



ROZPAD BETA W UOGÓLNIONYM MODELU FUNKCJONAŁU GĘSTOŚCI

SEMINARIUM FIZYKI JĄDRA ATOMOWEGO, 18.05.2017

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INTRODUCTION

1. A few words about Nuclear Theory, Skyrme interaction and DFT theory
2. How to build realistic nuclear wave function upon a DFT state?

3. Gamow-Teller transitions; mirror nuclei $T=1/2$; GS to GS transitions, the quenching of g_A
4. Gamow-Teller response function
5. Gamow-Teller sum rule
6. Spin-orbit detector of single particle orbitals

STATUS IN THEORY OF NUCLEAR STRUCTURE

Physics of Hadrons

Degrees of Freedom



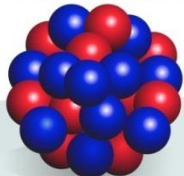
quarks, gluons



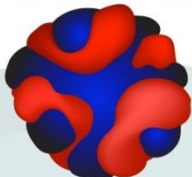
constituent quarks



baryons, mesons



protons, neutrons



nucleonic densities and currents



collective coordinates

Energy (MeV)

940
neutron mass

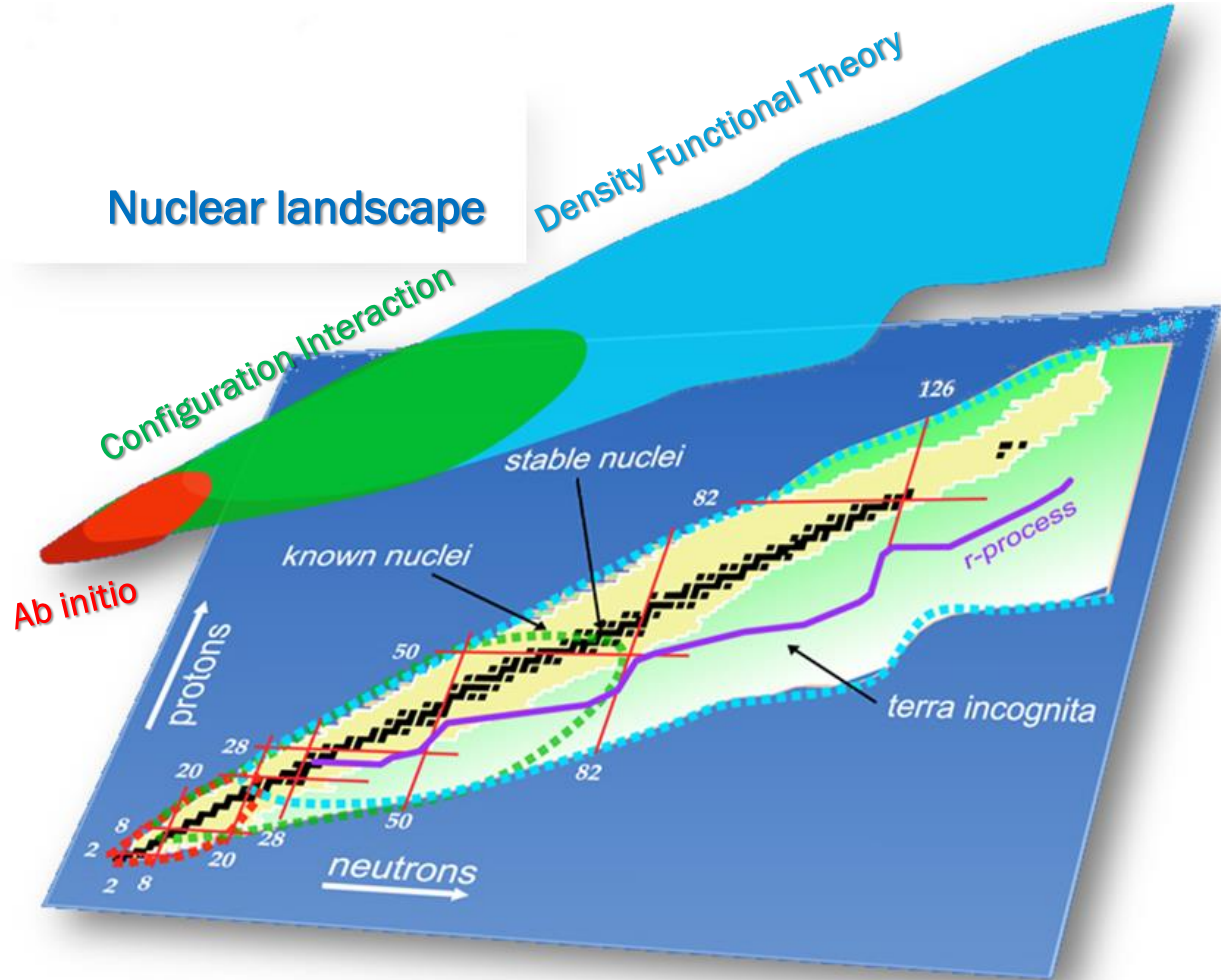
140
pion mass

8
proton separation
energy in lead

1.12
vibrational
state in tin

0.043
rotational
state in uranium

Physics of Nuclei



NSM & DFT

NSM

DFT-rooted
No-Core
Configuration-
Interaction

DFT

2-body Hamiltonian; correlations included after conf. mixing

2-body Hamiltonian; correlations included in deformed solution

Realistic wave function with laboratory-frame quantum numbers

Spontaneous symmetry breaking

Core approximation and restricted valence space;
huge matrices to diagonalize etc.

Not directly applicable to spectra and transition rates

Good for nuclear spectroscopy: spectra, transition rates etc.

Good for nuclear spectroscopy: quadrupole moments, masses radii etc.

1-body mean field hamiltonian to generate deformed configurations (DFT)

2-body part of Hamiltonian to mix deformed configurations

A model without neither a core nor the restricted valence space; it includes core-polarization effects

SKYRME NN INTERACTION

Effective interaction!!!

Necessary to separate long from short range (high momenta) physics.
 Due to the hard core, one needs to replace short distance by local correcting potentials

Ultraviolet cut-off

$v_{SK}(q) = v_{SK}(0) + v'_{SK}(0)q^2 + \dots$ ← Low energy nuclear physics is independent on high momenta transfer

$$\begin{aligned}
 v_{SK}(\vec{r}_1 - \vec{r}_2) = & t_0(1+x_0\hat{P}_\sigma)\delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2}t_1(1+x_1\hat{P}_\sigma)[\overleftarrow{k}^2\delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2)\overrightarrow{k}^2] \\
 & + t_2(1+x_2\hat{P}_\sigma)\overleftarrow{k}\delta(\vec{r}_1 - \vec{r}_2)\overrightarrow{k} + \frac{1}{6}t_3(1+x_3\hat{P}_\sigma)\rho^\alpha\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)\delta(\vec{r}_1 - \vec{r}_2) \\
 & + iW_0(\sigma_1 + \sigma_2)(\overleftarrow{k} \times \delta(\vec{r}_1 - \vec{r}_2)\overrightarrow{k})
 \end{aligned}$$

NN Skyrme interaction


Spin-Orbit term

Description of 3-body terms

FOUNDATION OF DENSITY FUNCTIONAL THEORY

HK theorem: Energy is a functional of density.

$$E_{SHF}[\rho] = T + E_{SK} + E_C$$

 long range physics
 Exact Coulomb interaction

$$E_{SHF}[\rho] = \frac{\hbar^2}{2m} \left(1 - \frac{1}{A}\right) \int \tau_0(\vec{r}) d\vec{r} + \sum_{t=0,1} \int [\mathcal{H}_t^e + \mathcal{H}_t^o] d\vec{r} + \frac{e^2}{2} \int \frac{\rho_p(\vec{r}_1)\rho_p(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|} d\vec{r}_1 d\vec{r}_2 - \frac{e^2}{2} \int \frac{\rho_p(\vec{r}_1, \vec{r}_2)\rho_p(\vec{r}_2, \vec{r}_1)}{|\vec{r}_1 - \vec{r}_2|} d\vec{r}_1 d\vec{r}_2$$

$$\mathcal{H}_t^e = C_t^\rho \rho_t^2 + C_t^{\Delta\rho} \rho_t \Delta \rho_t + C_t^\tau \rho_t \tau_t + C_t^J \vec{J}_t^2 + C_t^{\nabla J} \rho_t \vec{\nabla} \cdot \vec{J}_t$$

$$\mathcal{H}_t^o = C_t^s \vec{s}_t^2 + C_t^{\Delta s} \vec{s}_t \Delta \vec{s}_t + C_t^T \vec{s}_t \vec{T}_t + C_t^j \vec{j}_t^2 + C_t^{\nabla j} \vec{s}_t (\vec{\nabla} \times \vec{j}_t)$$

10 linearly independent parameters

Variation over density



MEAN FIELDS

Kohn-Sham equation

$$(\hat{T} + V_{KS}(\vec{r}))\phi_i(\vec{r}) = E_i\phi_i(\vec{r})$$

HOW TO CONSTRUCT REALISTIC WAVE FUNCTION?

Restoration of spontaneously broken symmetries

$$|\varphi_i; IMK; TT_z\rangle = \frac{1}{\sqrt{\mathcal{N}}} \hat{P}_{T_z T_z}^T \hat{P}_{MK}^I |\varphi_i\rangle$$

Isospin and K-quantum number mixing

$$|\varphi_i; I_n M; T_z\rangle = \sum_{n, K, T \geq |T_z|} a_{nIT}^{\varphi_i} |\varphi_i; I^\pi MK; TT_z\rangle$$

NCCI method:

$$|\Phi, IM; T_z\rangle = \sum_i c_i |\varphi_i; IM; T_z\rangle$$

ELECTROWEAK CURRENT

$$j^\mu = \frac{1}{2} \bar{\Psi} \gamma^\mu (1 \mp \gamma^5) \Psi$$

vector current
(Fermi decay)

$$\hat{O}_F = \tau$$

$$|H_{if}|^2 = \frac{g_V^2}{2J_i + 1} |M_F|^2 + \frac{g_A^2}{2J_i + 1} |M_{GT}|^2$$

CVC hypothesis

superallowed Fermi beta decay

axial-vector current
(Gamow-Teller decay)

$$\hat{O}_{GT} = \sigma \tau$$

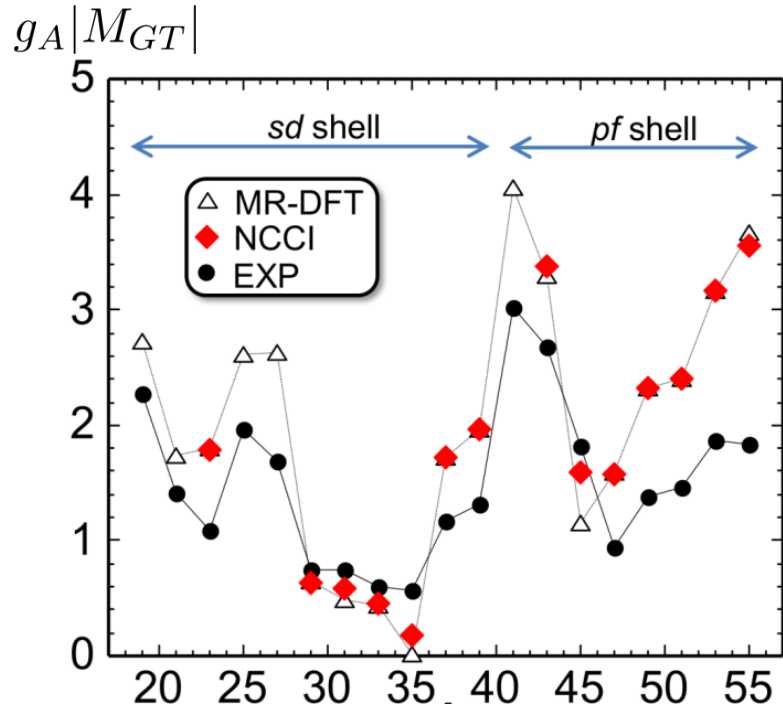
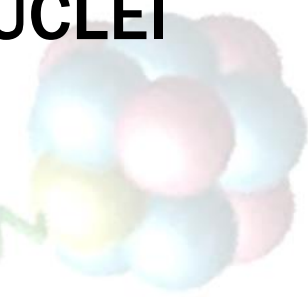
PCAC hypothesis

the quenching effect of g_A

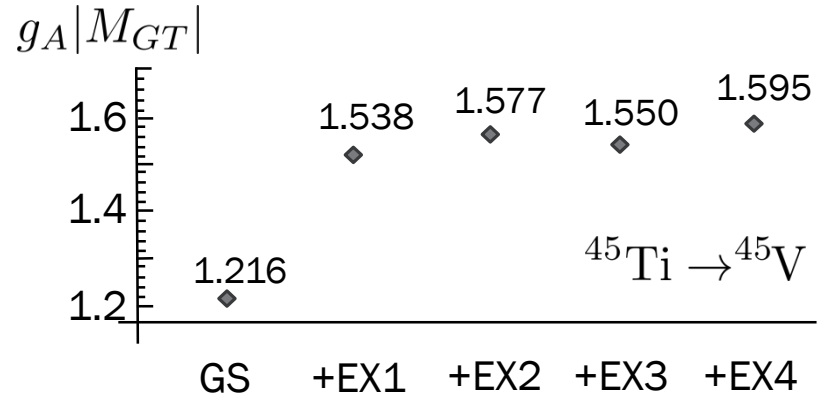
W. Satuła, et al,
Beta-decay study within multi-reference DFT
Phys. Rev. Lett. **106** 132502 (2011)

M. Konieczka, et al,
Beta-decay study within multi-reference DFT
Phys. Rev.C **93** 042501(R) (2016)

GAMOW TELLER BETA DECAY – MIRROR NUCLEI GS TO GS TRANSITIONS IN NCCI



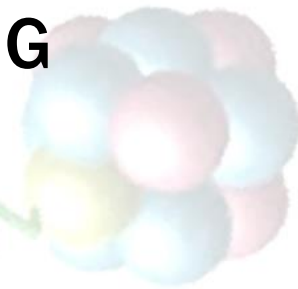
NCCI calculations usually with
3 -5 configurations included



Convergence at MR-DFT !!!

Except for $A=45$ transition configuration mixing
does not change the MR-DFT result !!!

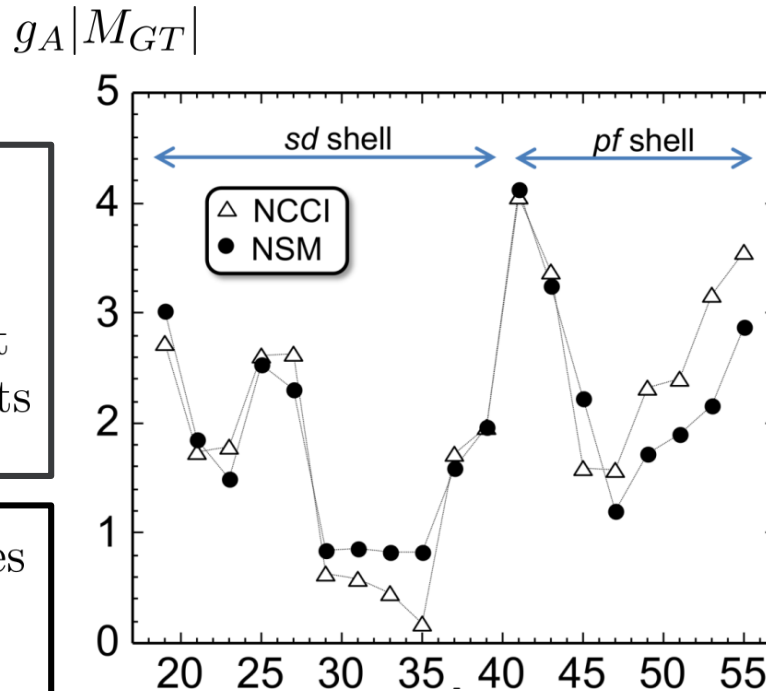
QUENCHING OF AXIAL-VECTOR COUPLING CONSTANT



NCCI vs NSM

NCCI is free from a core approximation and takes into account core polarization effects

Different model spaces
Different treatment of correlations



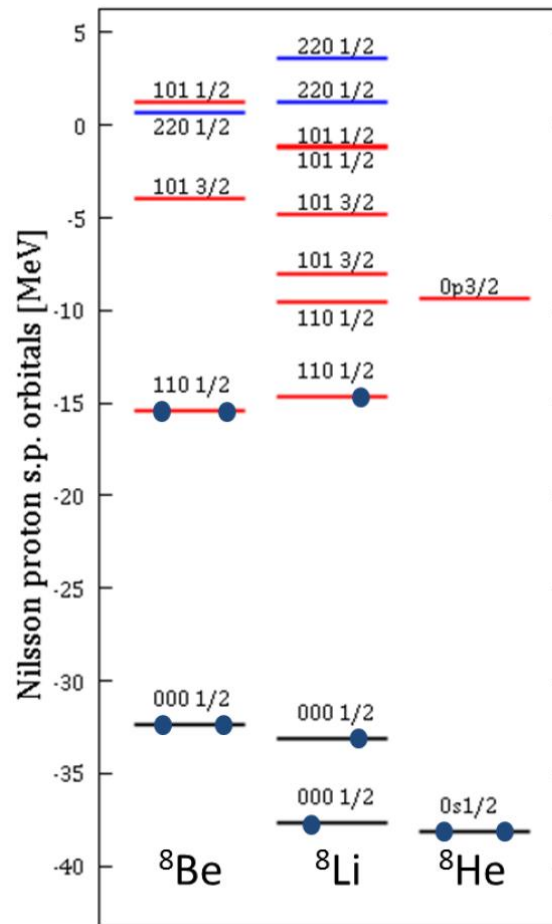
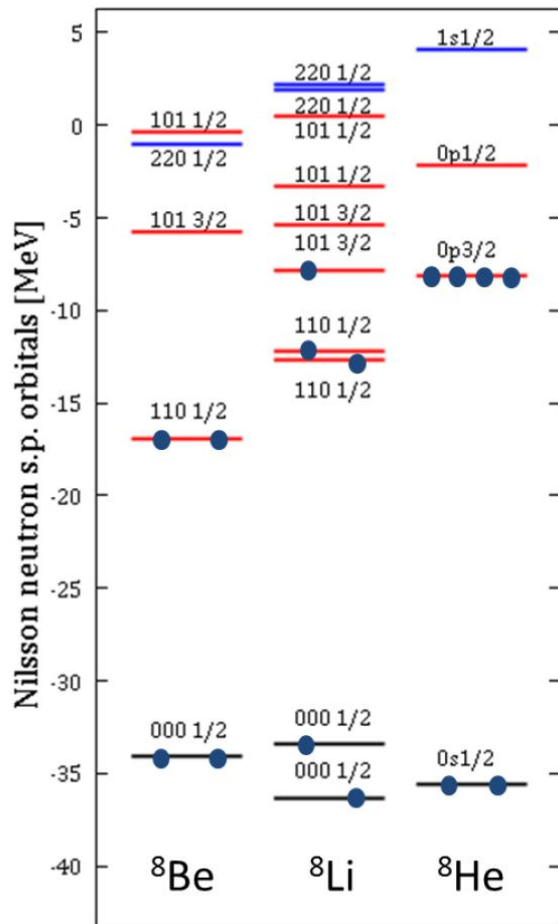
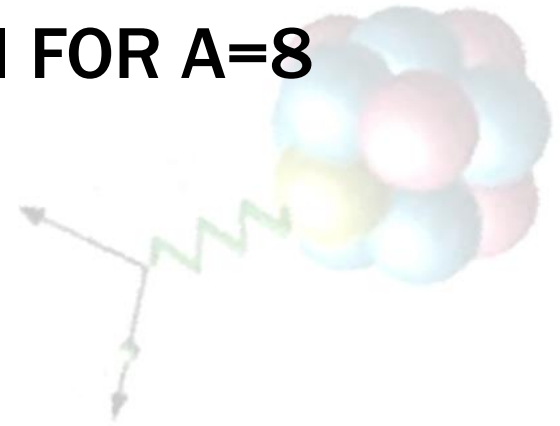
Quenching – possible solutions:

- ~~◆ non-nucleonic degree of freedom~~
- ~~◆ The effect of core approximation~~
- ◆ two-body currents

$$g_A^{\text{eff}} = q g_A \quad \text{Quenching of } g_A \quad g_A \approx 1.2701$$

q	MR-DFT	NCCI	NSM
<i>sd</i> -shell	0.77	0.78	0.77
<i>pf</i> -shell	0.75	0.69	0.74

CONFIGURATION SPACE OF NCCI FOR A=8



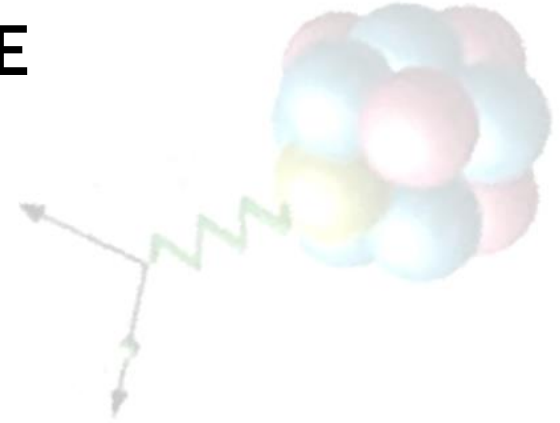
Isospin restoration



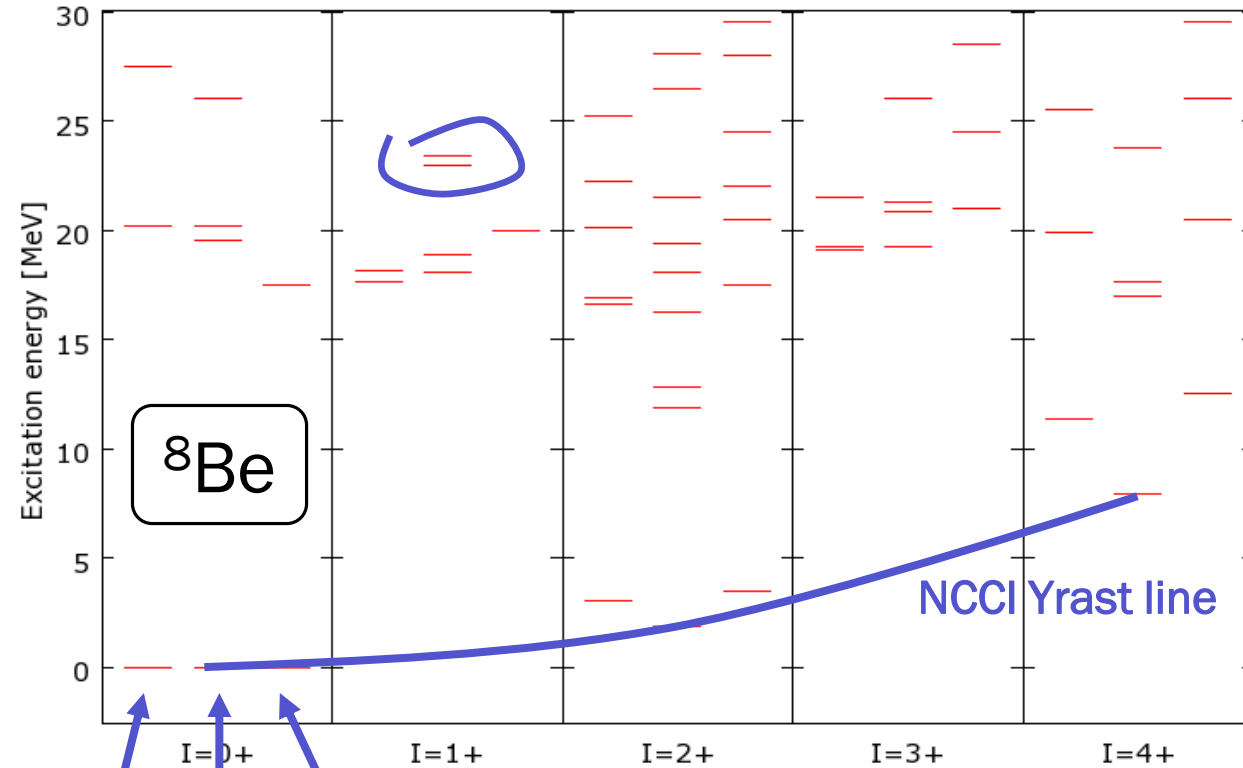
one nucleonic-type excitation
only in N=Z nuclei

8Be - 5 1p1h config.
8Li - 12 1p1h config.
8He - 5 1p1h config.

SPECTROSCOPY OF ^8Be



Unassigned doublet in the exp of about 23MeV



Characteristics of ^8Be GS

$$\diamond R_{4/2} = \frac{E_{I=4_1^+}}{E_{I=2_1^+}}$$

EXP	TH
3.75	3.77

- ♦ deformation parameters

$$\beta_2 = 0.69$$

$$r_x = 1.09\text{fm}, r_y = 1.09\text{fm}, r_z = 1.96\text{fm}$$

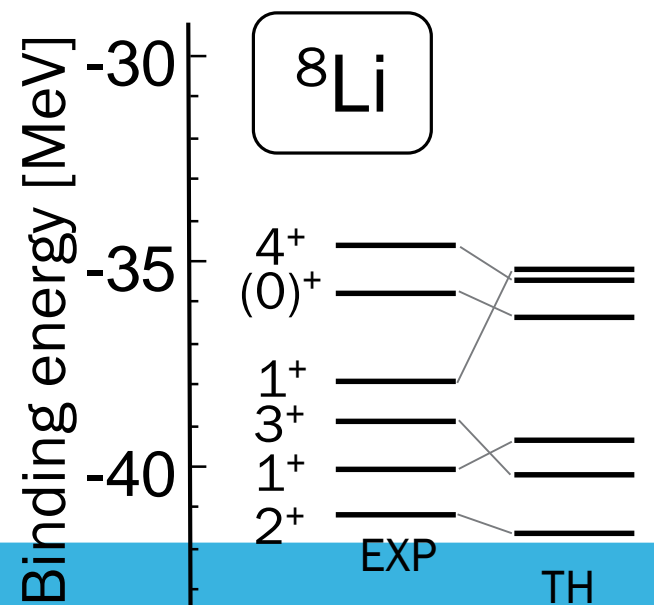
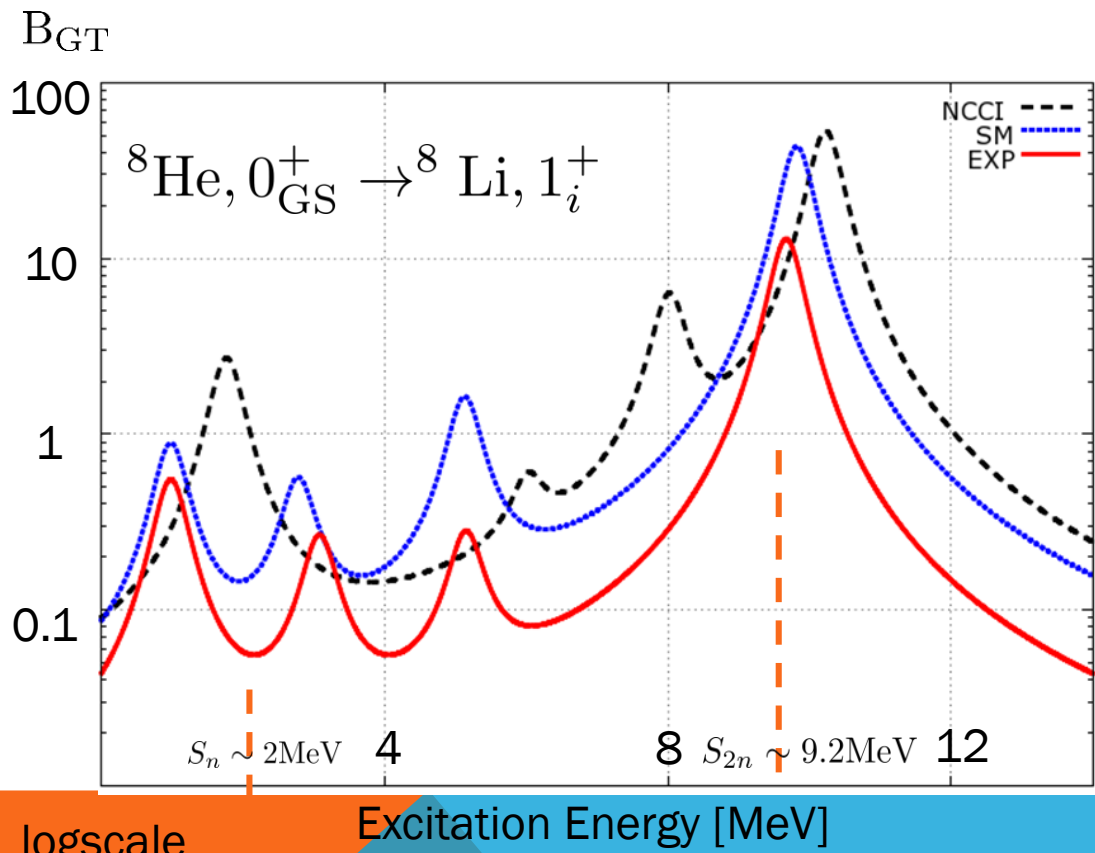
Root-mean-square radii

EXP M. Konieczka, et al, in preparation

AB-INITIO

P. Maris, et al, Phys. Rev.C **91** 014310 (2015)

GAMOW-TELLER RESPONSE OF ^8Li

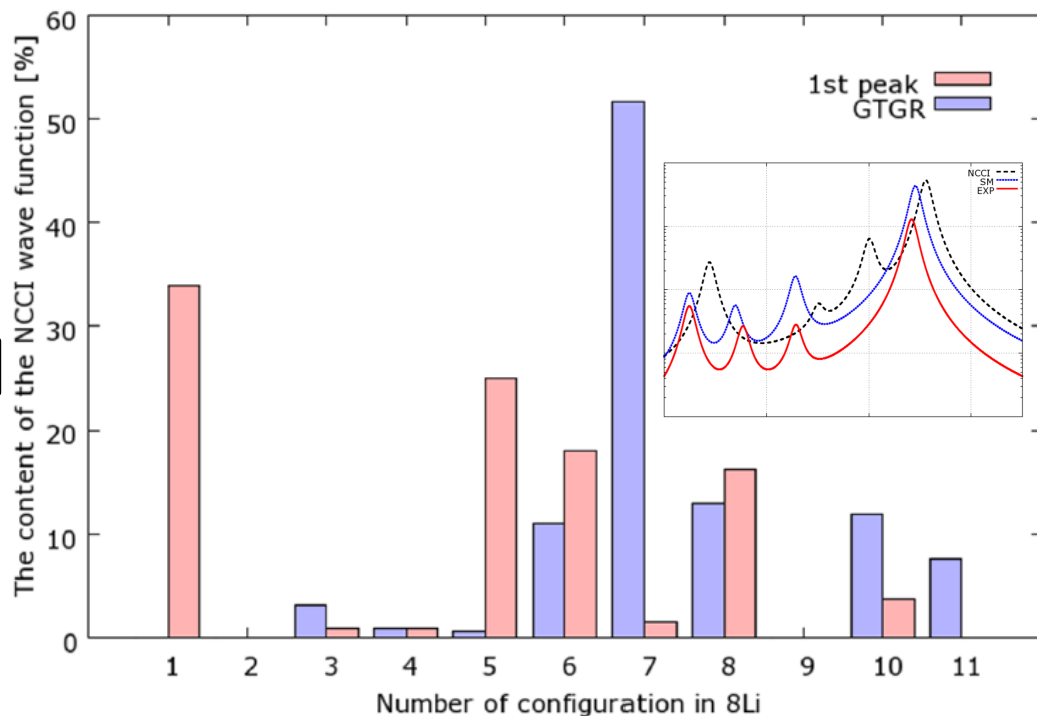


Shell-Model: F.C. Barker, E.K. Warburton, Nuc. Phys. A **487**, 269 (1988).

GAMOW-TELLER RESPONSE OF 8LI

i	${}^8\text{Li}; \varphi_i\rangle$	E_{HF}	β_2	γ	K
1	$ \nu 101 3/2+\rangle \otimes \pi 110 1/2-\rangle$	-39.08	0.38	0°	1_Z
2	$ \nu 101 3/2+\rangle \otimes \pi 110 1/2+\rangle$	-39.03	0.36	0°	2_Z
3	$ \nu 101 1/2+\rangle \otimes \pi 110 1/2+\rangle$	-34.04	0.36	0°	1_Z
4	$ \nu 101 1/2-\rangle \otimes \pi 110 1/2+\rangle$	-33.44	0.35	0°	0_Z
5	$ \nu 110 1/2+\rangle \otimes \pi 110 1/2-\rangle$	-36.51	0.04	60°	0_Z
6	$ \nu 101 3/2-\rangle \otimes \pi 101 3/2+\rangle$	-35.68	0.03	0°	0_Y
7	$ \nu 101 1/2-\rangle \otimes \pi 110 1/2-\rangle$	-31.19	0.03	60°	1_Z
8	$ \nu 101 3/2-\rangle \otimes \pi 101 3/2+\rangle$	-35.54	0.03	0°	0_X
9	$ \nu 101 3/2+\rangle \otimes \pi 101 1/2+\rangle$	-32.34	0.12	0°	2_Z
10	$ \nu 101 3/2-\rangle \otimes \nu 110 1/2-\rangle$ $\otimes \nu 101 1/2+\rangle \otimes \pi 101 3/2+\rangle$	-29.25	0.02	60°	0_Y
11	$ \nu 101 3/2-\rangle \otimes \nu 110 1/2+\rangle$ $\otimes \nu 101 1/2+\rangle \otimes \pi 101 3/2+\rangle$	-29.06	0.03	60°	1_Y

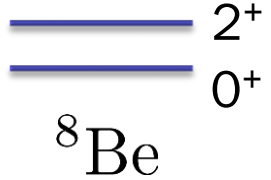
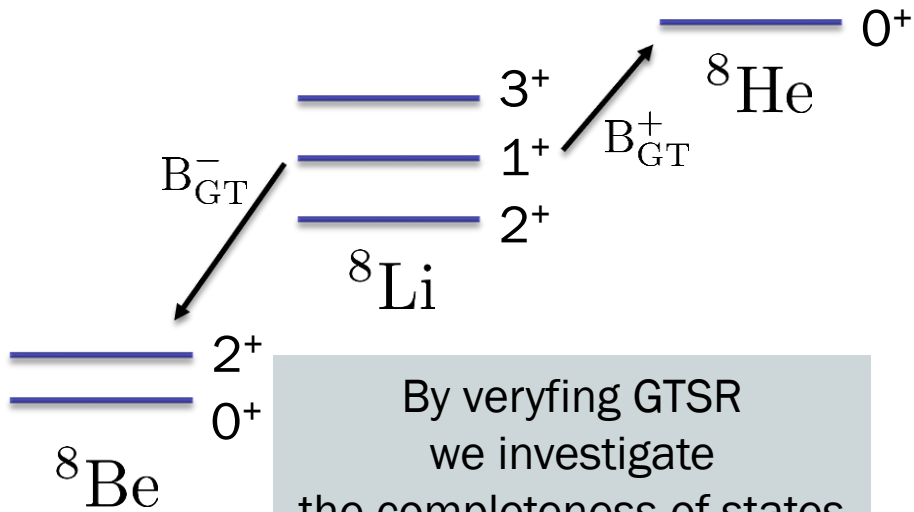
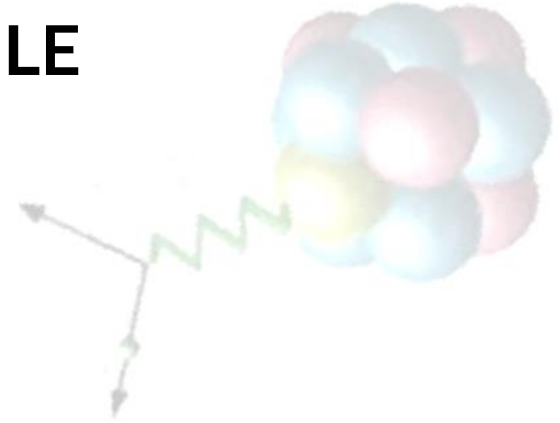
i	${}^8\text{He}; i\rangle$	E_{HF}	β_2	γ	$\langle j \rangle$
1	$\nu p_{3/2} \otimes \pi s_{1/2}$	-37.26	0	0°	0
2	$ \nu 101 3/2-\rangle^{-1} \otimes \nu 101 1/2+\rangle^1$	-32.47	0.14	0°	2_Z
3	$ \nu 101 3/2+\rangle^{-1} \otimes \nu 101 1/2-\rangle^1$	-30.81	0.03	60°	1_Y
4	$ \nu 110 1/2+\rangle^{-1} \otimes \nu 101 1/2+\rangle^1$	-30.04	0.03	60°	0_Y
5	$ \nu 110 1/2+\rangle^{-1} \otimes \nu 101 1/2-\rangle^1$	-29.13	0.02	0°	1_Z



NCCI gives an opportunity to get the physical meaning of a GTR function in terms of mean field configurations !!!

GAMOW-TELLER SUM RULE

$$\sum_f \left[B_{GT}^-(I_i^\pi \rightarrow I_f^\pi) - B_{GT}^+(I_i^\pi \rightarrow I_f^\pi) \right] = 3(N - Z)$$

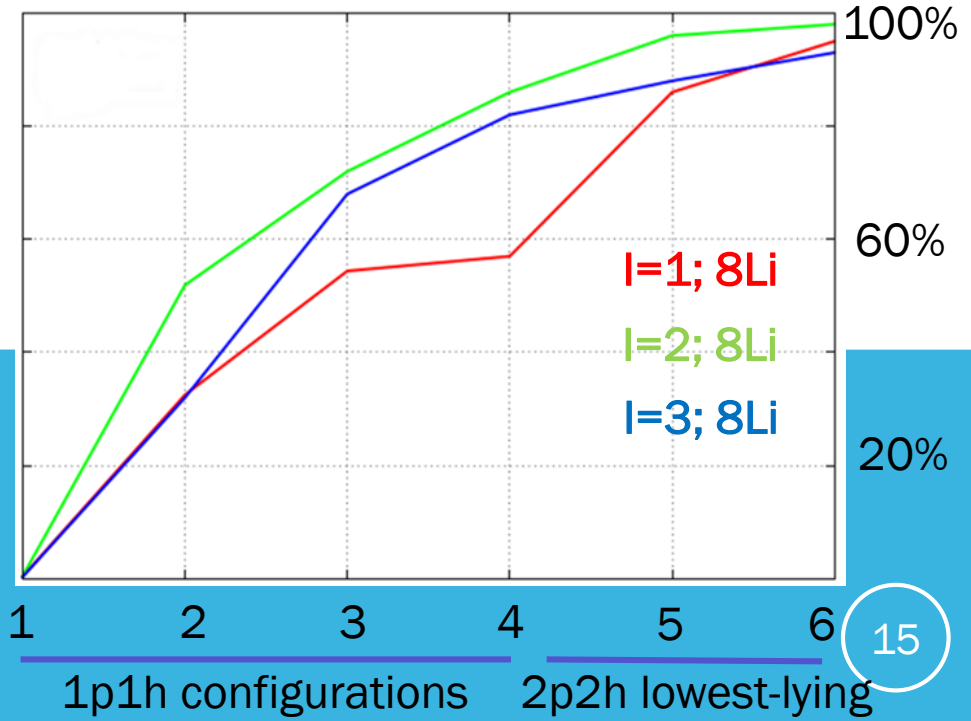


By verifying GTSR we investigate the completeness of states with given multipole

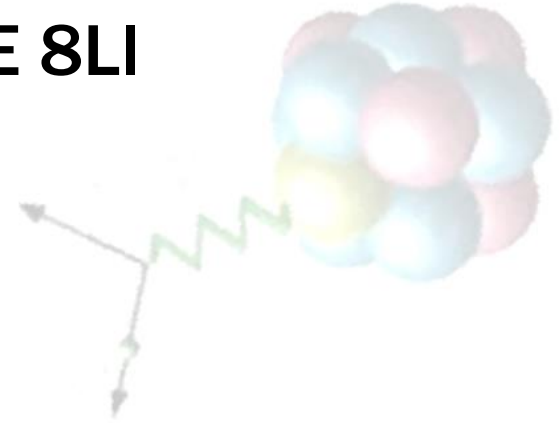
We keep all conf in ^8Li and change only those in ^8Be and ^8He

1 1p1h configuration in ^8Be cannot be converged; lowest-lying 2p2h are covering the missing correlations

Sum rule saturation against number of conf in ^8Be

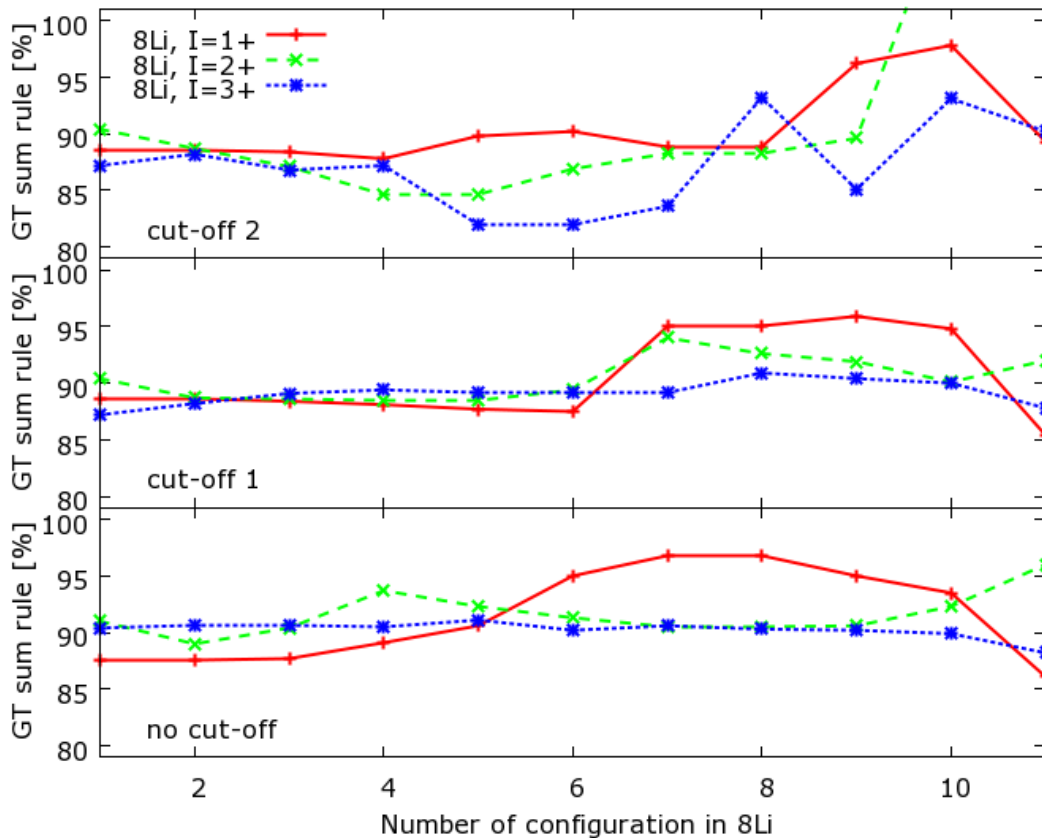


GAMOW-TELLER SUM RULE 8Li



$$\sum_f \left[B_{GT}^-(I_i^\pi \rightarrow I_f^\pi) - B_{GT}^+(I_i^\pi \rightarrow I_f^\pi) \right] = 3(N - Z)$$

Gamow-Teller sum rule saturation
against number of configurations in 8Li



Cut-off on very low (0.001)
eigenvalues of norm matrix

No cut-off

GAMOW-TELLER RESPONSE OF ^{24}Mg

$$^{24}\text{Al}, 4_{\text{GS}}^+ \rightarrow ^{24}\text{Mg}, (3_i^+, 4_i^+, 5_i^+)$$

Isospin restoration

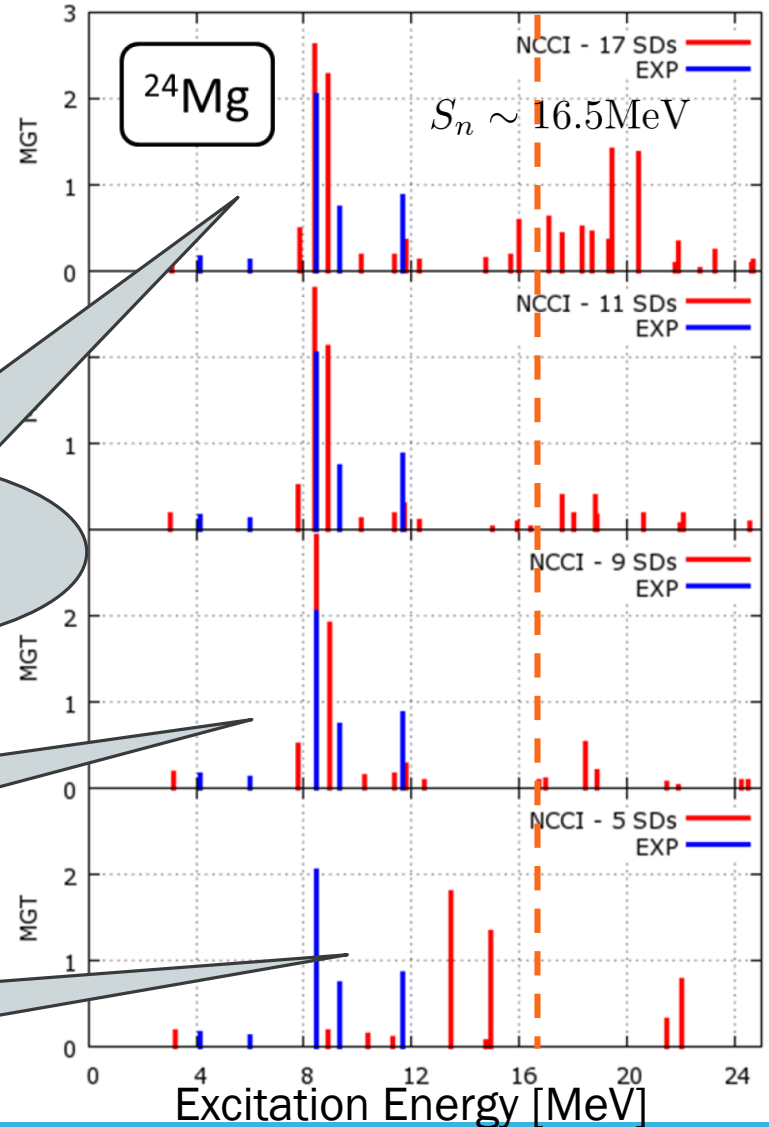
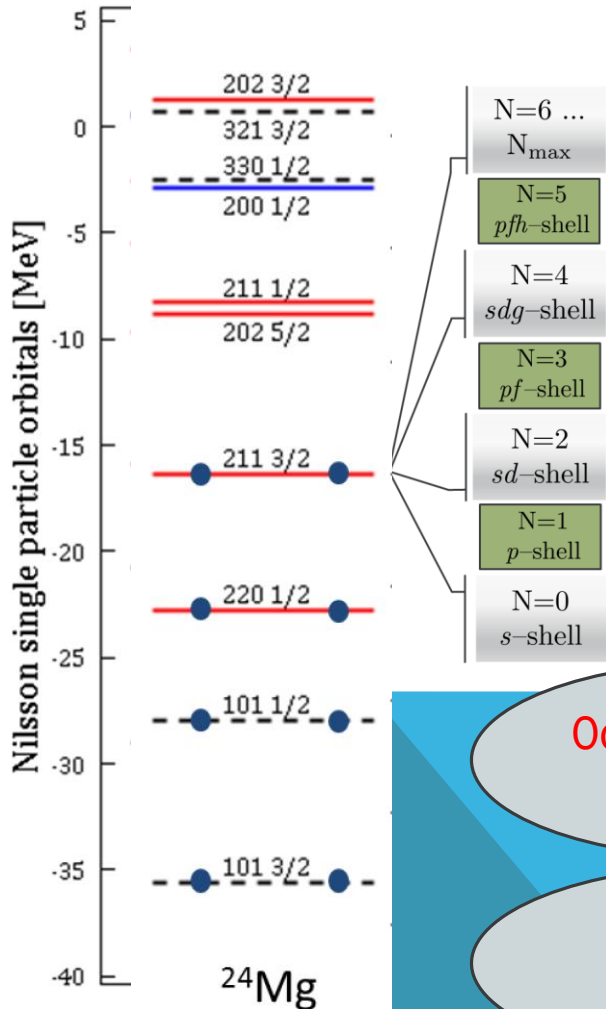
neutron excitation
only in N=Z nuclei

In overall 17
1p1h configurations

entire sd-shell

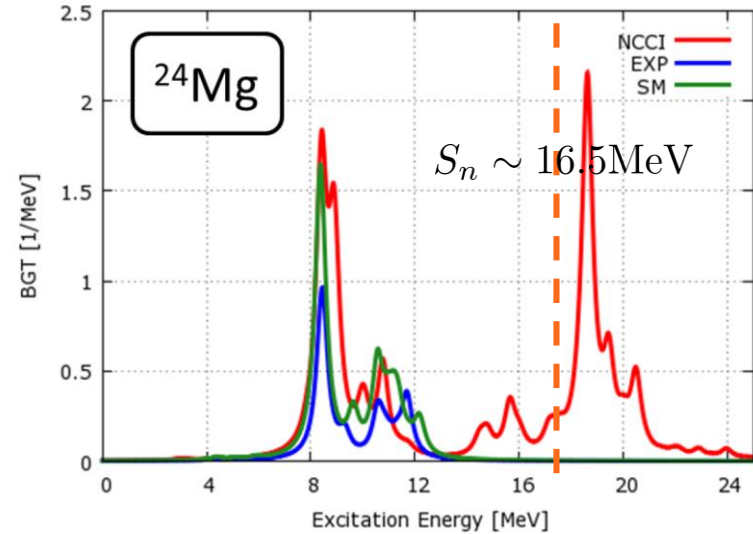
$0d_{5/2} + 211 \frac{1}{2}$
($0d_{3/2}$)

$0d_{5/2}$ only

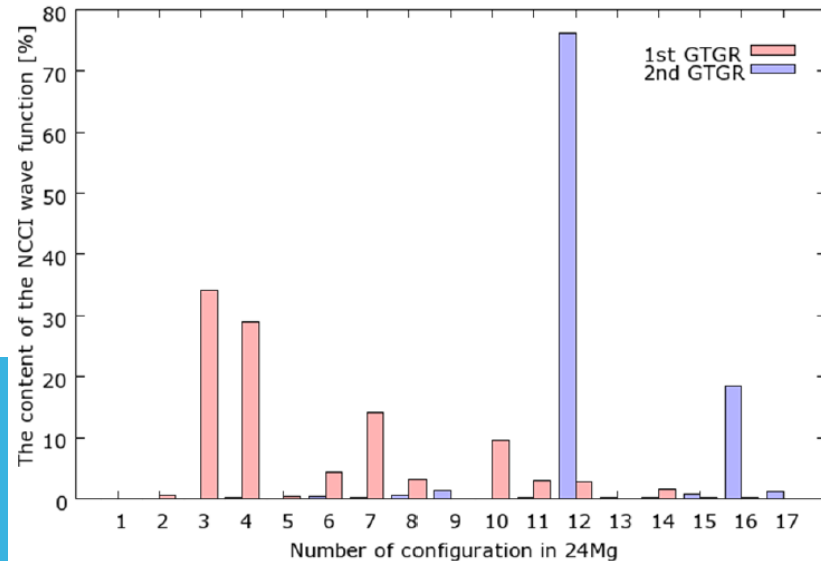


GAMOW-TELLER RESPONSE OF ^{24}Mg

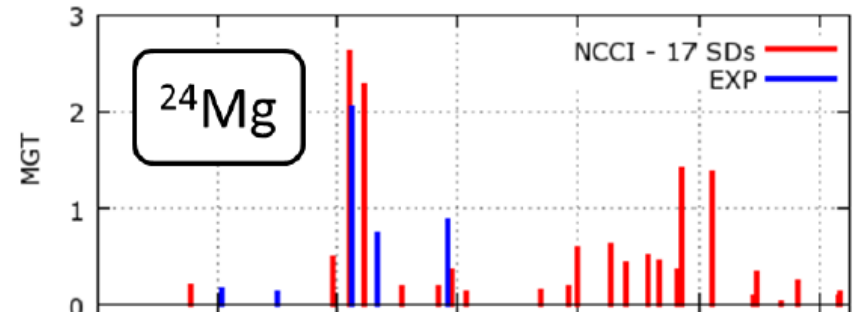
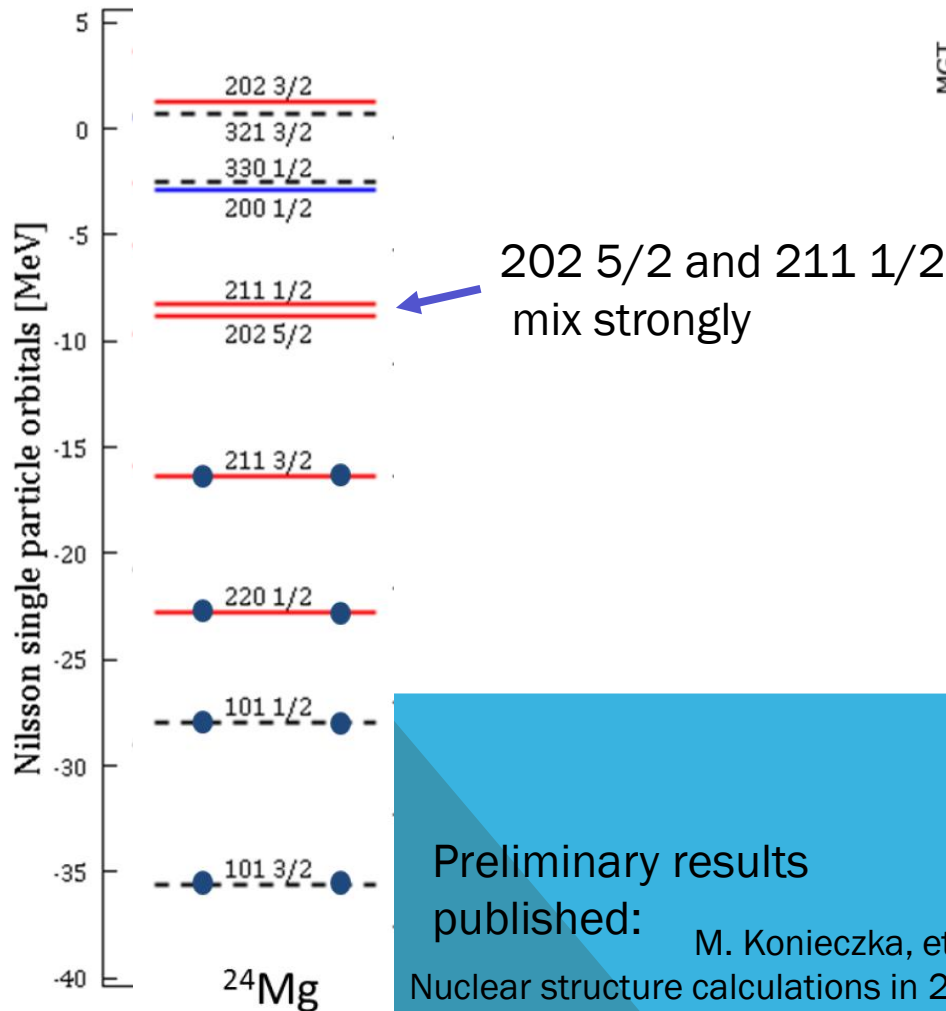
i	$ ^{24}\text{Mg}; i\rangle$	E_{HF}	β_2	γ	K
1	g.s.	-194.33	0.42	0°	0
2	$ \nu 211 3/2-\rangle^{-1} \otimes \nu 202 5/2-\rangle^1$	-187.92	0.34	0°	1_Z
3	$ \nu 211 3/2+\rangle^{-1} \otimes \nu 202 5/2-\rangle^1$	-187.25	0.34	0°	4_Z
4	$ \nu 211 3/2+\rangle^{-1} \otimes \nu 211 1/2-\rangle^1$	-187.46	0.43	0°	2_Z
5	$ \nu 211 3/2-\rangle^{-1} \otimes \nu 211 1/2-\rangle^1$	-184.89	0.40	0°	1_Z
6	$ \nu 220 1/2-\rangle^{-1} \otimes \nu 202 5/2-\rangle^1$	-183.34	0.24	0°	2_Z
7	$ \nu 220 1/2+\rangle^{-1} \otimes \nu 202 5/2-\rangle^1$	-183.27	0.23	0°	3_Z
8	$ \nu 211 3/2+\rangle^{-1} \otimes \nu 200 1/2+\rangle^1$	-181.79	0.36	0°	1_Z
9	$ \nu 211 3/2+\rangle^{-1} \otimes \nu 200 1/2-\rangle^1$	-181.50	0.34	0°	2_Z
10	$ \nu 220 1/2+\rangle^{-1} \otimes \nu 211 1/2-\rangle^1$	-181.99	0.35	0°	1_Z
11	$ \nu 220 1/2-\rangle^{-1} \otimes \nu 211 1/2-\rangle^1$	-180.78	0.33	0°	0_Z
12	$ \nu 211 3/2-\rangle^{-1} \otimes \nu 202 3/2+\rangle^1$	-178.83	0.34	0°	3_Z
13	$ \nu 211 3/2+\rangle^{-1} \otimes \nu 202 3/2+\rangle^1$	-177.16	0.33	0°	0_Z
14	$ \nu 220 1/2-\rangle^{-1} \otimes \nu 200 1/2-\rangle^1$	-177.04	0.27	0°	0_Z
15	$ \nu 220 1/2+\rangle^{-1} \otimes \nu 200 1/2-\rangle^1$	-176.94	0.25	0°	1_Z
16	$ \nu 220 1/2-\rangle^{-1} \otimes \nu 202 3/2+\rangle^1$	-174.00	0.25	0°	2_Z
17	$ \nu 211 3/2+\rangle^{-1} \otimes \nu 202 3/2+\rangle^1$	-173.47	0.24	0°	1_Z



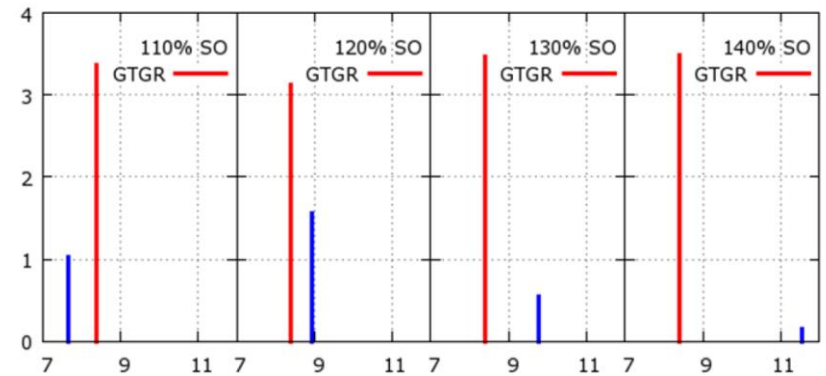
The content of GT resonances



SPIN-ORBIT DETECTOR FOR SP ORBITALS ?



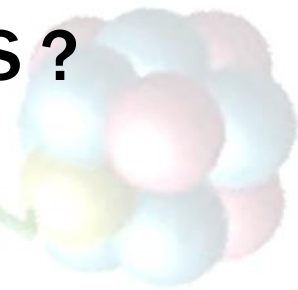
The change of peaks with the change of the strength of the spin-orbit interaction



The change of peaks with the change of the strength of the spin-orbit interaction

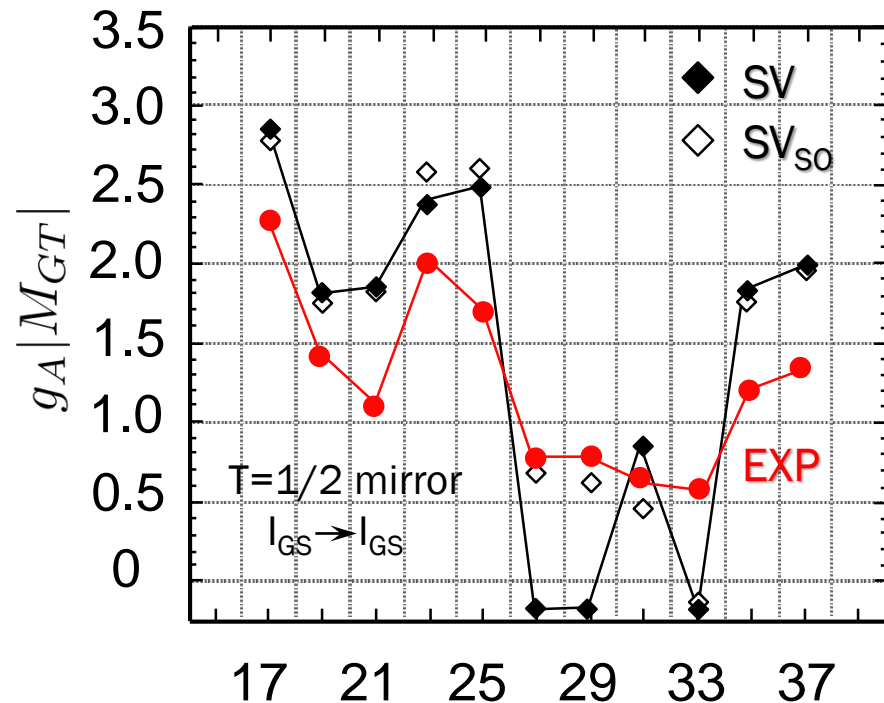
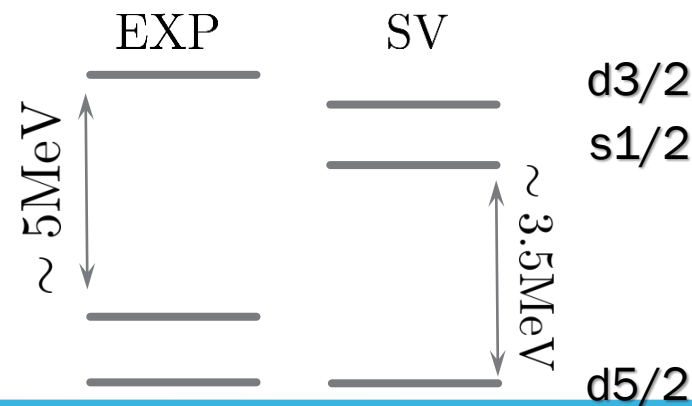
Preliminary results
published: M. Konieczka, et al,
Nuclear structure calculations in ^{20}Ne with NCCI model
Acta Phys. Pol. B48 293 (2017)

SPIN-ORBIT DETECTOR FOR SP ORBITALS ?

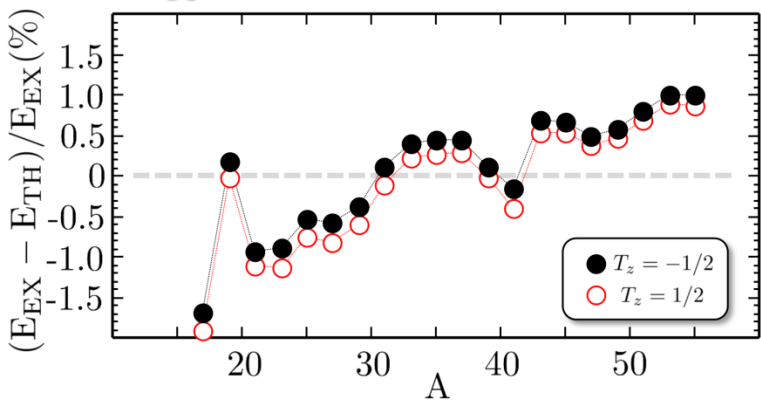


destructive interference
of $1s_{1/2}$ and $0d_{3/2}$ subshell

Single particle energies (sd shell) in ^{17}O



SV_{s0} - Spin-orbit tuning



$$\mathcal{M}_{GT}^{sp}(1s_{1/2}1s_{1/2}) = \sqrt{2} \approx 1.4$$

$$\mathcal{M}_{GT}^{sp}(0d_{3/2}0d_{3/2}) = -\frac{2}{\sqrt{5}} \approx -0.9$$

SUMMARY AND OUTLOOK

MR-DFT and NCCI results show that

1. MR-DFT and NCCI results are fully consistent in Fermi MEs
2. MR-DFT and NCCI results are fully consistent in GT ME calculations of mirror nuclei ($T=1/2$) except for $A=45$ transition

MR-DFT seems to be sufficient to describe transitions between ground states

3. The quenching effect does not depend on the core approximation.

4. NCCI is able to capture general feature of Gamow-Teller response in ${}^8\text{Li}$.
5. Small amount of NCCI states is sufficient to satisfy 90% of GTSR
6. The spin-orbit int. may indicate the physical spacing between orbitals

VERY PRELIMINARY !!!

OTHER FIELDS OF INTENSIVE WORK

pn mixing correlations – PhD project of Paweł Bączyk
Zero-range tensor interaction
3D projection on isospin from HFB reference states
double beta decay, neutrino-nucleus scattering



ROZPAD BETA W UOGÓLNIONYM MODELU FUNKCJONAŁU GĘSTOŚCI

SEMINARIUM FIZYKI JĄDRA ATOMOWEGO, 18.05.2017

Our team:

Wojciech Satuła

Jacek Dobaczewski

Markus Korteleinen

Paweł Bączyk

Dziękuję za uwagę!