



# WYBRANE PROBLEMY SYNTEZY JĄDER SUPERCIEŹKICH

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NATIONAL  
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SEMINARIUM FIZYKI JĄDRA ATOMOWEGO - FUW  
23.03.2023

A close-up photograph of a hand wearing a blue nitrile glove holding a glass test tube. The test tube contains a vibrant green liquid. The test tube is held over a periodic table of elements, which is slightly out of focus in the background. The lighting is bright, highlighting the green color of the liquid and the texture of the glove.

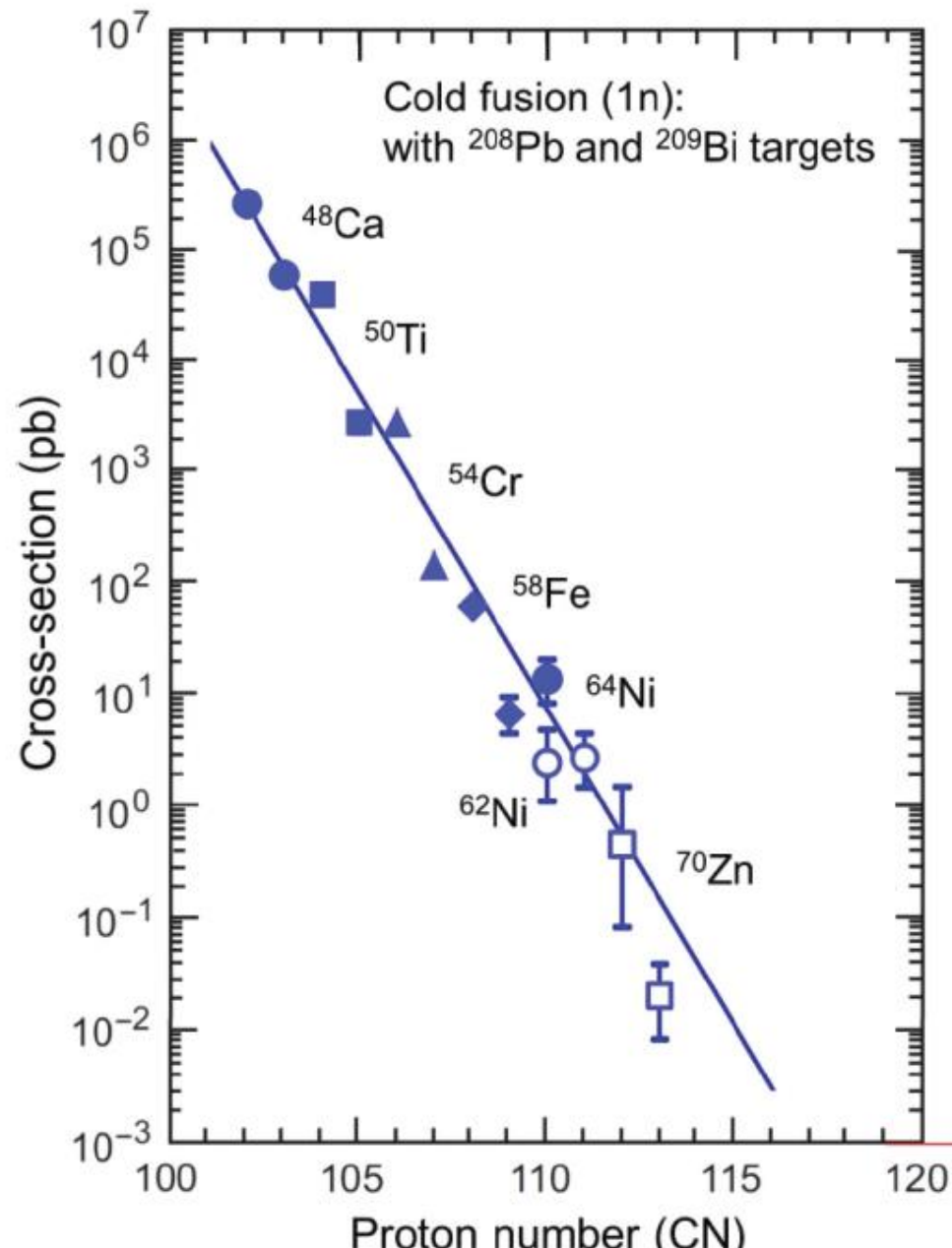
## Outline:

- New elements & reaction mechanism
- Capture - deformation & orientation of HI
- Fusion - stochastic nature & centrifugal effect
- Survival - fission barrier & density of states.

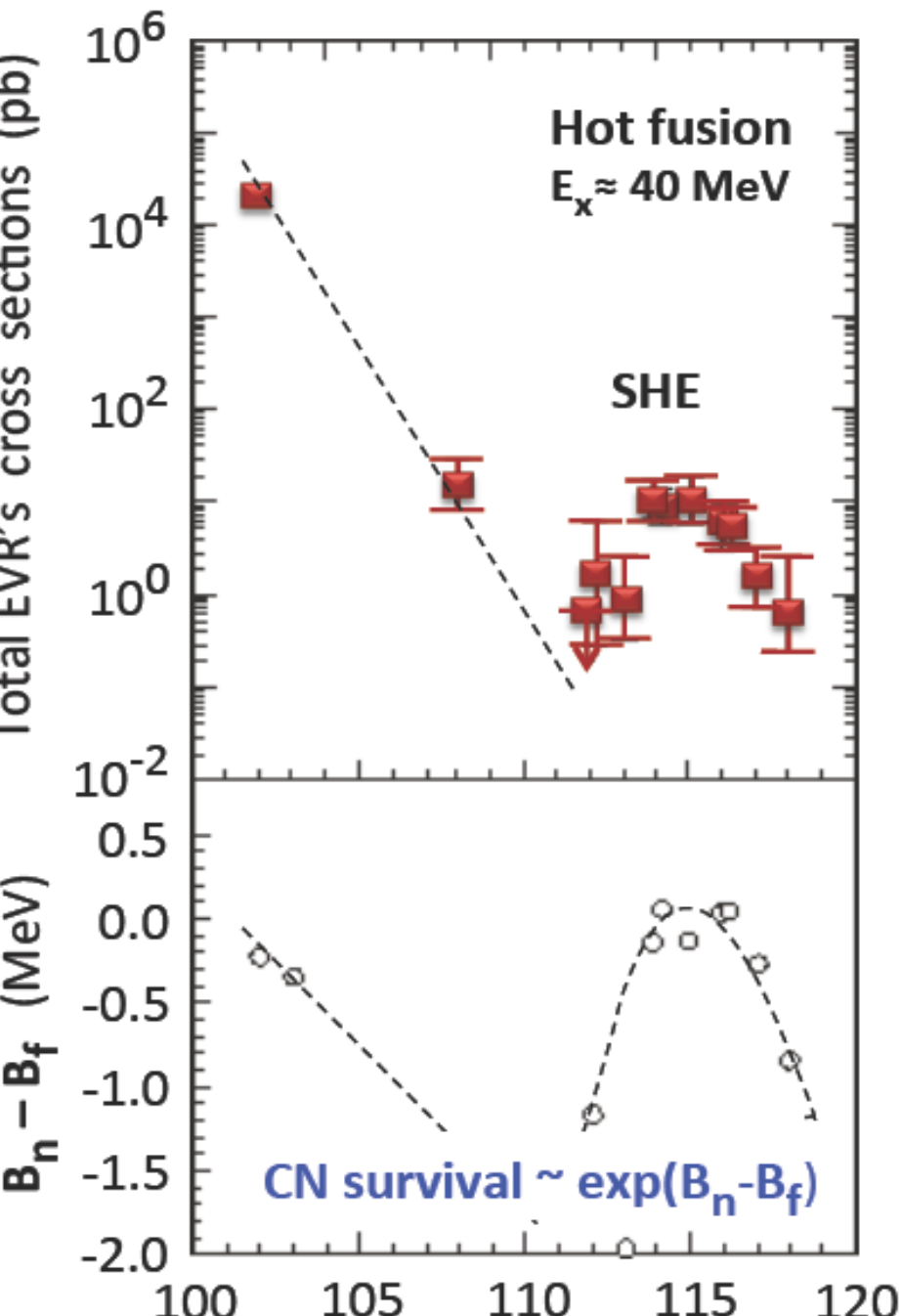
# SYNTHESIS SCENARIOS

## COLD ( $102 < Z < 113$ )

- the strongly bound target nuclei ( $^{208}\text{Pb}$  or  $^{209}\text{Bi}$ ) are bombarded with projectiles ranging from Ca to Zn;
- the excitation energy of the resulting compound nucleus is usually in the range of 10 to 20 MeV;
- as the target-projectile symmetry increases, the compound nucleus production cross section decreases.



# SYNTHESIS SCENARIOS



## HOT ( $112 < Z < 118$ )

- the deformed actinide target-nuclei (from U to Cm) are bombarded with a doubly magic  $^{48}\text{Ca}$  projectile;
- the excitation energy of the resulting compound nucleus is usually in the range of 30 to 40 MeV, and the dominant evaporation channels are 3n and 4n channels;
- the evaporation residue cross sections do not show any strong dependence on the target-projectile symmetry and are at the picobarn level.

# IUPAC Periodic Table of the Elements

1 <b>H</b> hydrogen 1.0080 ± 0.0002																	18 <b>He</b> helium 4.0026 ± 0.0001						
3 <b>Li</b> lithium 6.94 ± 0.06	4 <b>Be</b> beryllium 9.0122 ± 0.0001																	13 <b>B</b> boron 10.81 ± 0.02	14 <b>C</b> carbon 12.011 ± 0.002	15 <b>N</b> nitrogen 14.007 ± 0.001	16 <b>O</b> oxygen 15.999 ± 0.001	17 <b>F</b> fluorine 18.998 ± 0.001	10 <b>Ne</b> neon 20.180 ± 0.001
11 <b>Na</b> sodium 22.990 ± 0.001	12 <b>Mg</b> magnesium 24.305 ± 0.002																	13 <b>Al</b> aluminium 26.982 ± 0.001	14 <b>Si</b> silicon 28.085 ± 0.001	15 <b>P</b> phosphorus 30.974 ± 0.001	16 <b>S</b> sulfur 32.06 ± 0.02	17 <b>Cl</b> chlorine 35.45 ± 0.01	18 <b>Ar</b> argon 39.95 ± 0.16
19 <b>K</b> potassium 39.098 ± 0.001	20 <b>Ca</b> calcium 40.078 ± 0.004	21 <b>Sc</b> scandium 44.956 ± 0.001	22 <b>Ti</b> titanium 47.867 ± 0.001	23 <b>V</b> vanadium 50.942 ± 0.001	24 <b>Cr</b> chromium 51.996 ± 0.001	25 <b>Mn</b> manganese 54.938 ± 0.001	26 <b>Fe</b> iron 55.845 ± 0.002	27 <b>Co</b> cobalt 58.933 ± 0.001	28 <b>Ni</b> nickel 58.693 ± 0.001	29 <b>Cu</b> copper 63.546 ± 0.003	30 <b>Zn</b> zinc 65.38 ± 0.02	31 <b>Ga</b> gallium 69.723 ± 0.001	32 <b>Ge</b> germanium 72.630 ± 0.008	33 <b>As</b> arsenic 74.922 ± 0.001	34 <b>Se</b> selenium 78.971 ± 0.008	35 <b>Br</b> bromine 79.904 ± 0.003	36 <b>Kr</b> krypton 83.798 ± 0.002						
37 <b>Rb</b> rubidium 85.468 ± 0.001	38 <b>Sr</b> strontium 87.62 ± 0.01	39 <b>Y</b> yttrium 88.906 ± 0.001	40 <b>Zr</b> zirconium 91.224 ± 0.002	41 <b>Nb</b> niobium 92.906 ± 0.001	42 <b>Mo</b> molybdenum 95.95 ± 0.01	43 <b>Tc</b> technetium [97]	44 <b>Ru</b> ruthenium 101.07 ± 0.02	45 <b>Rh</b> rhodium 102.91 ± 0.01	46 <b>Pd</b> palladium 106.42 ± 0.01	47 <b>Ag</b> silver 107.87 ± 0.01	48 <b>Cd</b> cadmium 112.41 ± 0.01	49 <b>In</b> indium 114.82 ± 0.01	50 <b>Sn</b> tin 118.71 ± 0.01	51 <b>Sb</b> antimony 121.76 ± 0.01	52 <b>Te</b> tellurium 127.60 ± 0.03	53 <b>I</b> iodine 126.90 ± 0.01	54 <b>Xe</b> xenon 131.29 ± 0.01						
55 <b>Cs</b> caesium 132.91 ± 0.01	56 <b>Ba</b> barium 137.33 ± 0.01	57-71 lanthanoids	72 <b>Hf</b> hafnium 178.49 ± 0.01	73 <b>Ta</b> tantalum 180.95 ± 0.01	74 <b>W</b> tungsten 183.84 ± 0.01	75 <b>Re</b> rhenium 186.21 ± 0.01	76 <b>Os</b> osmium 190.23 ± 0.03	77 <b>Ir</b> iridium 192.22 ± 0.01	78 <b>Pt</b> platinum 195.08 ± 0.02	79 <b>Au</b> gold 196.97 ± 0.01	80 <b>Hg</b> mercury 200.59 ± 0.01	81 <b>Tl</b> thallium 204.38 ± 0.01	82 <b>Pb</b> lead 207.2 ± 1.1	83 <b>Bi</b> bismuth 208.98 ± 0.01	84 <b>Po</b> polonium [209]	85 <b>At</b> astatine [210]	86 <b>Rn</b> radon [222]						
87 <b>Fr</b> francium [223]	88 <b>Ra</b> radium [226]	89-103 actinoids	104 <b>Rf</b> rutherfordium [261]	105 <b>Db</b> dubnium [268]	106 <b>Sg</b> seaborgium [269]	107 <b>Bh</b> bohrium [270]	108 <b>Hs</b> hassium [271]	109 <b>Mt</b> meitnerium [272]	110 <b>Ds</b> darmstadtium [281]	111 <b>Rg</b> roentgenium [282]	112 <b>Cn</b> copernicium [285]	113 <b>Nh</b> nihonium [286]	114 <b>Fl</b> flerovium [289]	115 <b>Mc</b> moscovium [290]	116 <b>Lv</b> livermorium [293]	117 <b>Ts</b> tennessine [294]	118 <b>Og</b> oganesson [294]						

Key:  
atomic number  
**Symbol**  
name  
abridged standard  
atomic weight



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57 <b>La</b> lanthanum 138.91 ± 0.01	58 <b>Ce</b> cerium 140.12 ± 0.01	59 <b>Pr</b> praseodymium 140.91 ± 0.01	60 <b>Nd</b> neodymium 144.24 ± 0.01	61 <b>Pm</b> promethium [145]	62 <b>Sm</b> samarium 150.36 ± 0.02	63 <b>Eu</b> europium 151.96 ± 0.01	64 <b>Gd</b> gadolinium 157.25 ± 0.03	65 <b>Tb</b> terbium 158.93 ± 0.01	66 <b>Dy</b> dysprosium 162.50 ± 0.01	67 <b>Ho</b> holmium 164.93 ± 0.01	68 <b>Er</b> erbium 167.26 ± 0.01	69 <b>Tm</b> thulium 168.93 ± 0.01	70 <b>Yb</b> ytterbium 173.05 ± 0.02	71 <b>Lu</b> lutetium 174.97 ± 0.01
89 <b>Ac</b> actinium [227]	90 <b>Th</b> thorium 232.04 ± 0.01	91 <b>Pa</b> protactinium 231.04 ± 0.01	92 <b>U</b> uranium 238.03 ± 0.01	93 <b>Np</b> neptunium [237]	94 <b>Pu</b> plutonium [244]	95 <b>Am</b> americium [243]	96 <b>Cm</b> curium [247]	97 <b>Bk</b> berkelium [247]	98 <b>Cf</b> californium [251]	99 <b>Es</b> einsteinium [252]	100 <b>Fm</b> fermium [257]	101 <b>Md</b> mendelevium [258]	102 <b>No</b> nobelium [259]	103 <b>Lr</b> lawrencium [262]

For notes and updates to this table, see [www.iupac.org](http://www.iupac.org). This version is dated 4 May 2022.  
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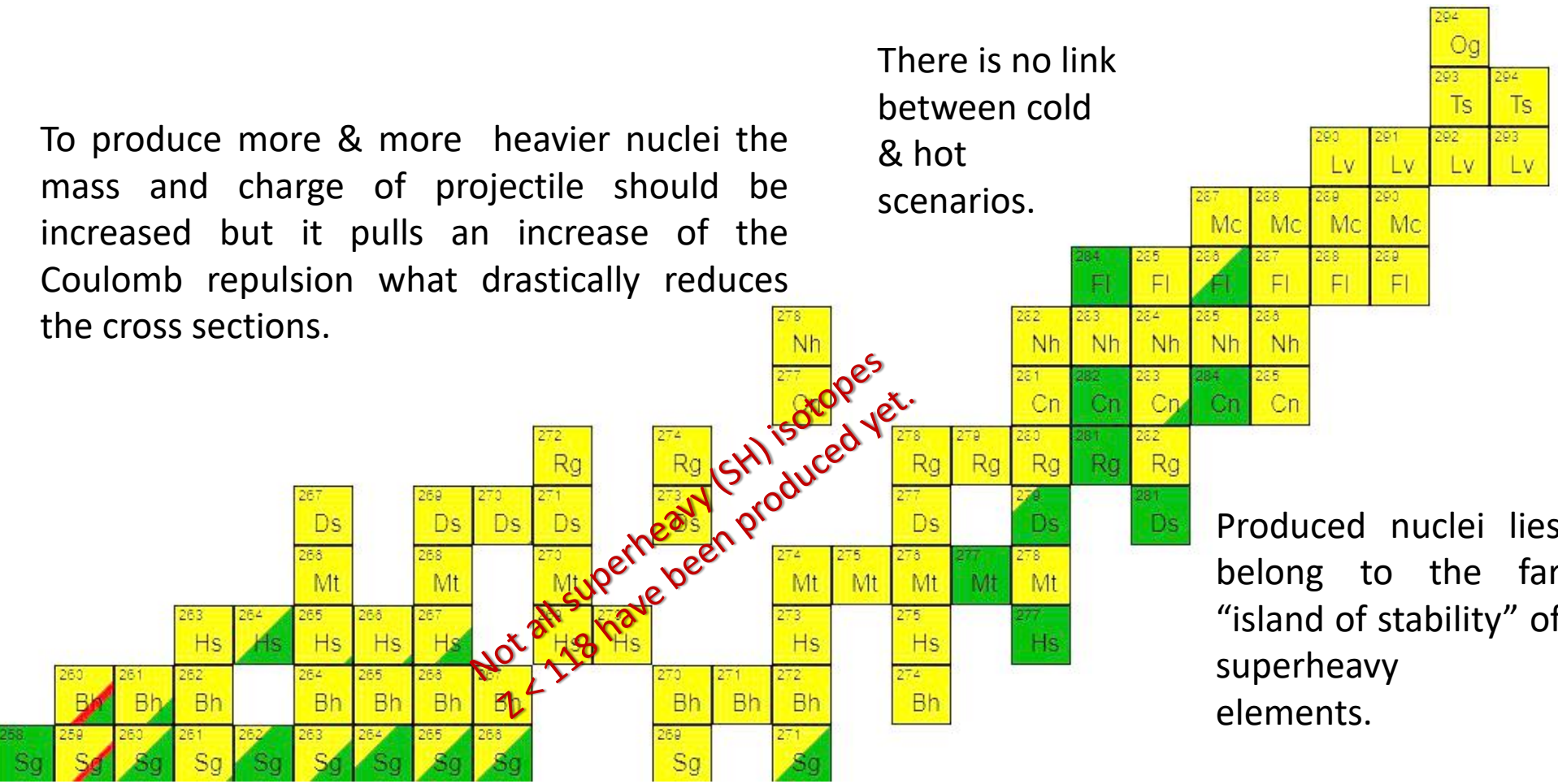
**BUT**

Attempts of going beyond the reactions Act. +  $^{48}\text{Ca}$  by using heavier projectiles like  $^{50}\text{Ti}$ ,  $^{54}\text{Cr}$ ,  $^{58}\text{Fe}$ , and  $^{64}\text{Ni}$  gave no results so far.

All heavier actinides with  $Z > 98$  live too short that one could perform target with them.

To produce more & more heavier nuclei the mass and charge of projectile should be increased but it pulls an increase of the Coulomb repulsion what drastically reduces the cross sections.

There is no link between cold & hot scenarios.

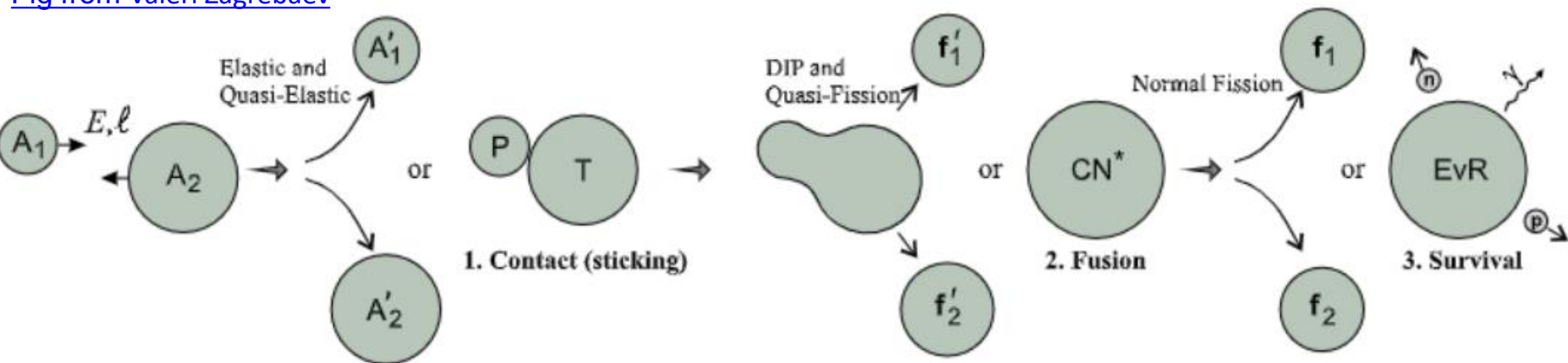


the overarching goal of the models is:

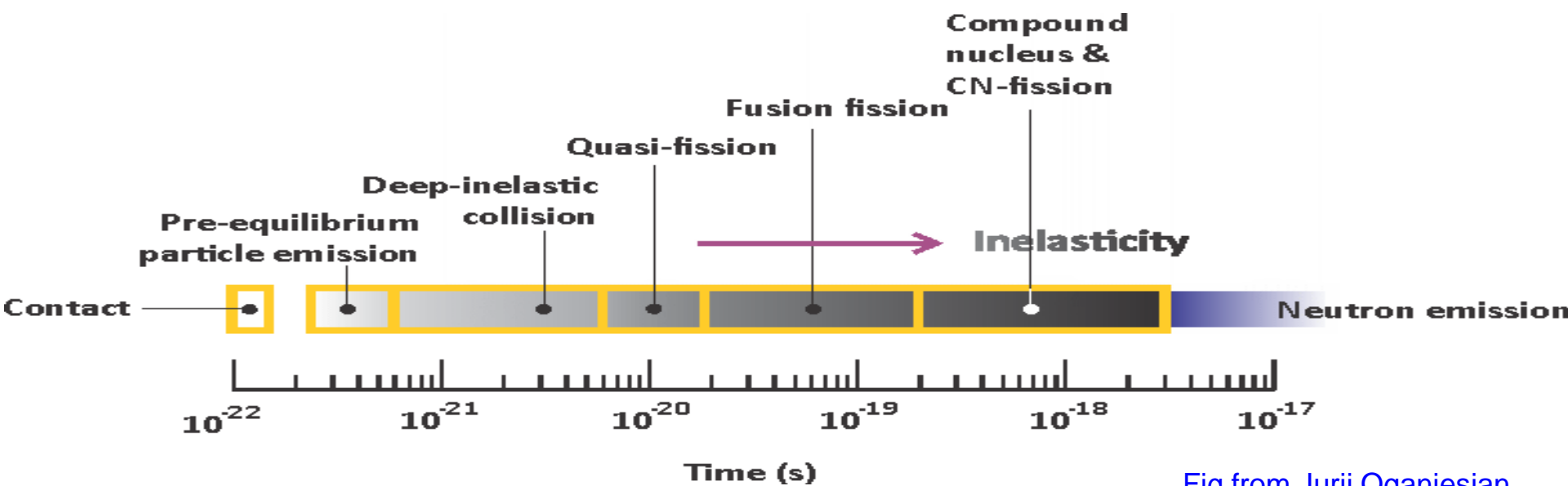
to determine the most appropriate **projectile-target combination**

to predict the **optimal bombardment energy** in the entrance channel at which the **production cross-section is the most significant**

to explain the **fusion process** in heavy-ion collisions



$$\sigma(\text{synthesis}) = \pi \hat{\lambda}^2 \sum_{l=0}^{l_{\max}} (2l+1) T_l \times P_l(\text{fusion}) \times P_{xn}^l(\text{survive})$$







## The Fusion-by-Diffusion model as a tool to calculate cross sections for the production of superheavy nuclei

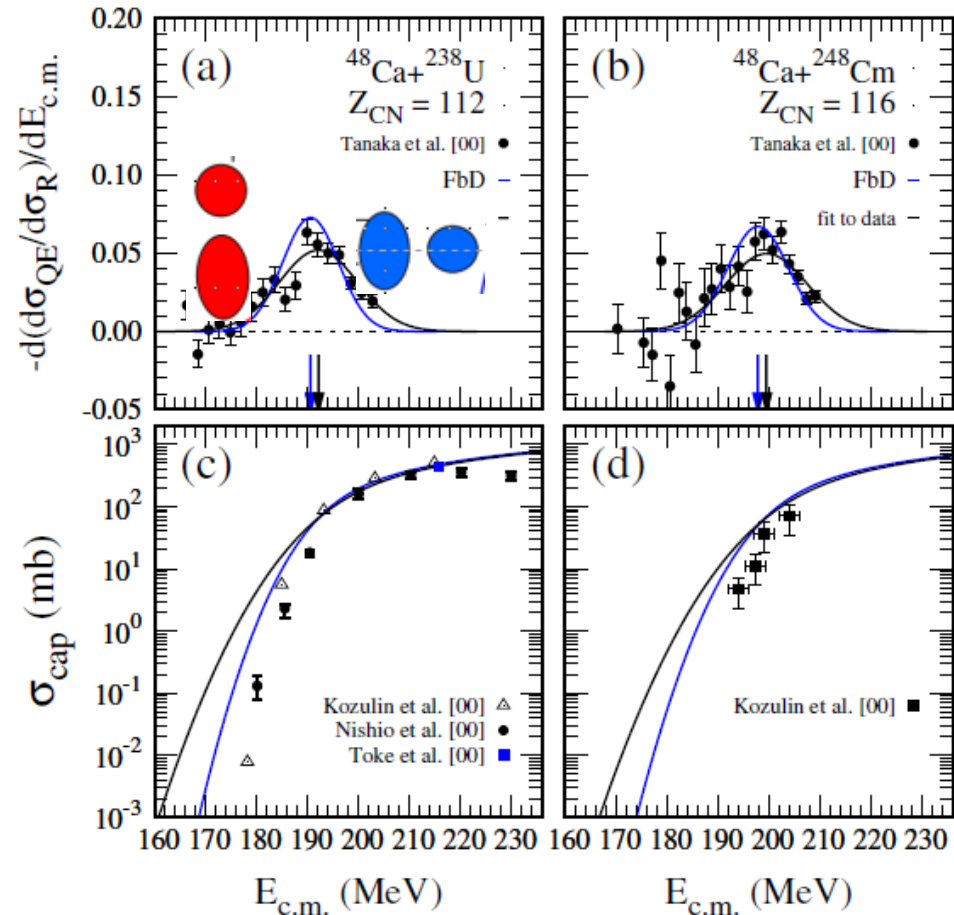
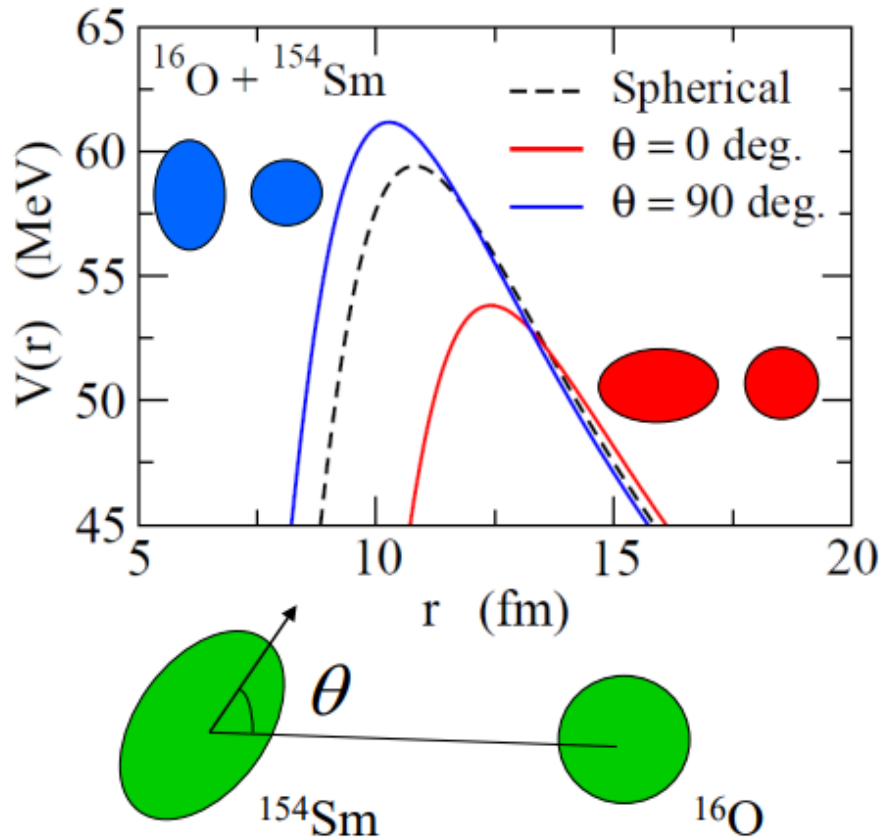
T. Cap<sup>1,a</sup>, M. Kowal<sup>1,b</sup>, K. Siwek-Wilczyńska<sup>2,c</sup>

<sup>1</sup> National Centre for Nuclear Research, Pasteura 7, 02-093 Warsaw, Poland

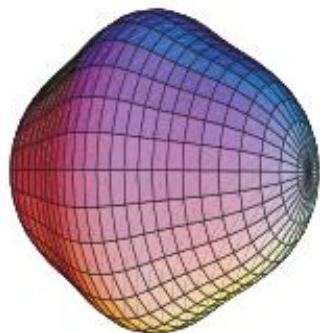
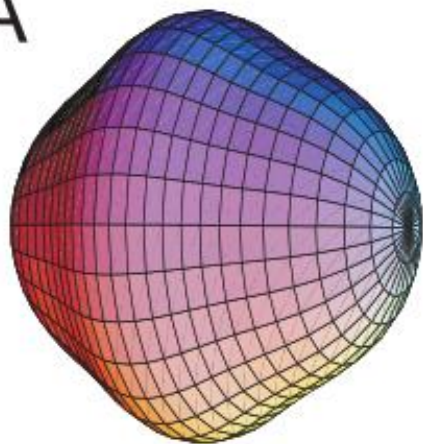
<sup>2</sup> Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland

$$V_c = \frac{e^2}{4\pi\epsilon_0} \frac{Z_a Z_b}{R_a + R_b}$$

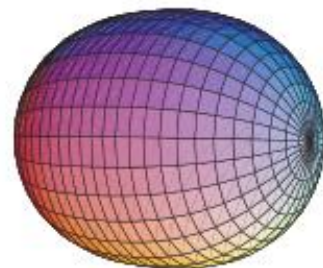
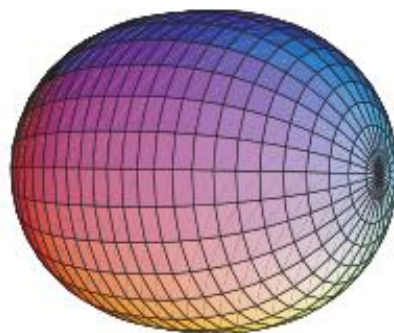
$$\sigma(\text{synthesis}) = \pi \lambda^2 \sum_{l=0}^{l_{\max}} (2l+1) T_l \times P_l(\text{fusion}) \times P_{\text{xn}}^l(\text{survive})$$



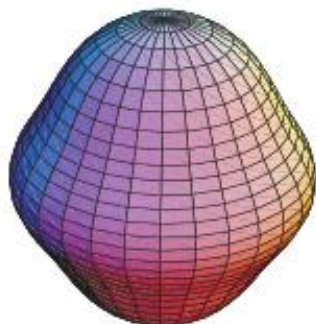
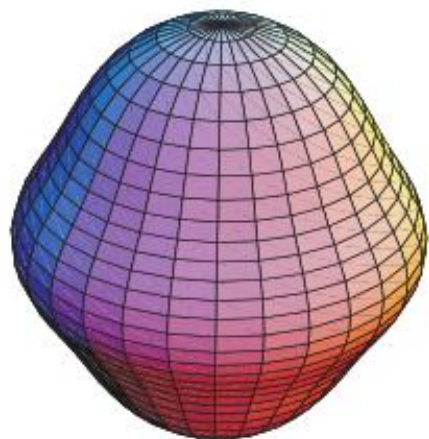
A



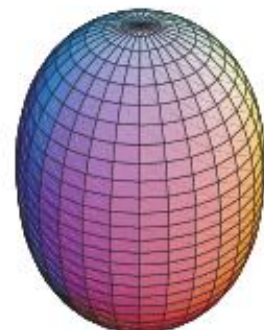
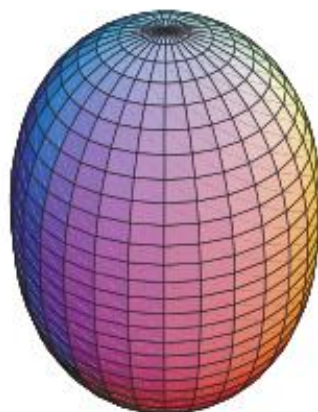
C



B

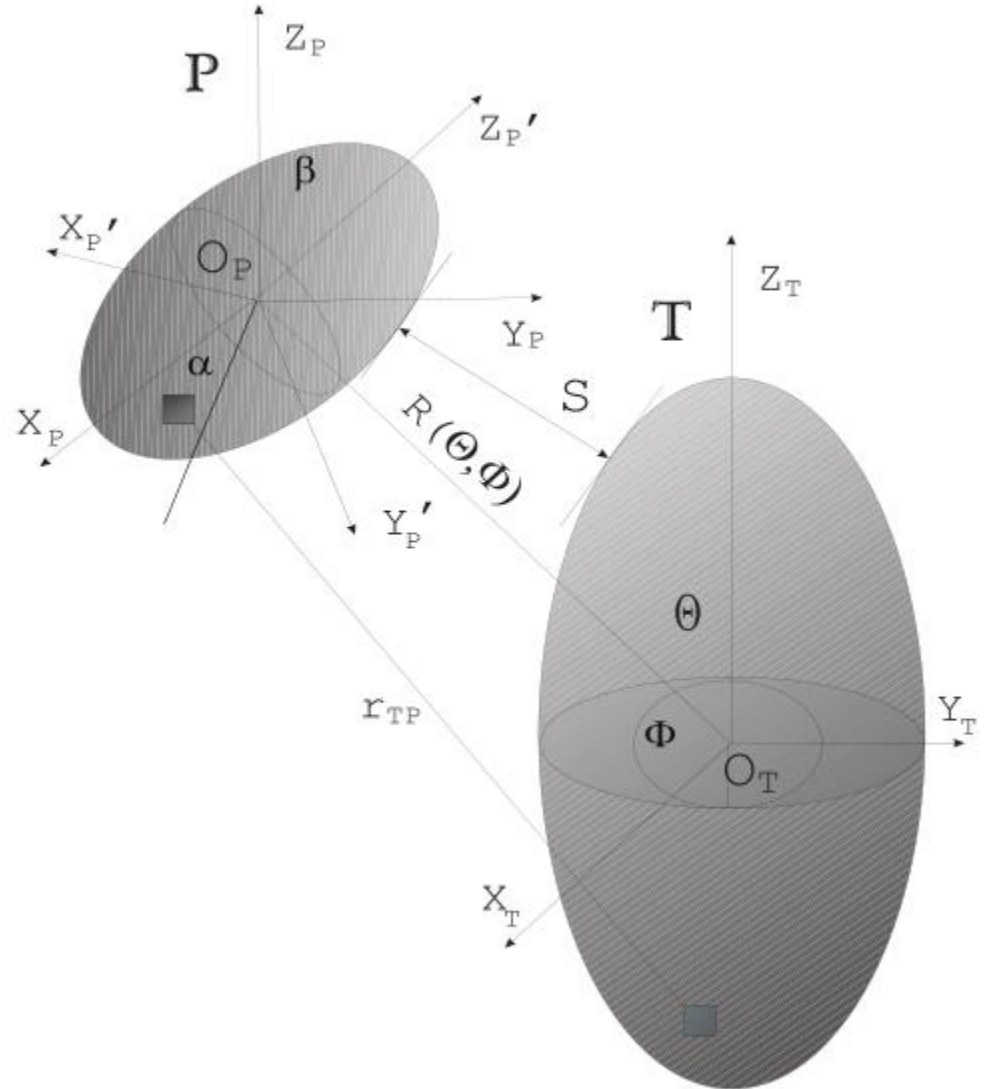


D



$$V_{TOT}(R, \Theta) = V_C(R, \Theta) + V_N(R, \Theta) + V_L(R) .$$

$$\vec{R} = \begin{cases} R \sin(\Theta) \cos(\Phi) \\ R \sin(\Theta) \sin(\Phi) \\ R \cos(\Theta) \end{cases}$$



# Electrostatic Interaction

$$V_C = \int_{V_P} \int_{V_T} \frac{\rho_T(\vec{r}_T) \rho_P(\vec{r}_P)}{r_{TP}} d^3 r_T d^3 r_P$$

$$\frac{1}{r_{TP}} = \sum_{l_T}^{\infty} \sum_{l_P}^{\infty} \frac{(-1)^{l_P} r_T^{l_T} r_P^{l_P}}{R^{l_T+l_P+1}} \sum_{m_T=-l_T}^{l_T} \sum_{m_P=-l_P}^{l_P} C_{l_T, l_P}^{m_T, m_P} Y_{l_T+l_P}^{-(m_T+m_P)}(\Theta, \Phi) \\ \times Y_{l_T}^{m_T}(\vartheta_T, \varphi_T) Y_{l_P}^{m_P}(\vartheta_P, \varphi_P),$$

$$Q_{l_{T(P)}}^{m_{T(P)}}(\vartheta_{T(P)}, \varphi_{T(P)}) = \int_{V_{T(P)}} \rho_{T(P)}(r_{T(P)}) r_{T(P)}^{l_{T(P)}} Y_{l_{T(P)}}^{m_{T(P)}}(\vartheta_{T(P)}, \varphi_{T(P)}) d\tau_{T(P)}$$

$$V_C = \sum_{l_T}^{\infty} \sum_{l_P}^{\infty} \frac{(-1)^{l_P}}{R^{l_T+l_P+1}} \sum_{m_T=-l_T}^{l_T} \sum_{m_P=-l_P}^{l_P} \sum_{m'_P=-l_P}^{l_P} C_{l_T, l_P}^{m_T, m_P} Y_{l_T+l_P}^{-(m_T+m_P)}(\Theta, \Phi) \\ \times Q_{l_T}^{m_T}(\vartheta_T, \varphi_T) D_{m'_P, m_P}^{l_P}(\alpha, \beta, \gamma) Q_{l_P}^{m'_P}(\vartheta'_P, \varphi'_P).$$

# Nuclear Interaction

$$V_N = 4 \pi \gamma K \psi(s)$$

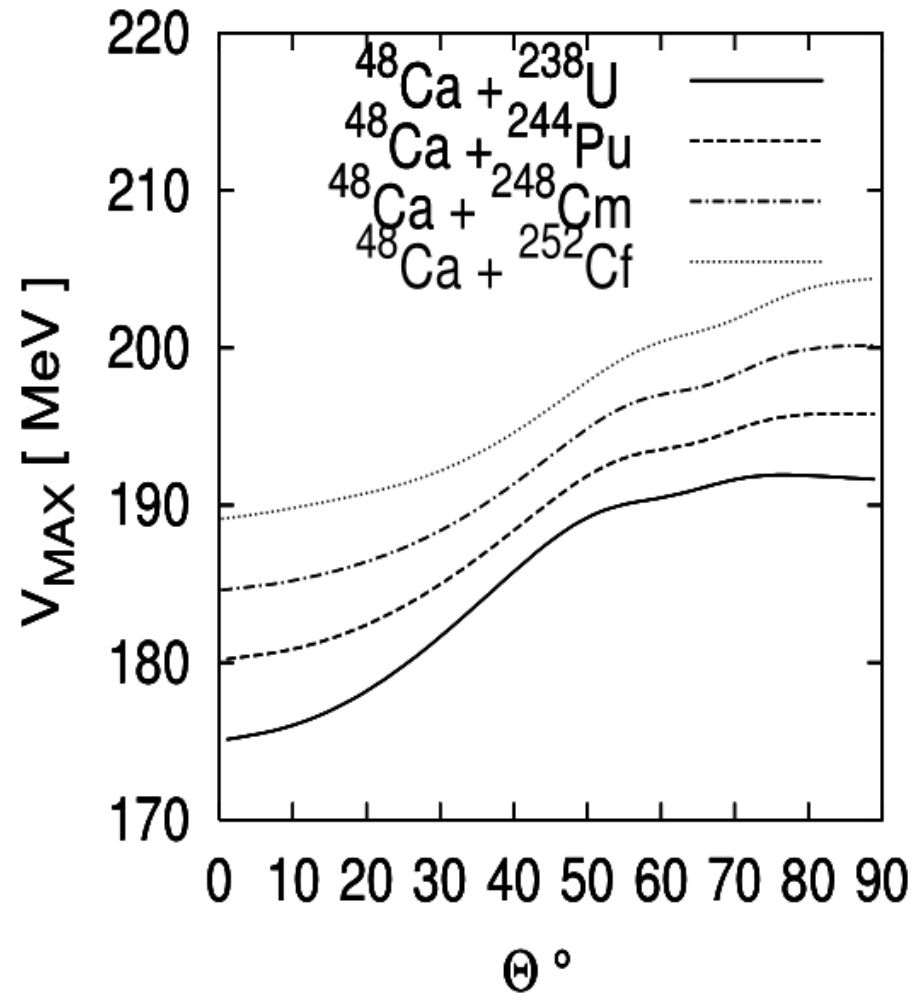
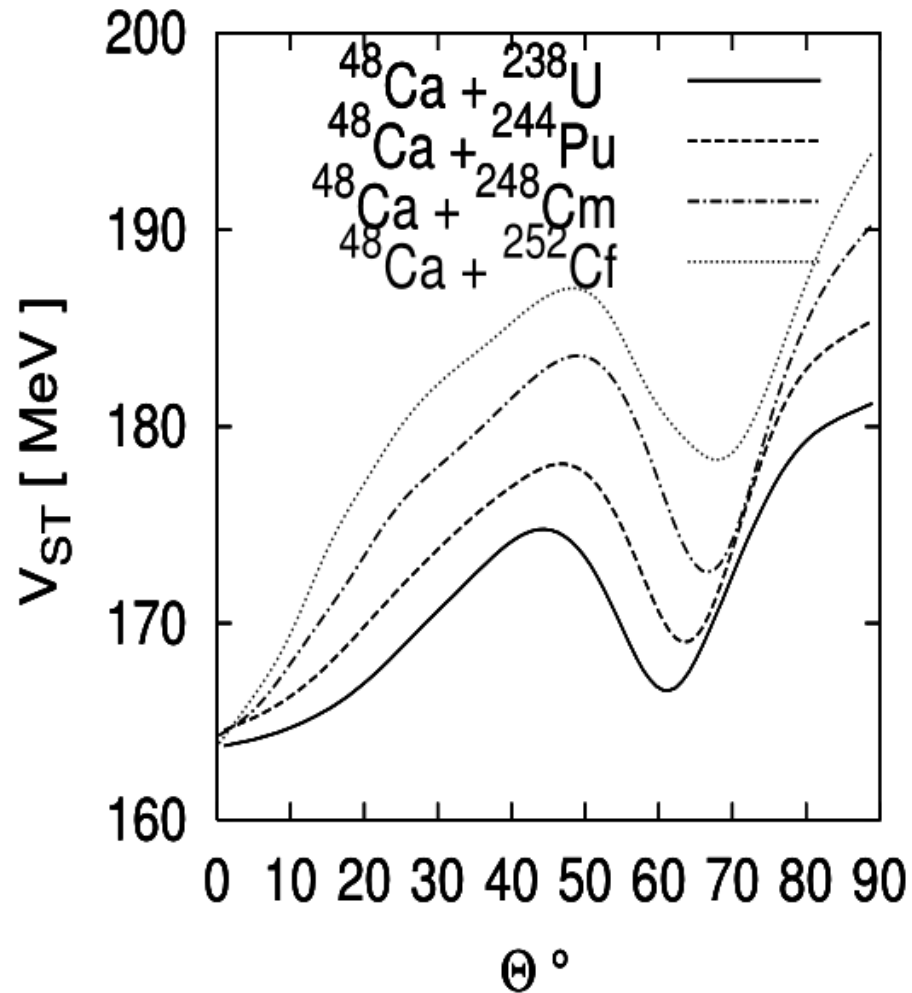
$$\frac{1}{K^2} = \frac{1}{R_{1T}R_{1P}} + \frac{1}{R_{2T}R_{2P}} + \frac{1}{R_{1T}R_{2P}} + \frac{1}{R_{2T}R_{1P}}$$

$$\begin{aligned} s = \min[ & (R \sin(\vartheta) \cos(\varphi) + R_P(\vartheta_P, \varphi_P) \sin(\vartheta_P) \cos(\varphi_P) - R_T(\vartheta_T, \varphi_T) \sin(\vartheta_T) \cos(\varphi_T))^2 \\ & + (R \sin(\vartheta) \sin(\varphi) + R_P(\vartheta_P, \varphi_P) \sin(\vartheta_P) \sin(\varphi_P) - R_T(\vartheta_T, \varphi_T) \sin(\vartheta_T) \sin(\varphi_T))^2 \\ & + (R \cos(\vartheta) + R_P(\vartheta_P, \varphi_P) \cos(\vartheta_P) - R_T(\vartheta_T, \varphi_T) \cos(\vartheta_T))^2 ]^{\frac{1}{2}} \end{aligned} \quad (4.24)$$

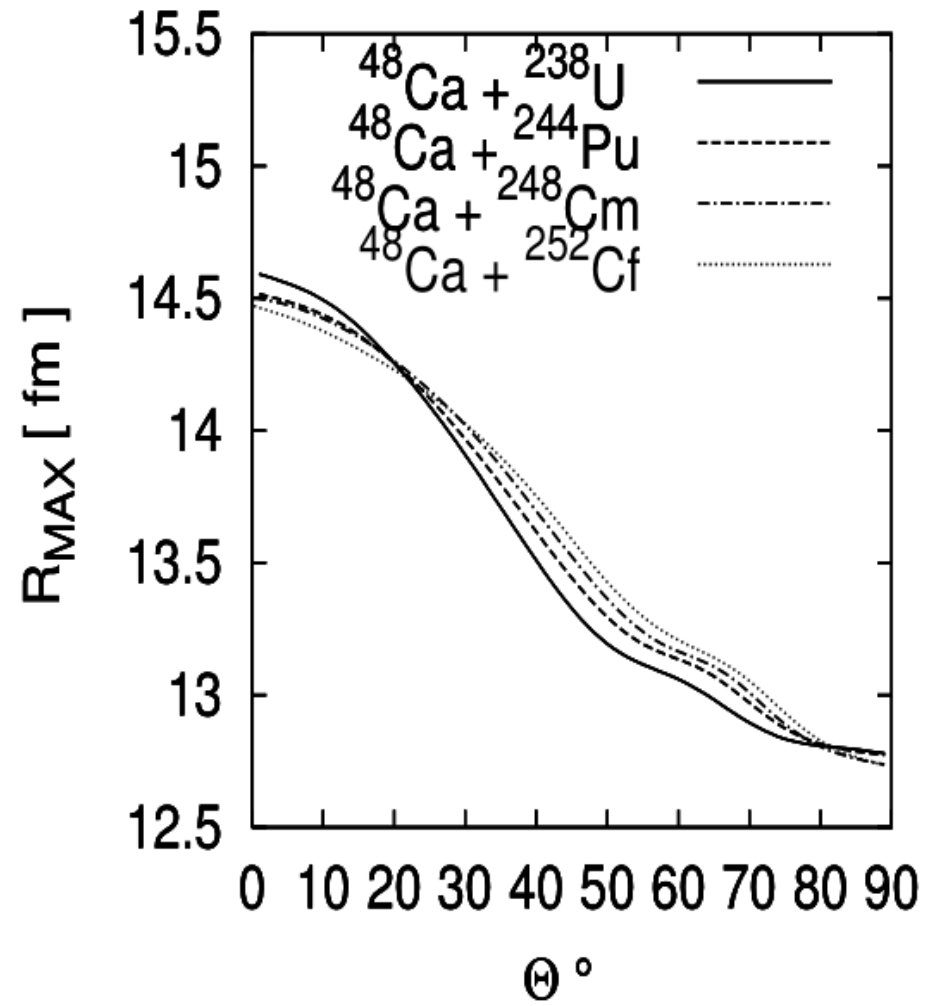
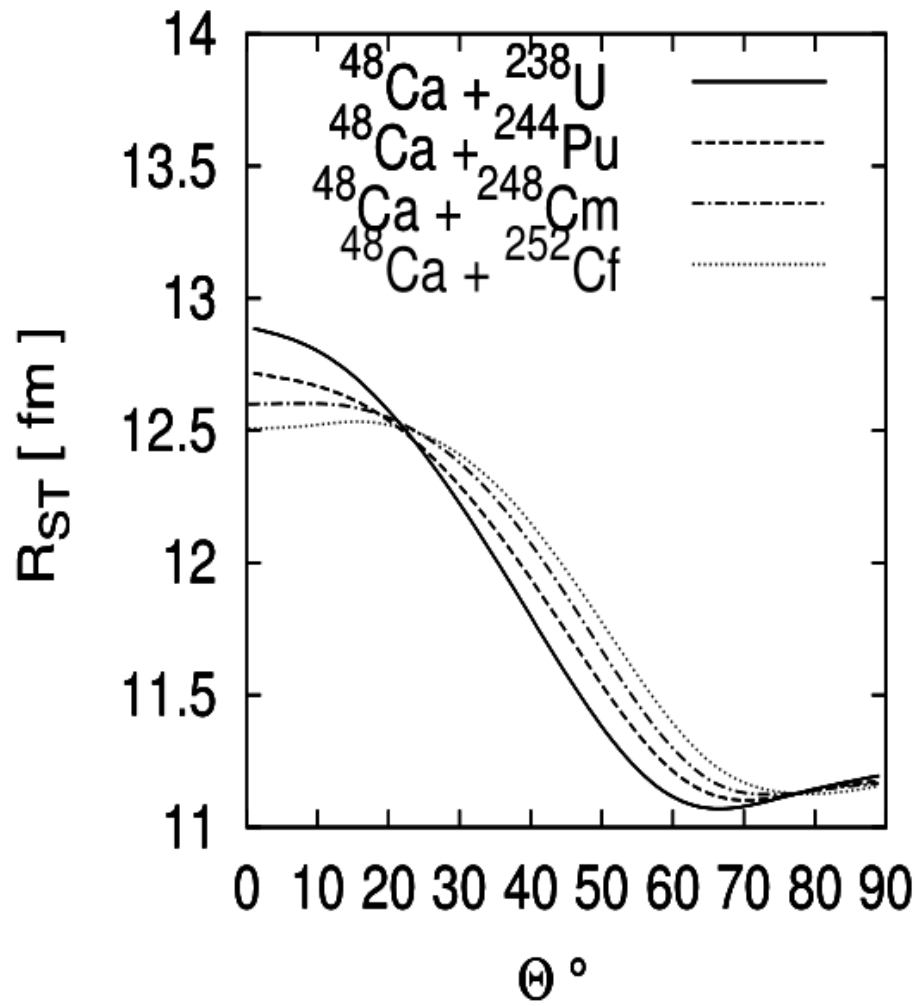
$$\psi(s) = -0.1333 + \sum_n^5 \frac{c_n}{n+1} (2.5 - s)^{n+1}, \quad 0 < s < 2.5$$

$$\psi(s) = -0.09551 \cdot \exp\left[\frac{(2.75 - s)}{0.7176}\right], \quad s > 2.5$$

# Entrance barrier parameters



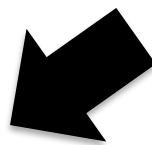
# Entrance barrier parameters



$$\sigma(\text{synthesis}) = \pi \hat{\lambda}^2 \sum_{l=0}^{l_{\max}} (2l+1) T_l \times P_l(\text{fusion}) \times P_{xn}^{\ell}(\text{survive})$$

Basic assumptions:

- After the capture stage, **the internal mass asymmetrical conditional saddle point** - the fusion barrier - must be overcome by the system.
- The **stochastic nature of the fusion process** is accompanied by **dissipation of energy and angular momentum**.
- All nucleons are transformed from projectile to the target by a **diffusion process**.



**Fusion - By - Diffusion - Smoluchowski equation**

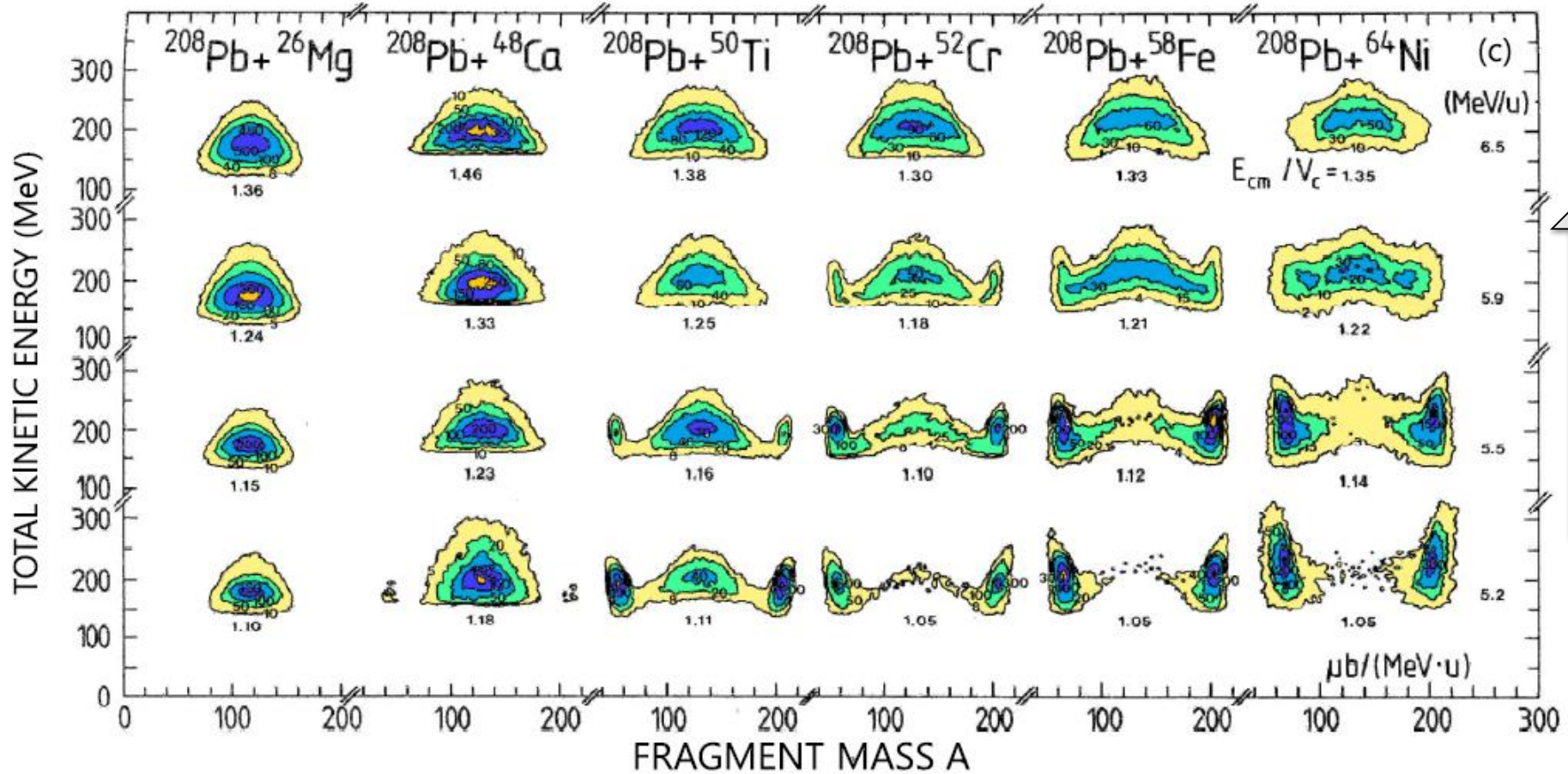
W. J. Świątecki, K. Siwek-Wilczyńska, and J. Wilczyński  
Phys. Rev. C **71**, 014602 (2005).

**Di - Nuclear - System - master equation**

G.G. Adamian, N.V. Antonenko, Eur. Phys. J. A (2022).



Experimental studies of the competition between fusion and quasifission in the formation of heavy and superheavy nuclei, D.J. Hinde, M. Dasgupta, E.C. Simpson, [Progress in Particle and Nuclear Physics, Volume 118](#), May 2021, 103856.



Letter

Access by Problemow Jadrowych Institute

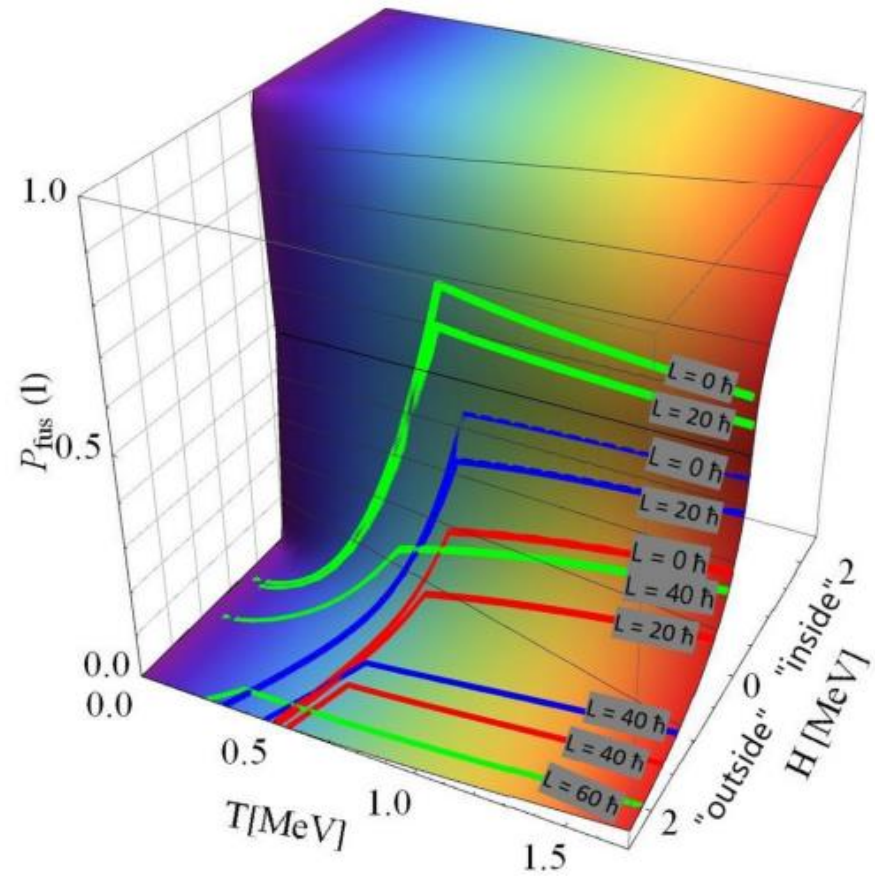
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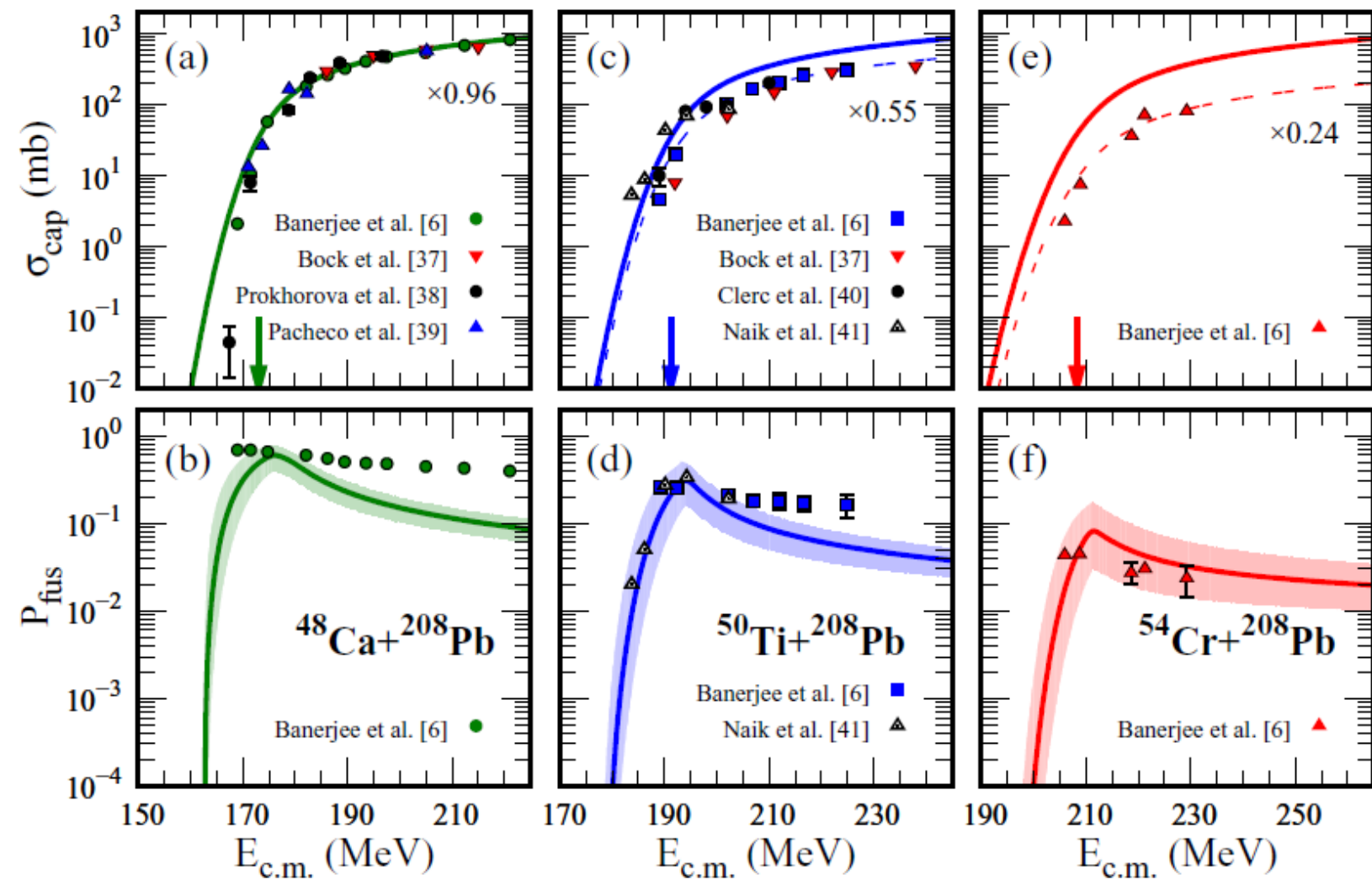
## Diffusion as a possible mechanism controlling the production of superheavy nuclei in cold fusion reactions

T. Cap, M. Kowal, and K. Siwek-Wilczyńska  
 Phys. Rev. C **105**, L051601 – Published 16 May 2022

Langevin in the overdamped limit:

$$P_{CN}(E) = \frac{1}{2} \left[ 1 - \operatorname{erf} \left( \frac{\Delta V}{T} \right) \right]$$

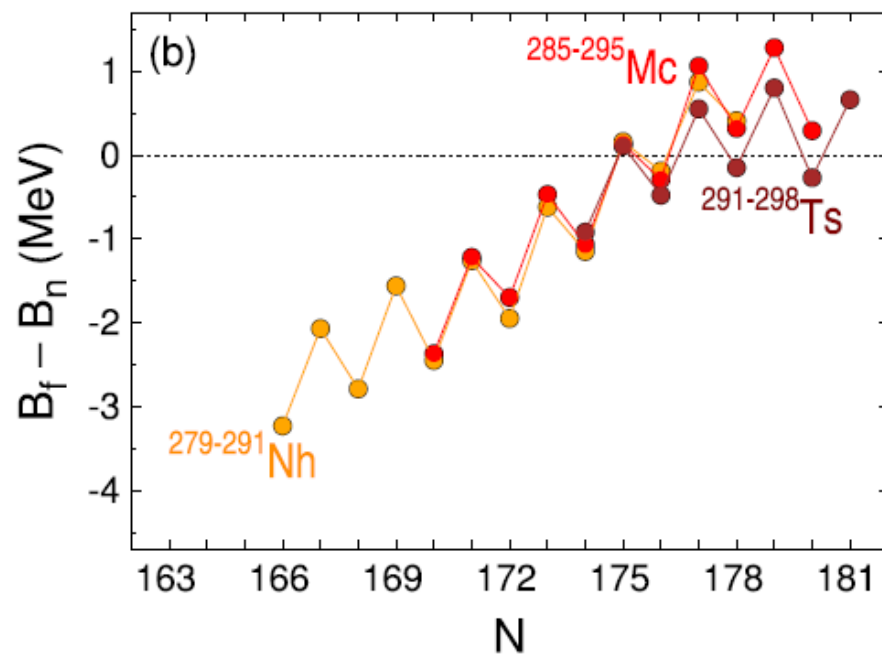
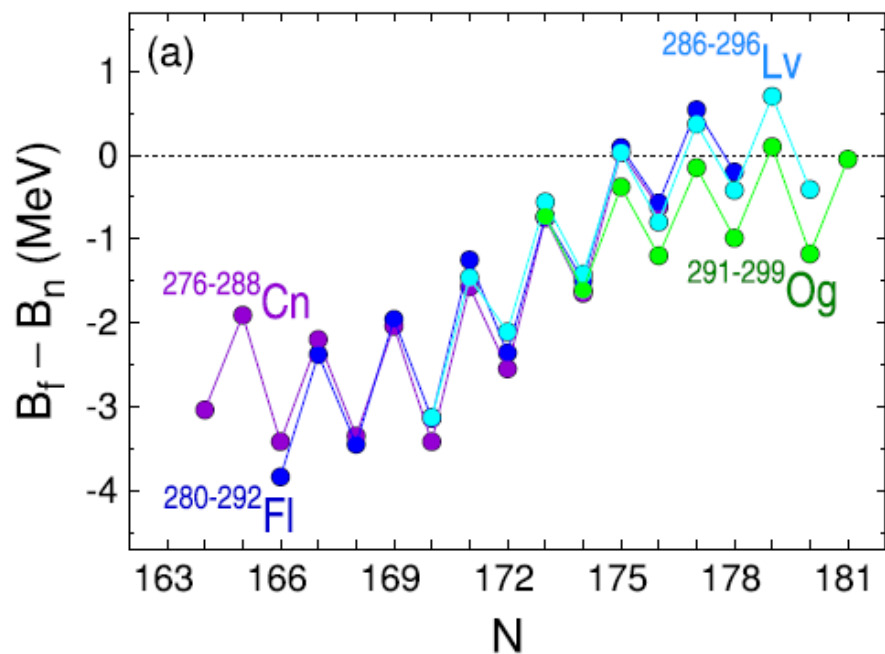




$$P_{\text{fus}} = \frac{1}{(l_{\text{max}} + 1)^2} \sum_{l=0}^{l_{\text{max}}} (2l + 1) P_{\text{fus}}(l),$$

$$\sigma(\text{synthesis}) = \pi \tilde{\lambda}^2 \sum_{l=0}^{l_{\max}} (2l+1) T_l \times P_l(\text{fusion}) \times P_{xn}^{\ell}(\text{survive})$$

$$\frac{\Gamma_n}{\Gamma_f} = \frac{gA^{2/3} \int_0^{U-B_n} \varepsilon \rho_{\text{GS}}(U - B_n - \varepsilon) d\varepsilon}{K_0 \int_0^{U-B_f} \rho_{\text{SP}}(U - B_f - \varepsilon) d\varepsilon}, \quad U > B_n > B_f > T \quad \longrightarrow \quad \frac{\Gamma_n}{\Gamma_f} = \frac{4mR_0^2}{\hbar^2} \exp\left(-\frac{B_n - B_f}{T}\right).$$

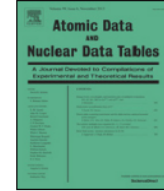




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## Atomic Data and Nuclear Data Tables

journal homepage: [www.elsevier.com/locate/adt](http://www.elsevier.com/locate/adt)



### Properties of heaviest nuclei with $98 \leq Z \leq 126$ and $134 \leq N \leq 192$

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<sup>b</sup>National Centre for Nuclear Research, Pasteura 7, 02-093 Warsaw, Poland



$$E_{tot}(\beta_{\lambda\mu}) = E_{macro}(\beta_{\lambda\mu}) + E_{micro}(\beta_{\lambda\mu})$$

$$E_{macro}(\beta_{\lambda\mu}) = \text{Yukawa} + \text{exponential}$$

$$E_{micro}(\beta_{\lambda\mu}) = \text{Woods - Saxon} + \text{pairing BCS}$$

$$R(\theta, \phi) = cR_0 \left\{ 1 + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{+\lambda} \beta_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right\}$$

I. Muntian, Z. Patyk and A. Sobiczewski, Acta Phys. Pol. B 32, 691 (2001).

H. J. Krappe, J. R. Nix and A. J. Sierk, Phys. Rev. C20, 992 (1979).

S. Cwiok, J. Dudek, W. Nazarewicz, J. Skalski and T. Werner, Comput. Phys. Commun. 46, 379 (1987).

