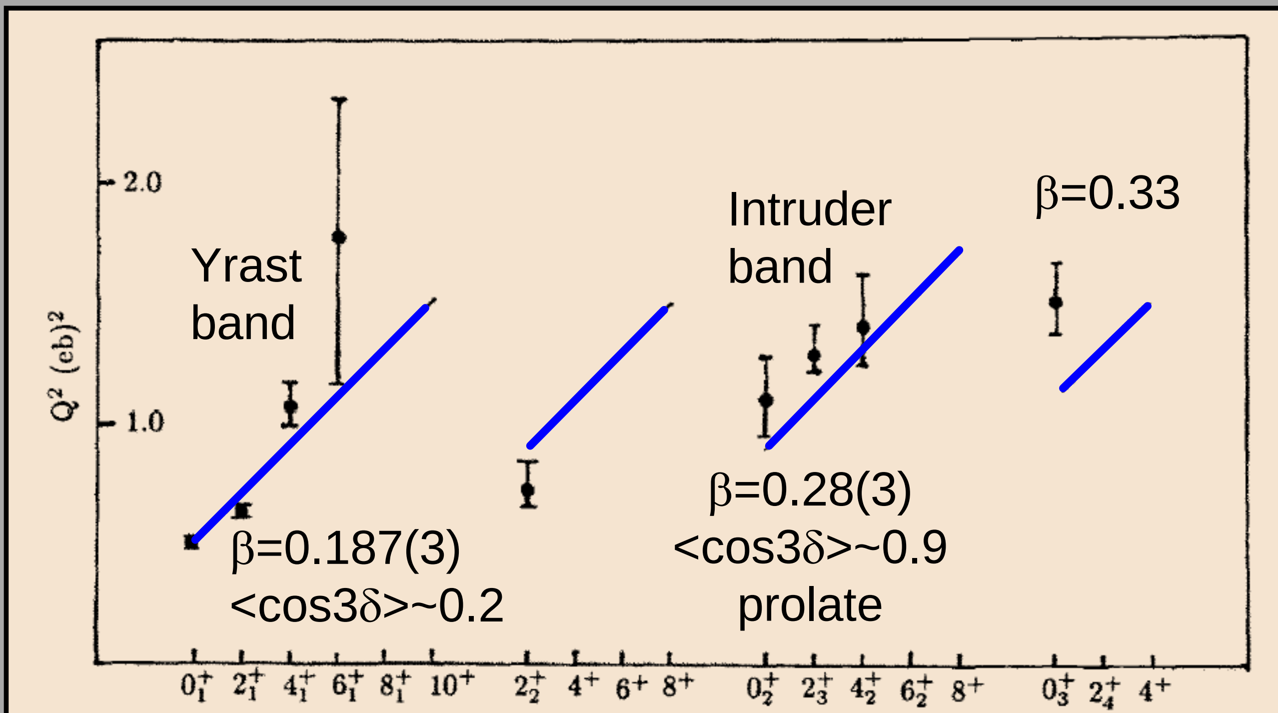
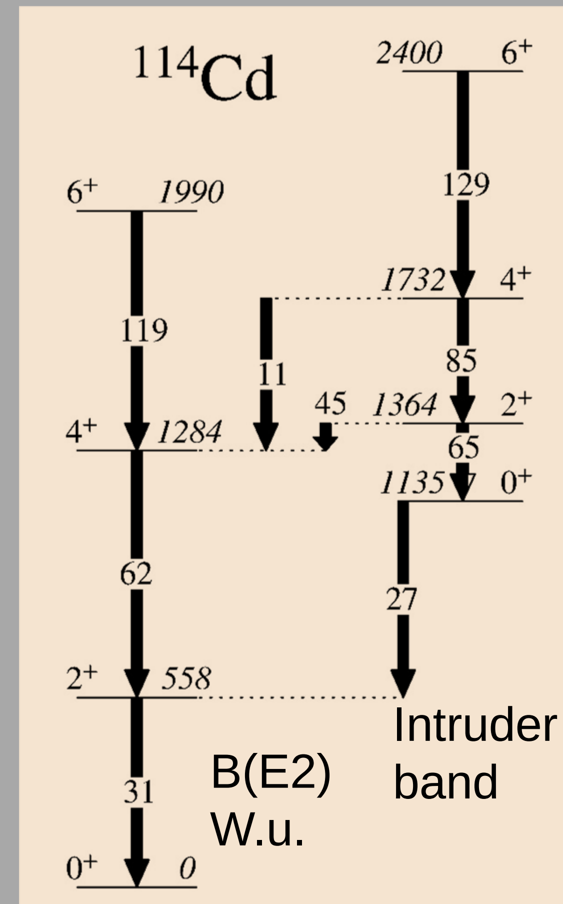
- Most detailed Coulex study to date on Cd isotopes [Fahlander, NPA **485**, 317 (1988)] ^{16}O, ^{40}Ca, ^{58}Ni, ^{208}Pb on ^{114}Cd





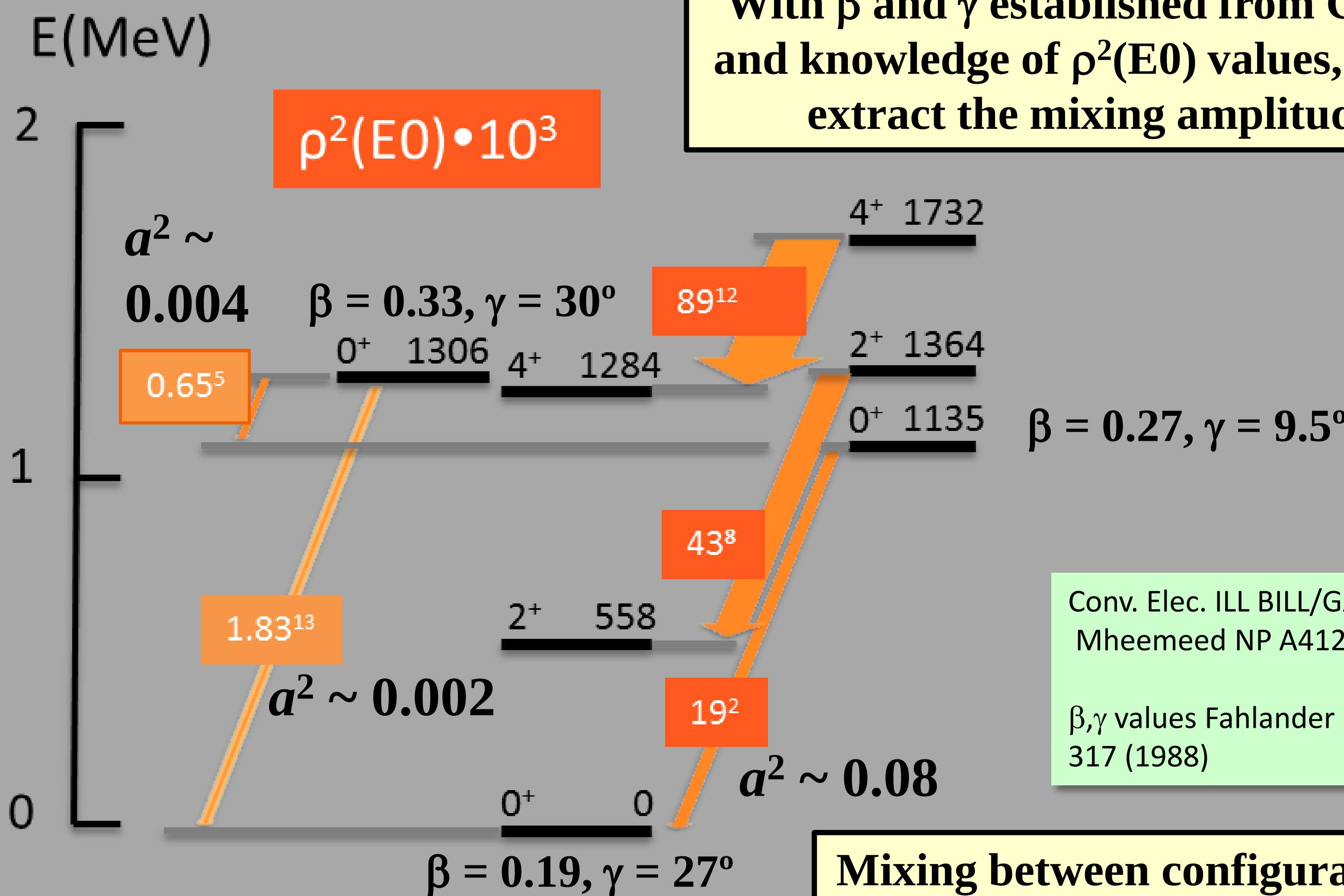
$$\frac{1}{\sqrt{5}}Q^2=\sum_i\langle 0\|M(E2)\|2_i\rangle\langle 2_i\|M(E2)\|0\rangle\begin{Bmatrix}2 & 2 & 0\\ 0 & 0 & 2\end{Bmatrix}$$

$$Q^2=\left(\frac{3}{4\pi}ZR_0^2\right)^2\beta_0^2$$

harmonic vibrator value

Analysis of 0$^+$ ρ^2(E0) values in ^{114}Cd

With β and γ established from Coulex, and knowledge of ρ^2(E0) values, we can extract the mixing amplitudes



Conv. Elec. ILL BILL/GAMS
Mheemeed NP A412 113 1984

β,γ values Fahlander NP A485, 317 (1988)

Mixing between configurations (generally) weak

What is the origin of the enhanced *E2* strength from the 0^+ intruder band head?

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- **While mixing is small, important consequences: Consider ^{114}Cd**

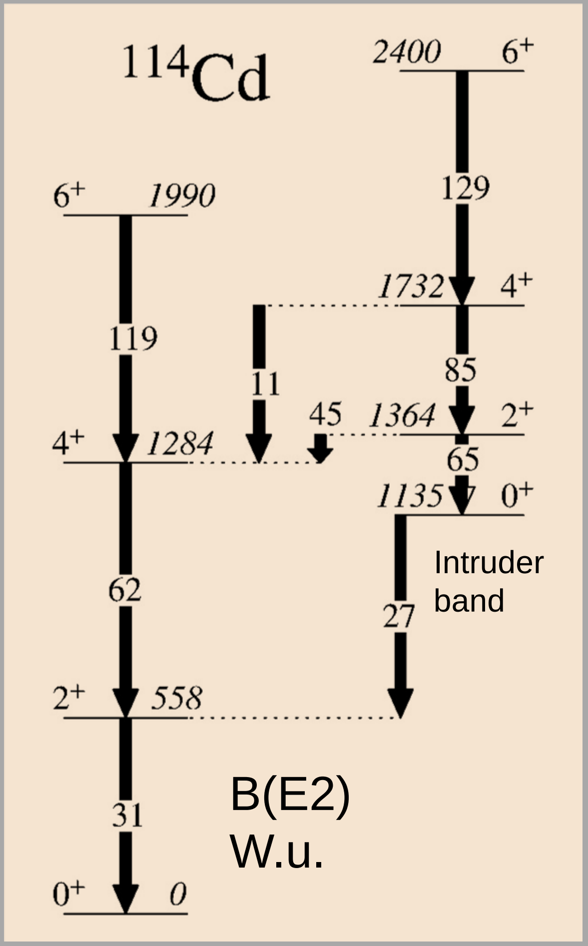
- **Write 0^+ wave functions**

$$|0^+_{gs}> = a|0^+_A> + b|0^+_B>$$

$$|0^+_I> = -b|0^+_A> + a|0^+_B>$$

- **Assume:**
  - **inband $2^+ \rightarrow 0^+$ transitions equal the observed (since weak mixing) $2_B^+ \rightarrow 0_B^+$ = 65 ± 9 W.u.**
  - **$2_B^+ \rightarrow 0_A^+ = 0$**

with admixture of 8% results in calculated $B(E2;0_2^+ \rightarrow 2_1^+)$ = 26 ± 4 W.u. consistent with observed value of 27.4 ± 1.7 W.u.



Important contribution to $0_2^+ \rightarrow 2_1^+$ *E*2 strength from mixing, although mixing is *weak*

Revealing the underlying structure

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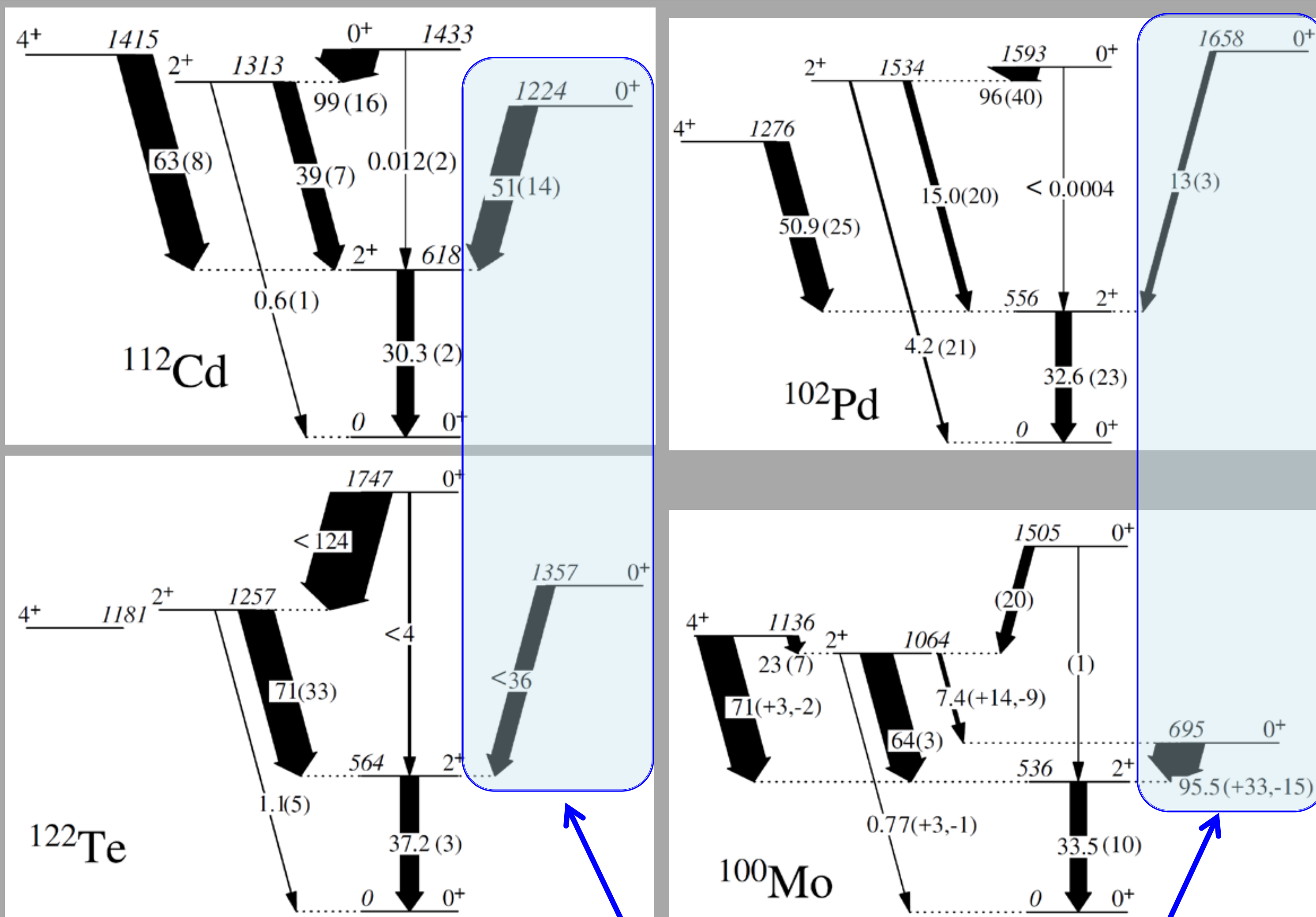


***B*(*E*2) in W.u.**

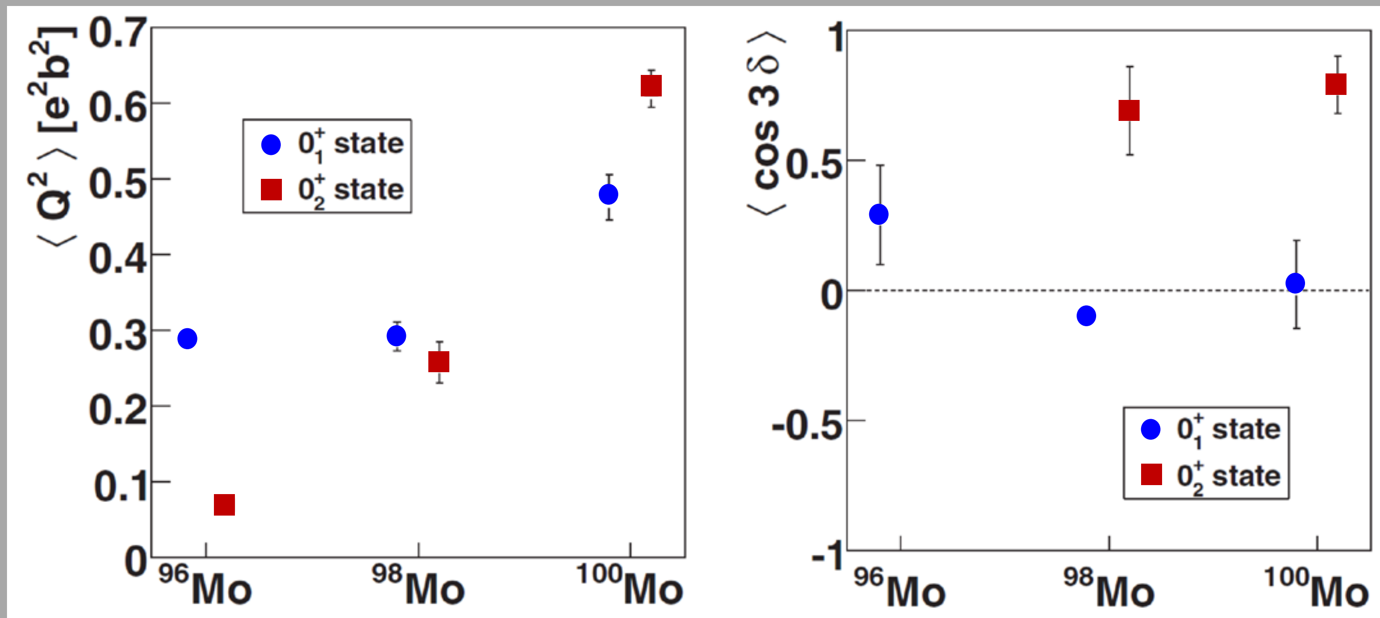
Shape coexisting structure

The favoured and enhanced decay of the excited non-intruder 0^+ state to the 2_2^+ state, rather than 2_1^+ state, common feature in Cd, Te, Pd, Ru, Mo, Xe,…

Structure of non-intruder states NOT that of a spherical vibrator

Extending towards Z=40 subshell – $^{98\text{-}102}$Mo U(5) candidates show clear shape coexistence

- **Detailed Coulomb excitation studies enable extraction of shape-invariants clearly indicating different shapes for 0_1^+ and 0_2^+ states**



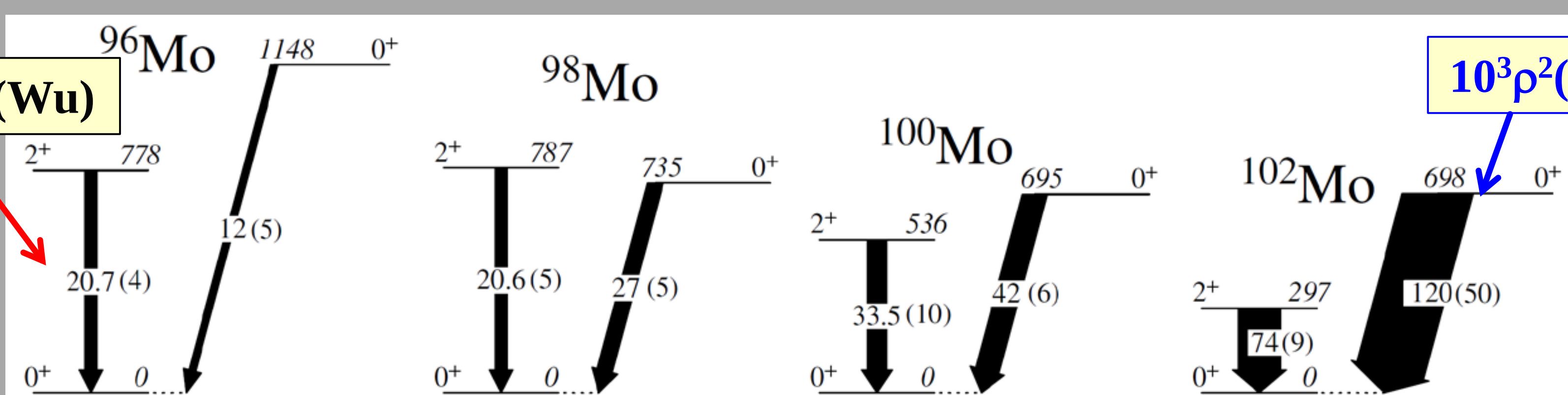
prolate

triaxial

oblate

Zielinska et al., NPA 712, 3 (2002), Wrzosek-Lipska et al., PRC 86 064305 (2012)

B(E2) (Wu)



$10^3\rho^2(E0)$

Are their any surviving candidates for near harmonic vibrational motion near $Z = 50$?

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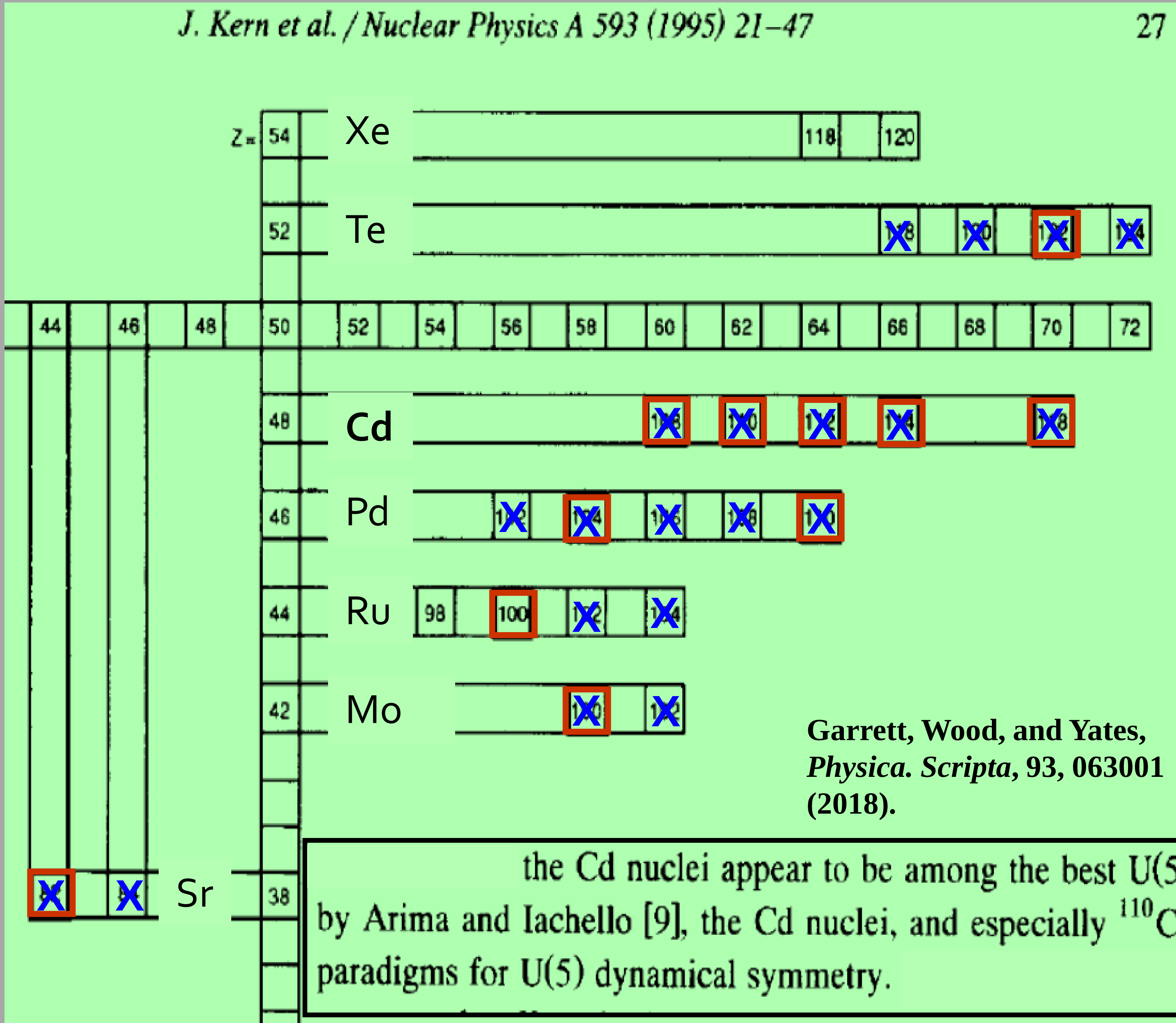

Combining data from multiple probes, we can rule our a harmonic vibrational picture for most of the vibrational candidates

If these nuclei are not spherical vibrators, what are their underlying structures?

Garrett, Wood, and Yates, *Physica. Scripta*, 93, 063001 (2018).

the Cd nuclei appear to be among the best U(5) candidates. As proposed by Arima and Iachello [9], the Cd nuclei, and especially ^{110}Cd, can still be regarded as paradigms for U(5) dynamical symmetry.

Cd 0^+ energy systematics

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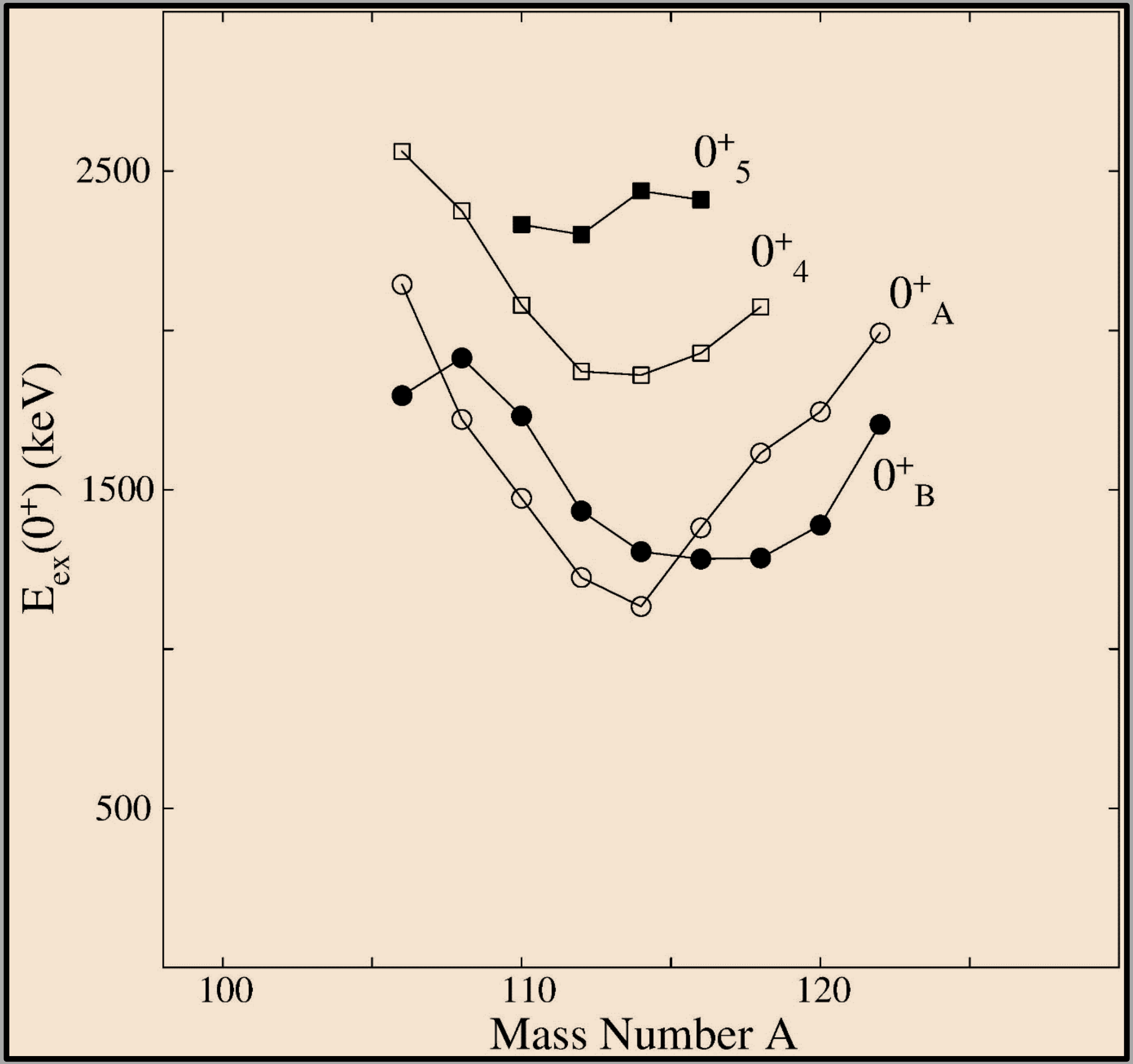
- 0_A^+ level was considered as $\pi 2p\text{-}4h$ intruder band head
- 0_B^+ level was considered as 2-phonon vibrational state mixed with intruder
- 0_4^+ also displays a "V"-shaped pattern
- Behaviour unexpected for phonon excitations

Garrett *et al.*, PRC **78**, 044307 (2008)



0_4^+ level preferentially decays to 2^+ intruder band member in the Cd isotopes

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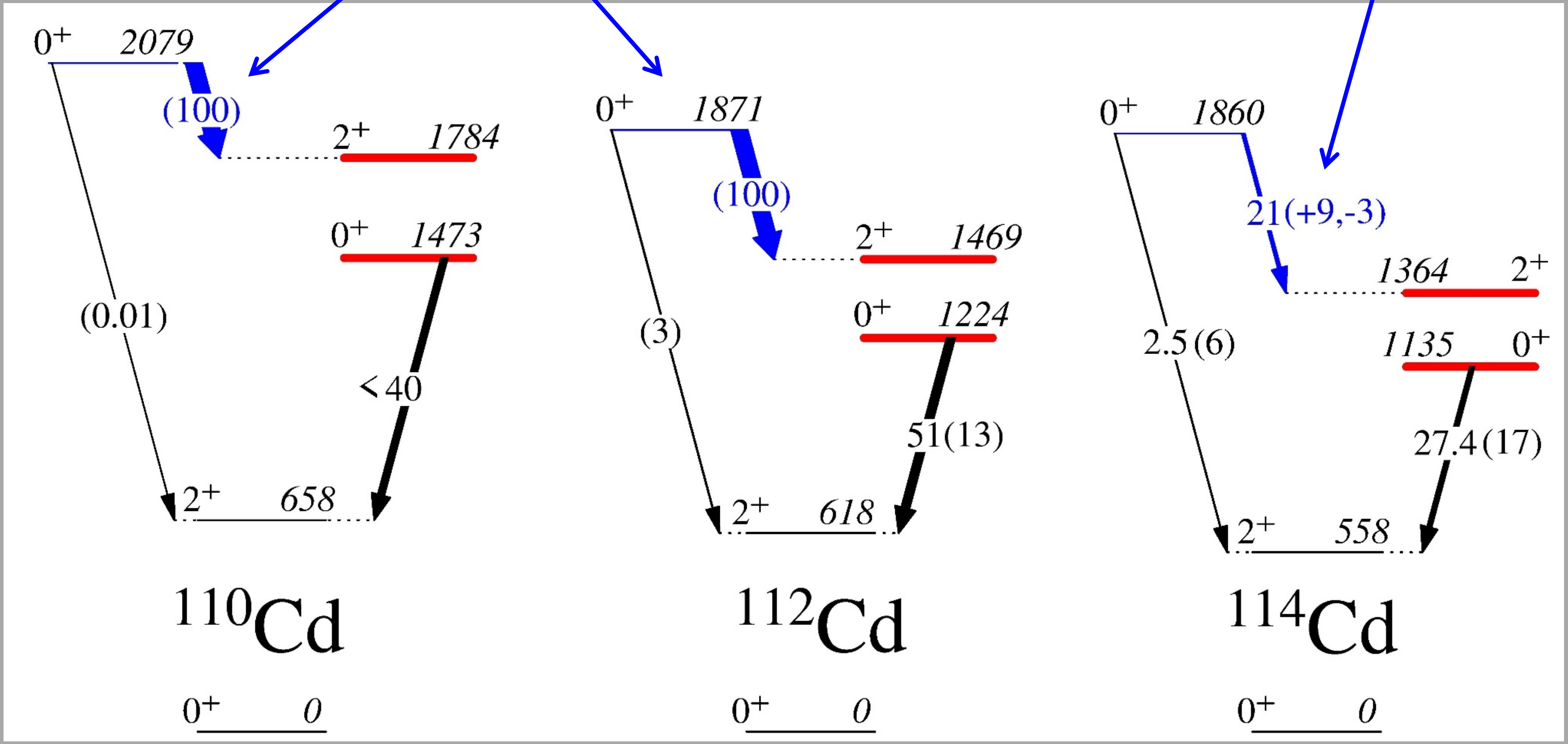
- **Enhanced $0_4^+ \rightarrow 2_3^+$ decay observed in ^{114}Cd**

Relative B(E2)values

Absolute B(E2) in W.u.



Cd isotopes systematically studied with β decay and (*n*,*n*′γ) reaction

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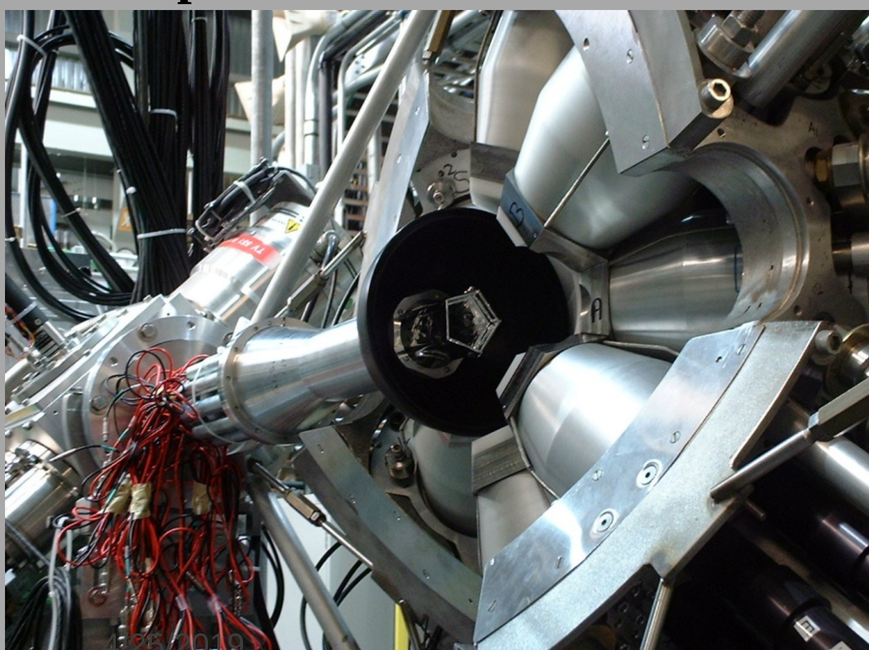
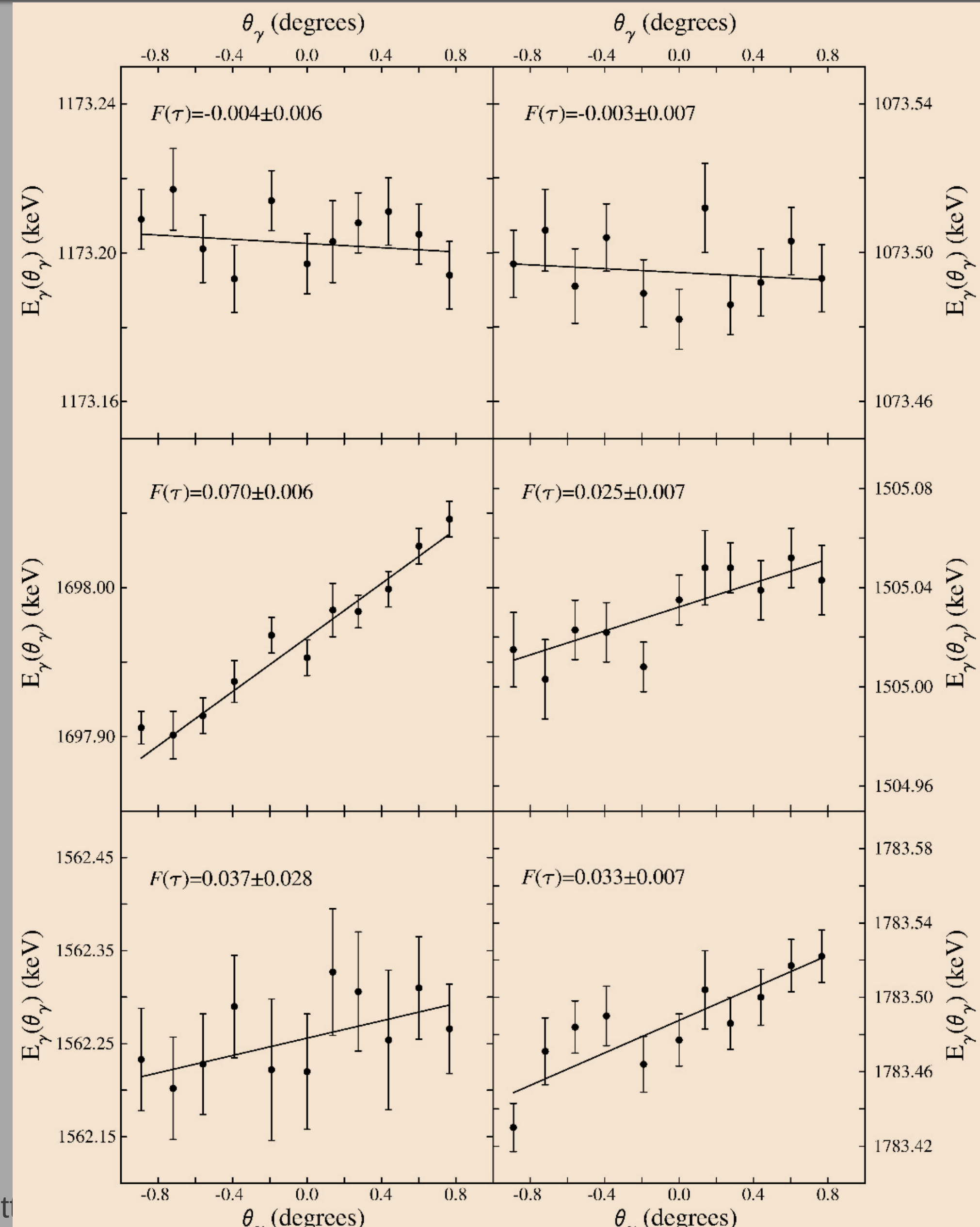
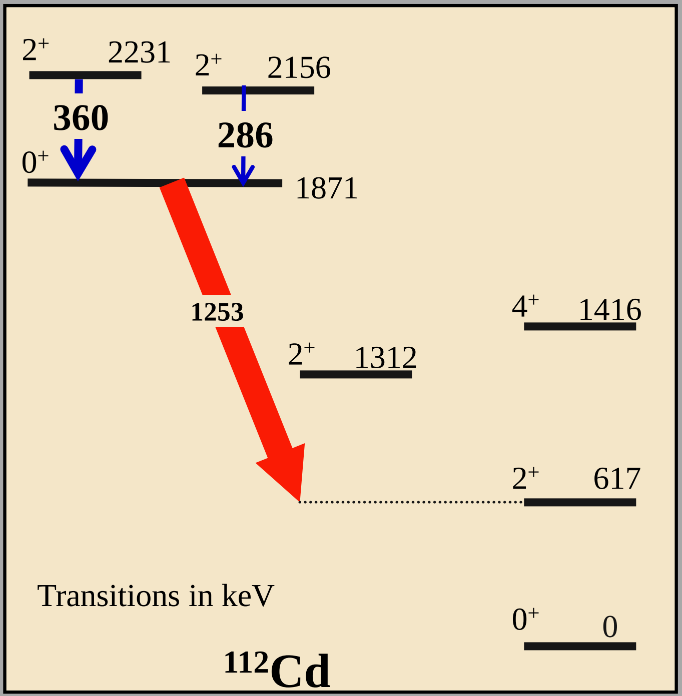
- **Stable even-even Cd isotopes studied with neutron inelastic scattering at University of Kentucky**
  - **Detailed spectroscopy of low-spin states**
  - **Lifetimes from DSAM**
- **^{110,112}Cd studied via β-decay using 8π spectrometer at TRIUMF-ISAC**

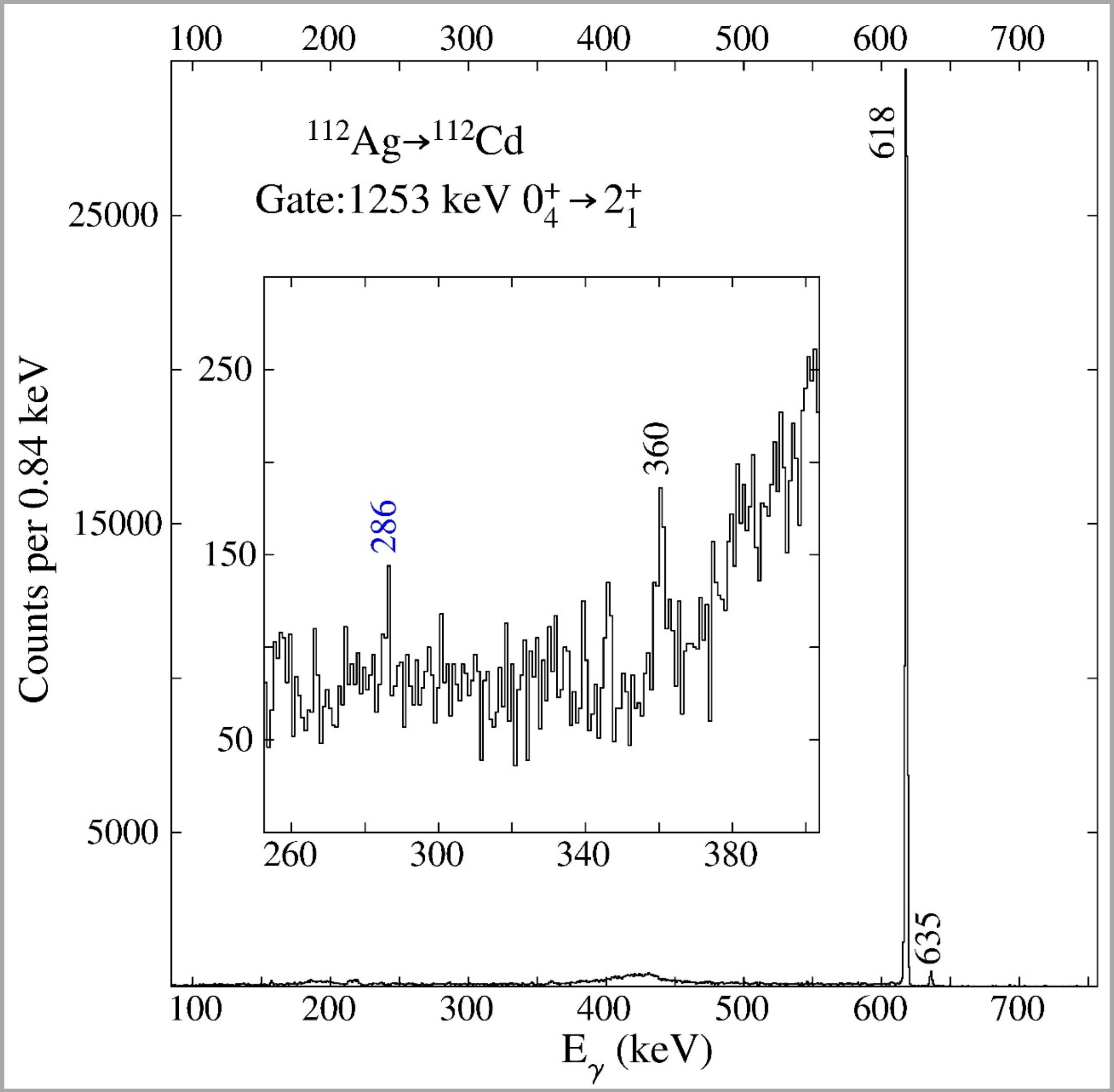




Search for very weak, low-energy γ branches in ^{112}Cd via β-decay

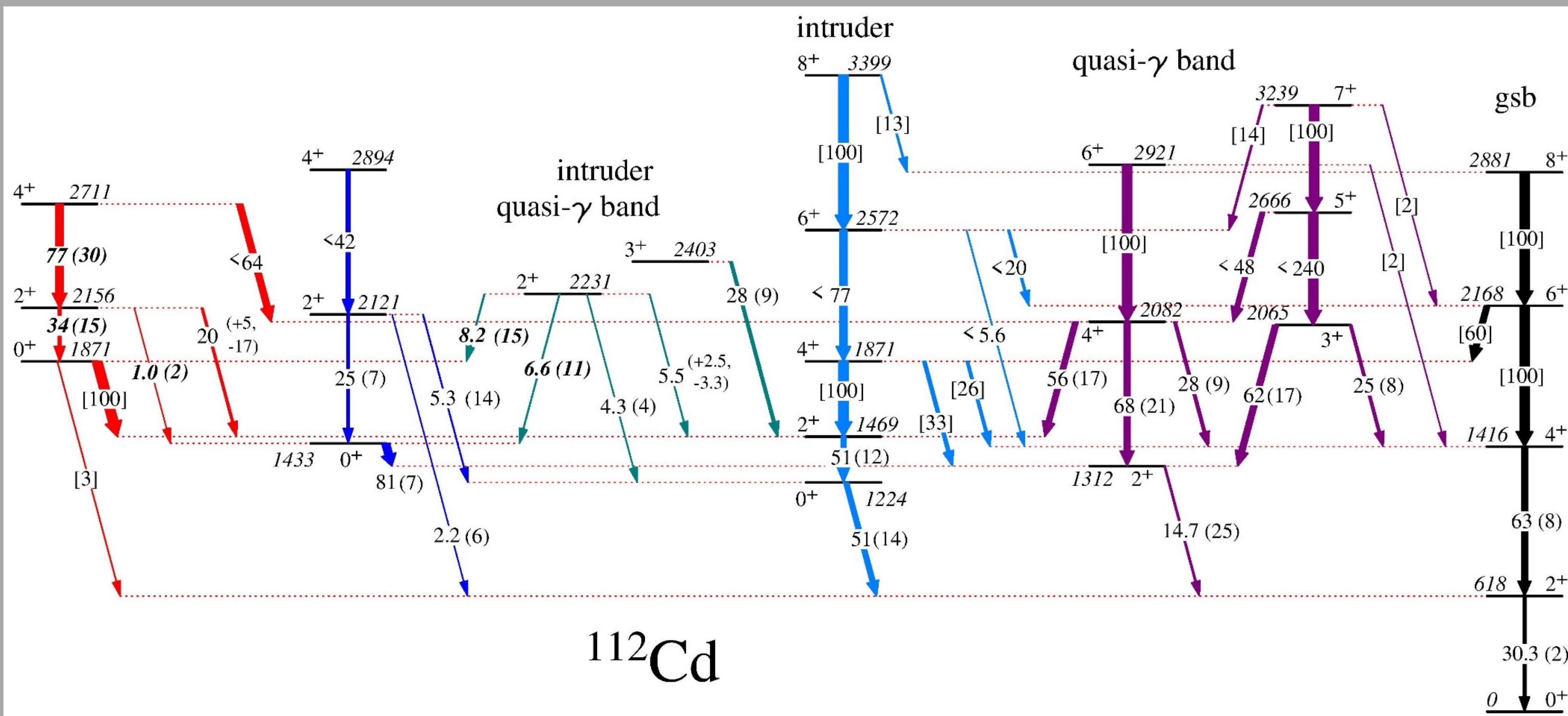


286-keV transition is a $7.9(33)\times10^{-4}$ γ branch from 2156-keV level, 34(15) W.u. transition

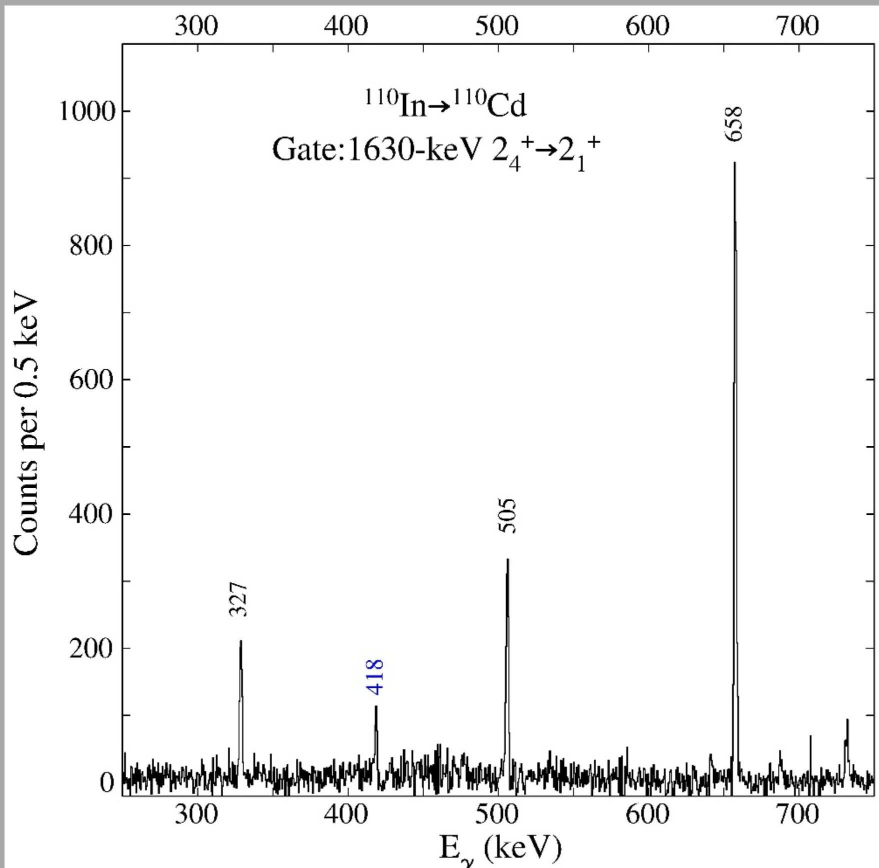


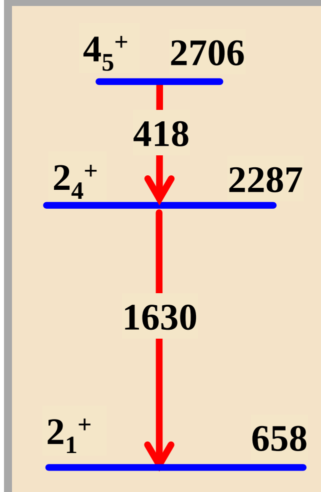
^{112}Cd band structure

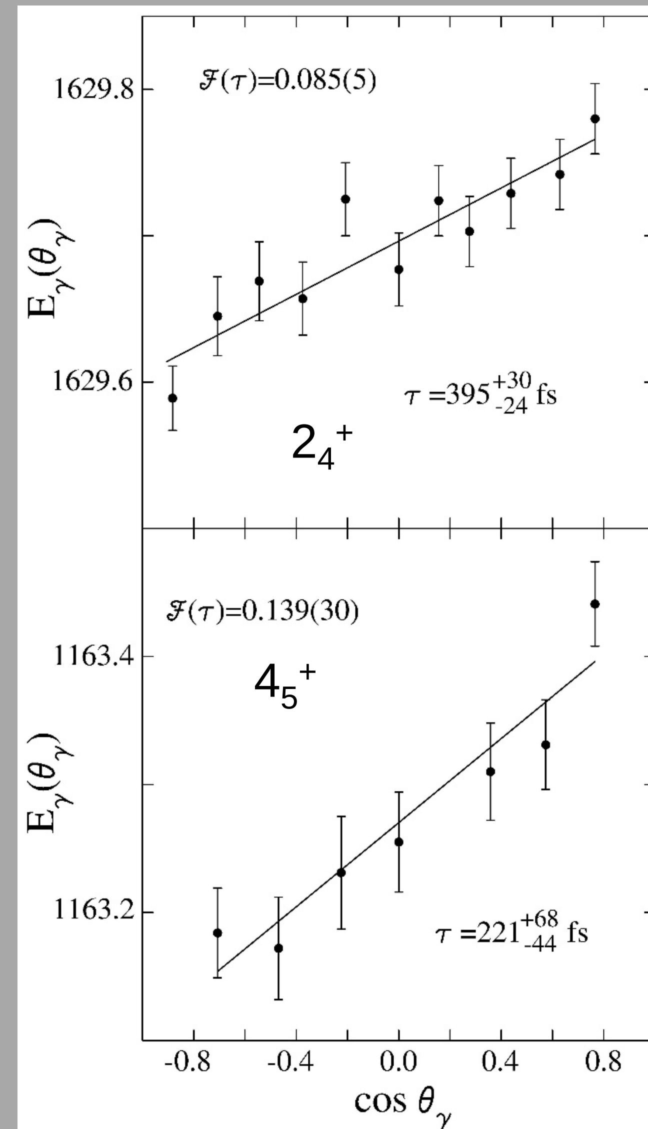
- Transitions labelled with *B(E2)* in W.u.
- Square brackets indicate relative *B(E2)* values
- Very weak transitions removed



Results from 7^+ ^{110}In decay, and lifetime from DSAM following INS



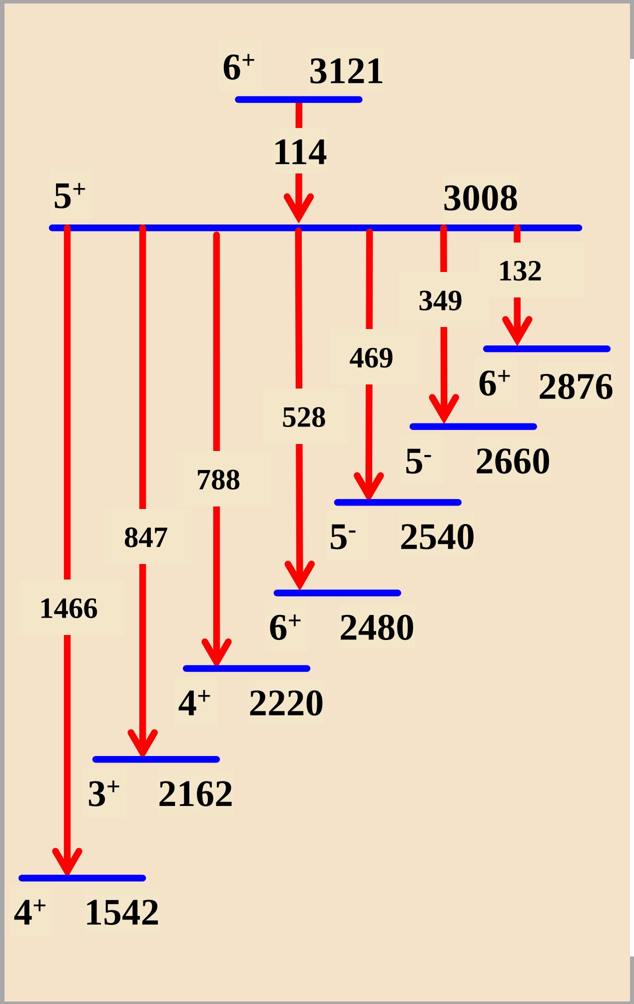


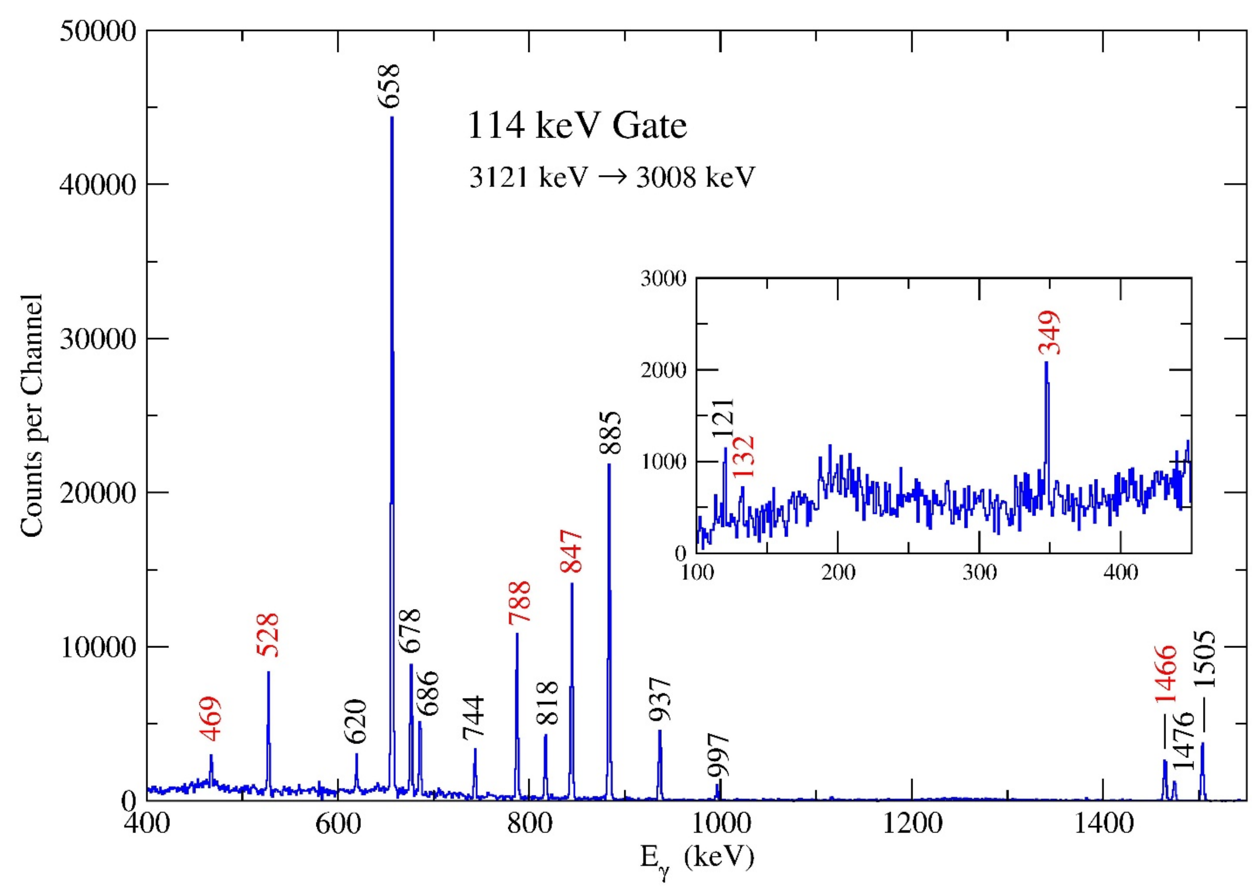


418-keV transition has branching ratio = 0.0057(3), resulting in $B(E2;4_5^+\rightarrow 2_4^+)$ = 55 ± 14W.u.

Establishment of new 3008 keV 5^+ level in ^{110}Cd

- **Results from 7^+ ^{110}In decay**





^{110}Cd band structure

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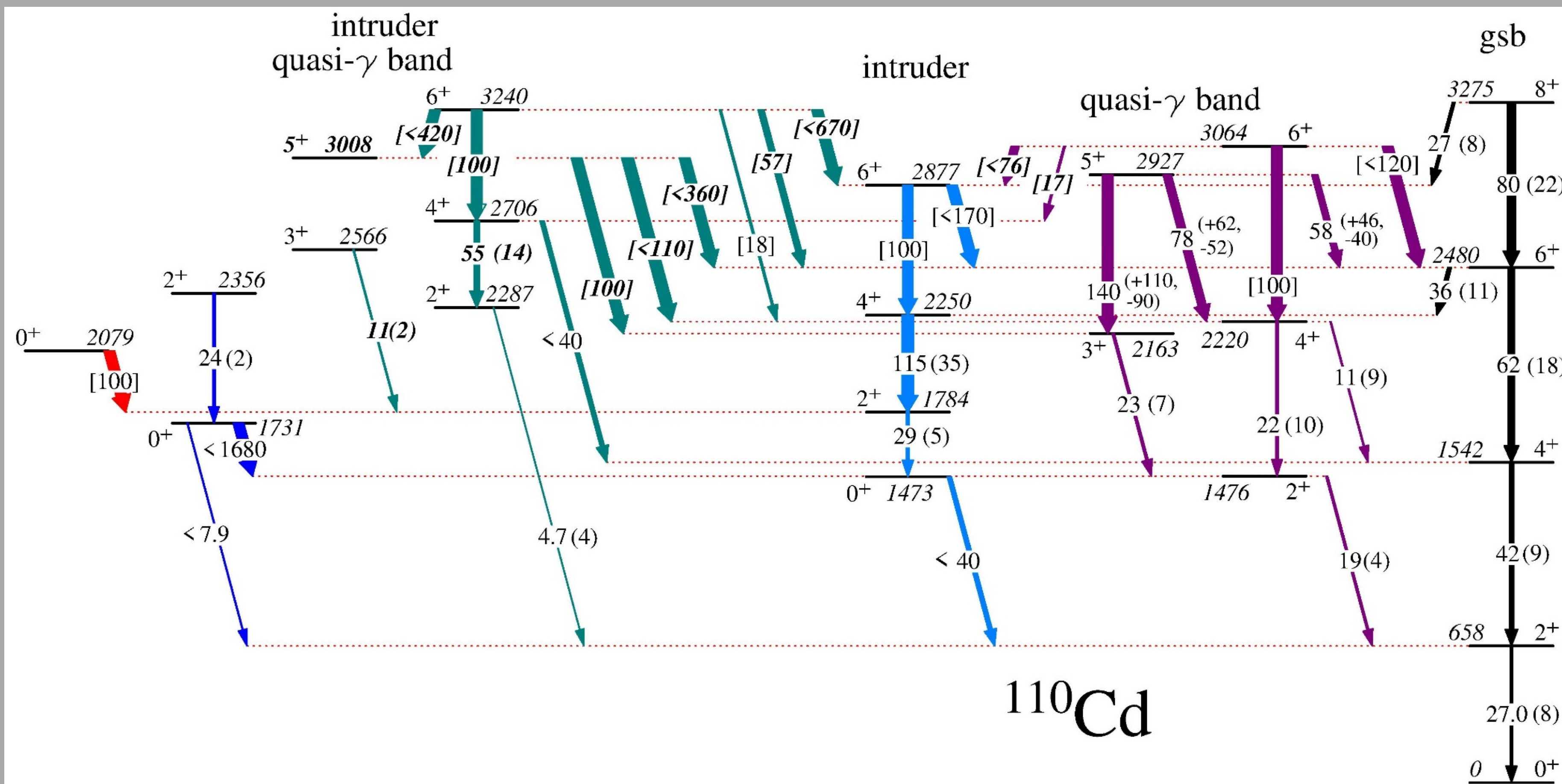
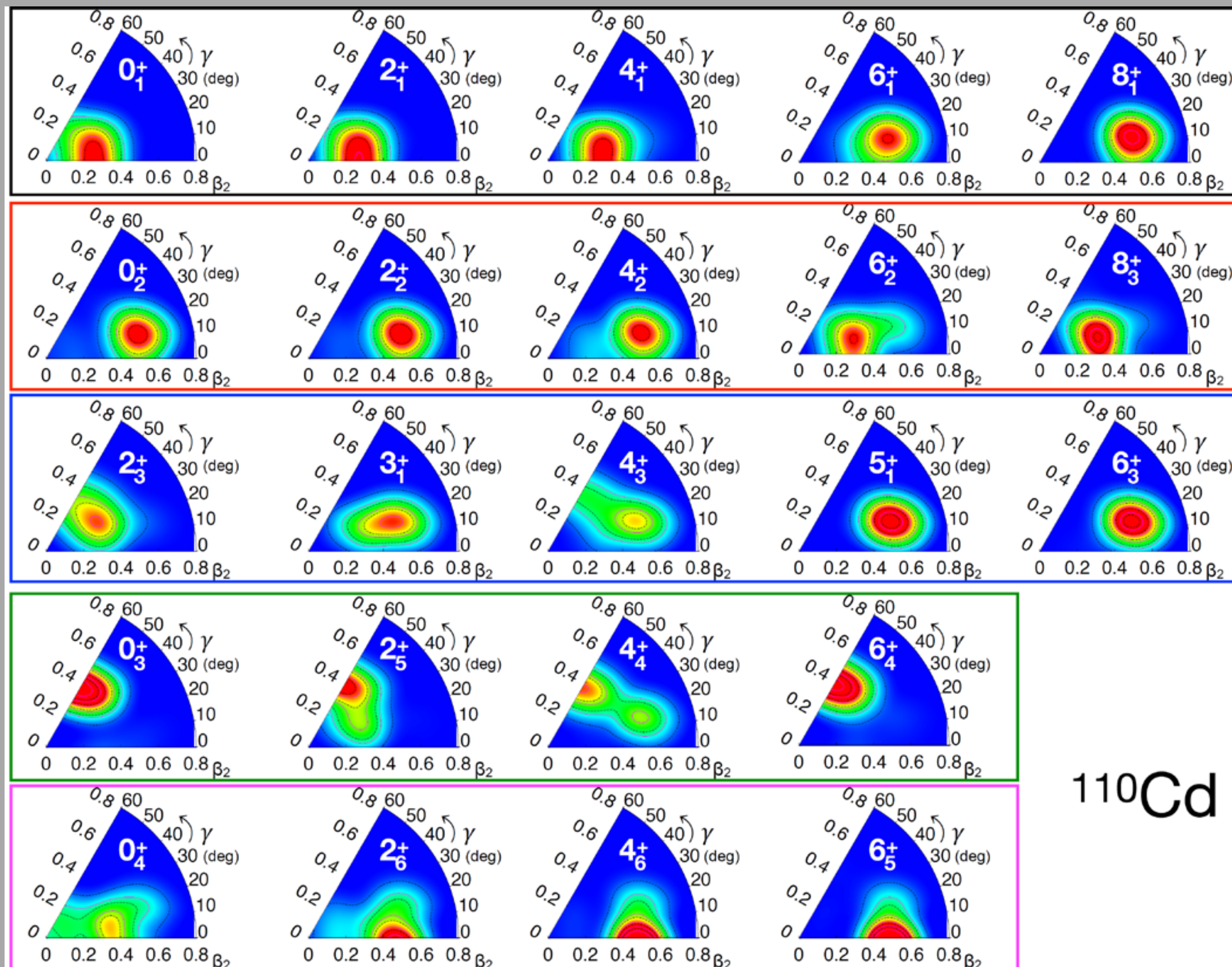
- Transitions labelled with *B(E2)* in W.u.
- Square brackets indicate relative *B(E2)* values
- Very weak transitions removed



Comparison to BMF calculations

- **BMF calculations using symmetry-conserving configuration method (SCCM) with Gogny D1S energy-density functional**
- **Exact angular momentum and particle number restoration**
- **Includes axial and non-axial shape mixing**
- **Occupation numbers for 0^+ states computed using the 0s, 0p, 1s0d, 1p0f, and 0g9/2 orbits as the reference to define the particle-hole structure for both protons and neutrons**

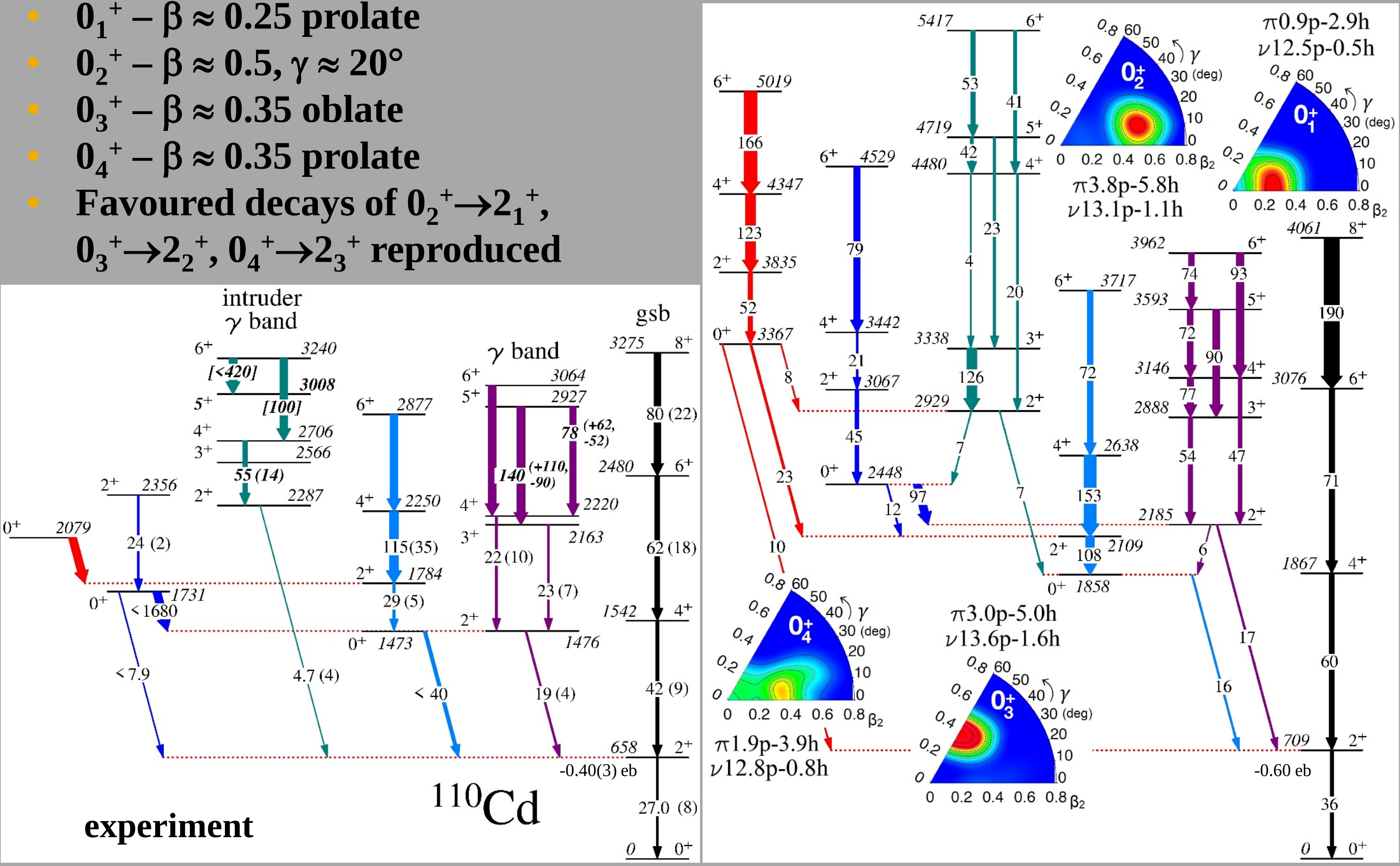
Wave function probability distributions in (β,γ) plane



Calculations by T. Rodriguez, Madrid

4 distinct shapes predicted for 0^+ bands

- $0_1^+ - \beta \approx 0.25$ prolate
- $0_2^+ - \beta \approx 0.5, \gamma \approx 20°$
- $0_3^+ - \beta \approx 0.35$ oblate
- $0_4^+ - \beta \approx 0.35$ prolate
- Favoured decays of $0_2^+ \rightarrow 2_1^+$, $0_3^+ \rightarrow 2_2^+$, $0_4^+ \rightarrow 2_3^+$ reproduced



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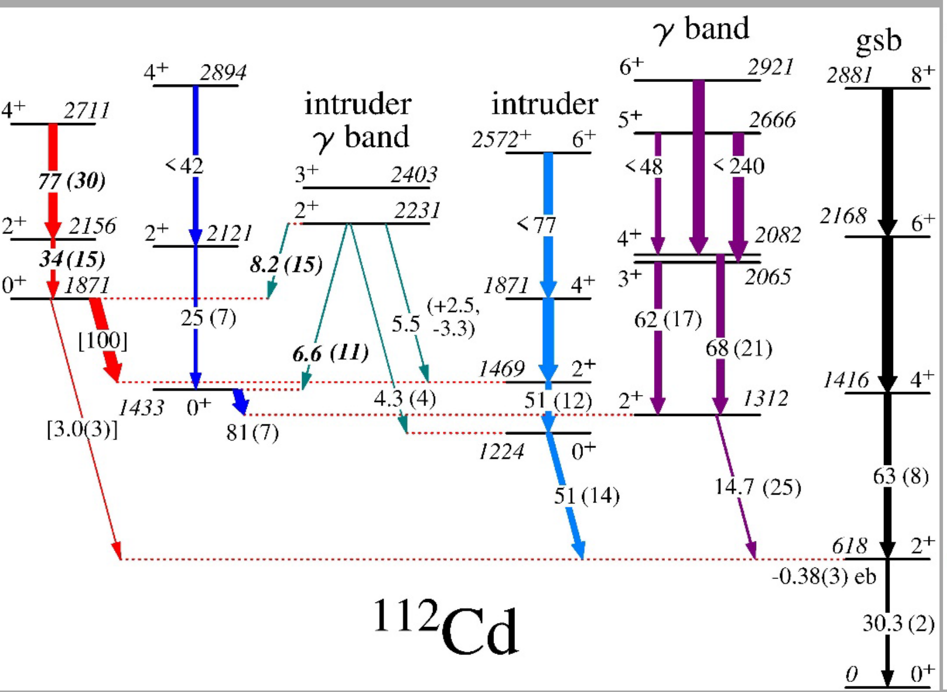

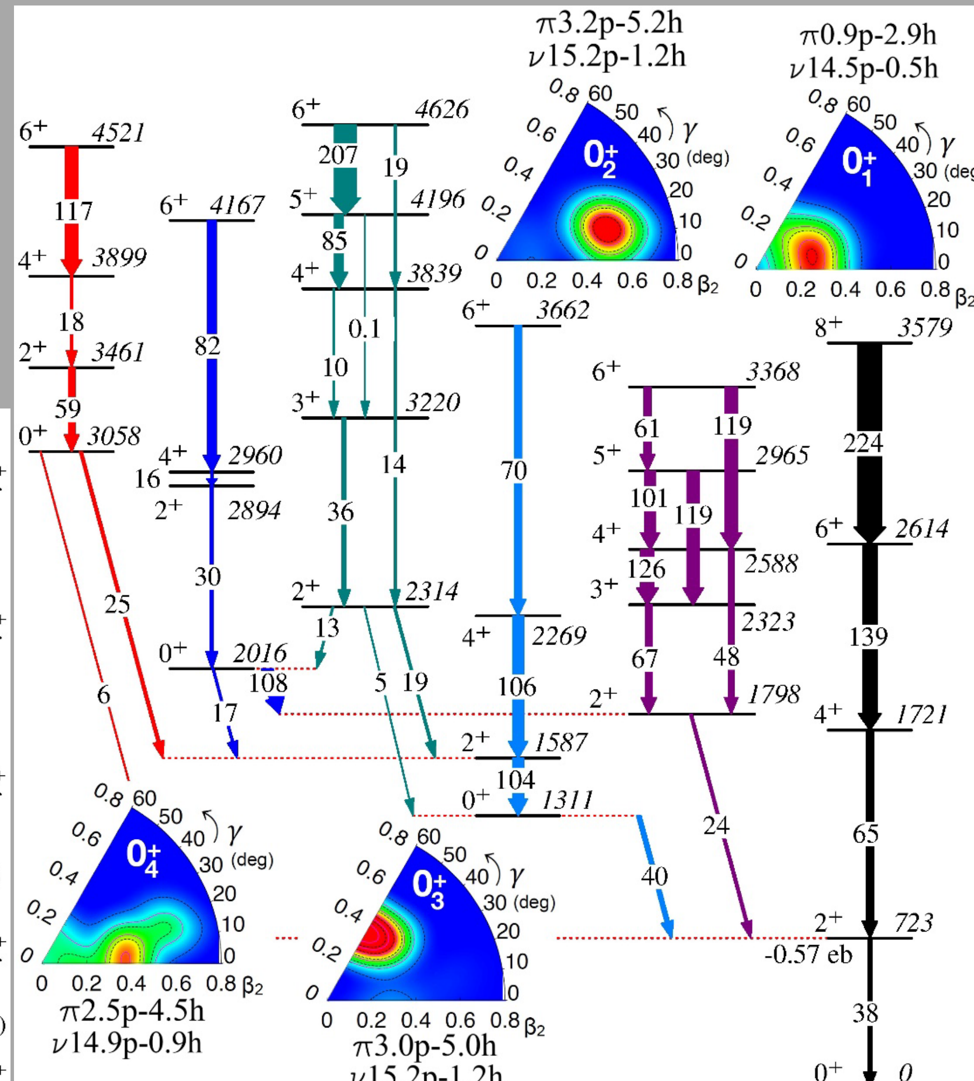

What do we conclude?

- **Description of the Cd isotopes as spherical vibrators – the paradigms of harmonic vibrational motion – fails**
  - **0^+ two-phonon strength appears nonexistent**
  - **We don't have a full 2-phonon triplet – *E*2 strength from 0^+ intruder band head to 2^+ one-phonon entirely consistent with weak mixing of the 0^+ states.**
  - **Only the yrast band appears to follow vibrational pattern in its *B*(*E*2)'s**
- **In many, if not all, of the vibrational candidate nuclei near Z=50, the 0_2^+ state is a shape-coexisting state**
  - **Proven conclusively with detailed Coulex studies in some cases**
- **BMF calculations suggest the intriguing possibility of multiple shape coexistence in the stable mid-shell Cd isotopes**
  - **This needs to be thoroughly tested – program under way to do this at HIL and LNL**

Summary

- **Existence of levels, and their energies are NOT a good indicator of vibrational structure**
  - **Often misleading and appear vibrational when they are not**
- ***B*(*E*λ) values are a necessary, but not sufficient, condition to establish multiphonon states**
- **Ideally, detailed Coulomb excitation and formation of rotational invariants are required to firmly establish vibrational nature**
- **Firm benchmarks need to be established in stable nuclei – when probed in detail, the best examples of vibrational nuclei are failing the test**
  - **Are their *any* nuclei that pass the stringent tests?**
- **Quadrupole vibrational phonon does not appear to be a "robust" boson – a fragile object that appears, at best, can only be coupled in an aligned manner**

Main collaborators

- Fribourg/Köln – Jan Jolie
- University of Kentucky – Steve Yates
- Georgia Tech – John Wood
- Warsaw – Kasia Wrzosek-Lipska
- Madrid – T. Rodriguez
- …and the students/post-docs at Guelph who have led the analysis: Katie Green, Alejandra Diaz Varela, Drew Jamieson, Andrew Finlay, Chandana Sumithrarachchi, and the rest of the University of Guelph group