Własności oktupolowej przestrzeni kolektywnej

Leszek Próchniak

Plan

- Wprowadzenie
- Własności przestrzeni oktupolowej
  - Układ wewnętrzny
  - Operatory: moment pędu, hamiltonian
- Uwagi końcowe

- Aparatura
- Oktupole w Warszawie i okolicach. Długa historia
  - J. Chwaszczewska, R. Kaczarowski, J. Rudzińska, W. Kurcewicz, and J. Żylicz. *Possible $K^{\pi}=2^{-}$ and $K^{\pi}=1^{-}$ Octupole Bands in ^{232}U*. Contrib. Intern. Symp. Nucl. Struct., Dubna, p.59 (1968).
  - J. Błocki and W. Kurcewicz. *Octupole Vibrations of Even Nuclei in the Transuranic Region*. Phys. Lett. B **30** (1969), 458.

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  - L. P. Gaffney et al., Nature, **497** (2013) 199

    ...
- Doświadczenie
- Teoria
  1. W stronę fizyki
  2. W stronę matematyki

2. W stronę matematyki

S.G. Rohoziński, *Oscillator basis for octupole collective motion in nuclei*, J.Phys. G Nuclear and Particle Physics, **4** (1978) 1075

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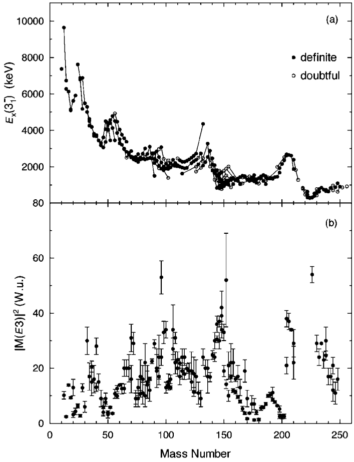
S.G. Rohoziński and L.P., *The octupole collective Hamiltonian. Does it follow the example of the quadrupole case?*, Walter Greiner Memorial Volume (2018).



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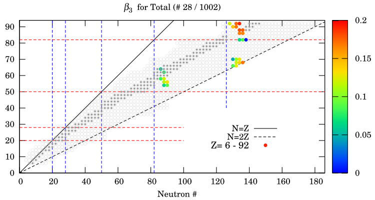

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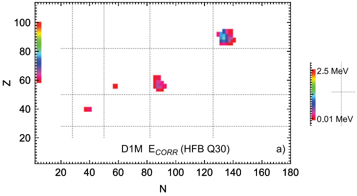


L. M. Robledo, 2015, J. Phys. G **42** 055109

7-wymiarowa nieprzywiedlna reprezentacja grupy O(3), $P = -I$
Zmienne zespolone $\alpha_{3\mu} = \alpha_\mu, \mu = -3, \dots, 3$ z warunkiem

$$\alpha_{-\mu} = (-)^\mu \alpha_\mu^*$$

Działanie grupy SO(3), $\boldsymbol{\omega} = (\phi, \theta, \psi)$ — kąty Eulera

$$\tilde{\alpha}_\mu = \sum_\nu \mathscr{D}^3_{\mu\nu}(\boldsymbol{\omega})\alpha_\nu, \qquad \mathscr{D}^3_{\mu\nu}(\boldsymbol{\omega}) = e^{i\mu\phi} d^3_{\mu\nu}(\theta) e^{i\nu\psi}$$

Dwa podstawowe przykłady

1.

$$r(\theta,\phi) = r_0\big(1 + \sum_\mu \alpha_\mu Y^*_{3\mu}(\theta,\phi)\big)$$

2.

$$\alpha_\mu \sim \langle Q_{3\mu} \rangle, \quad Q_{3\mu} \sim \sum_i r_i^3 Y_{3\mu}(\theta_i, \phi_i)$$

Zmienne rzeczywiste $a_0, a_m, b_m, m = 1, 2, 3$

$$\alpha_\mu = (a_\mu + i b_\mu)/\sqrt{2}$$
$$\alpha_0 = a_0$$
$$\alpha_{-\mu} = (-)^\mu (a_\mu - i b_\mu)/\sqrt{2}$$

Działanie grupy SO(3)

$$\tilde{a}_m = \sum_{k0} T^{++}_{mk}(\boldsymbol{\omega}) a_k + \sum_{k>0} T^{-+}_{mk}(\boldsymbol{\omega}) b_k$$
$$\tilde{b}_m = \sum_{k0} T^{+-}_{mk}(\boldsymbol{\omega}) a_k + \sum_{k>0} T^{--}_{mk}(\boldsymbol{\omega}) b_k$$

$$T^{\pm\pm}_{mk}(\boldsymbol{\omega}) \sim \left\{ \begin{array}{c} \cos m\phi \\ \sin m\phi \end{array} \right\} d^3_{mk}(\theta) \left\{ \begin{array}{c} \cos m\psi \\ \sin m\psi \end{array} \right\}$$

Jeszcze raz działanie grupy SO(3)

$$\tilde{\alpha}_\mu = D^{(+)}_{\mu 0}(\boldsymbol{\omega})a_0 + \sum_{k>0}\left(D^{(+)}_{\mu k}(\boldsymbol{\omega})a_k + D^{(-)}_{\mu k}(\boldsymbol{\omega})b_k\right)$$

$$\begin{aligned} D^{(+)}_{\mu 0} &= \mathscr{D}^3_{\mu 0} \\ D^{(+)}_{\mu k} &= \left(\mathscr{D}^3_{\mu k} + (-)^k \mathscr{D}^3_{\mu,-k}\right)/\sqrt{2} \\ D^{(-)}_{\mu k} &= i\left(\mathscr{D}^3_{\mu k} - (-)^k \mathscr{D}^3_{\mu,-k}\right)/\sqrt{2} \end{aligned}$$

$D^{(+)}_{\mu k}, D^{(-)}_{\mu k}$ tworzą macierz unitarną (tak jak $\mathscr{D}_{\mu\mu'}$)

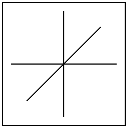
Analogiczne definicje dla kwadrupoli i innych multipoli

- Elipsoida, $\sum_{i,j=1}^{3} B_{ij} x_i x_j = 0$
- Tensory jako symetryczne, bezśladowe macierze kwadratowe 3×3, $\tilde{W} = RWR^T$
- Układ wewnętrzny: układ osi głównych elipsoidy, postać diagonalna macierzy symetrycznej

$$(\tilde{a}_{2m}, \tilde{b}_{2m}) \to (a_{20}, a_{22}, \boldsymbol{\omega})$$

$$\boldsymbol{\omega} \quad \longleftarrow \quad a_{21} = b_{21} = b_{22} = 0 \;\; \text{czyli} \;\; \alpha_{\pm 1} = 0, \alpha_2 = \alpha_{-2}$$

$$a_{20} = \beta \cos\gamma, \quad a_{22} = \beta \sin\gamma$$



Niezmienniczość ze względu na grupę O_h

Nieprzywiedlne reprezentacje O_h

IR	$A_1^\pm$	$A_2^\pm$	$E^\pm$	$F_1^\pm$	$F_2^\pm$
dim	1	1	2	3	3

QC

$$E^+: \quad a_{20}, a_{22}$$
$$F_2^+: \quad a_{21}, b_{21}, b_{22}$$

Przestrzeń oktupolowa

$$A_2^-: \quad b = b_3$$
$$F_1^-: \quad f_x = \sqrt{3/8}a_1 - \sqrt{5/8}a_3, \quad f_y = \sqrt{3/8}b_1 + \sqrt{5/8}b_3, \quad f_z = a_0$$
$$F_2^-: \quad g_x = \sqrt{5/8}a_1 + \sqrt{3/8}a_3, \quad g_y = -\sqrt{5/8}b_1 + \sqrt{3/8}b_3, \quad g_z = a_2$$

$$\alpha_\mu \to (a_m, b_m) \to (b, f_s, g_s)$$

$$\tilde{\alpha}_\mu = A_\mu(\boldsymbol{\omega})\, b + \sum_{s=x,y,z} [F_{\mu s}(\boldsymbol{\omega}) f_s + G_{\mu s}(\boldsymbol{\omega}) g_s]$$

Funkcje „kubiczne”

$$A_\mu(\boldsymbol{\omega}) = D^{(-)}_{\mu 2}(\boldsymbol{\omega})$$

$$F_{\mu x}(\boldsymbol{\omega}) = \sqrt{3/8}\, D^{(+)}_{\mu 1}(\boldsymbol{\omega}) - \sqrt{5/8}\, D^{(+)}_{\mu 3}(\boldsymbol{\omega}),$$

$$F_{\mu y}(\boldsymbol{\omega}) = \sqrt{3/8}\, D^{(-)}_{\mu 1}(\boldsymbol{\omega}) + \sqrt{5/8}\, D^{(-)}_{\mu 3}(\boldsymbol{\omega}),$$

$$F_{\mu z}(\boldsymbol{\omega}) = D^{(+)}_{\mu 0}(\boldsymbol{\omega})$$

$$G_{\mu x}(\boldsymbol{\omega}) = \sqrt{5/8}\, D^{(+)}_{\mu 1}(\boldsymbol{\omega}) + \sqrt{3/8}\, D^{(+)}_{\mu 3}(\boldsymbol{\omega})$$

$$G_{\mu y}(\boldsymbol{\omega}) = -\sqrt{5/8}\, D^{(-)}_{\mu 1}(\boldsymbol{\omega}) + \sqrt{3/8}\, D^{(-)}_{\mu 3}(\boldsymbol{\omega})$$

$$G_{\mu z}(\boldsymbol{\omega}) = D^{(+)}_{\mu 2}(\boldsymbol{\omega}).$$

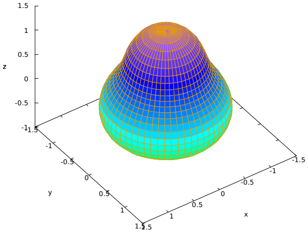
Oktupoloida

$$r(\theta,\phi) = r_0(1+\sum_\mu \alpha_\mu Y^*_{3\mu}(\theta,\phi)) = r_0\left(1 + b v_b(\theta,\phi) + \sum_s f_s v_{f_s}(\theta,\phi) + \sum_s g_s v_{g_s}(\theta,\phi)\right)$$

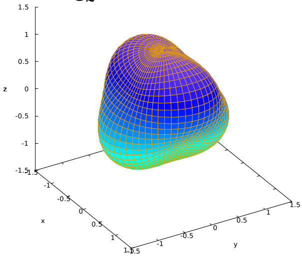
Przykład

$$v_{f_x} = \sqrt{2}\left(\sqrt{3/8}\mathrm{Re}\,Y_{31} - \sqrt{5/8}\mathrm{Re}\,Y_{33}\right) = \frac{\sqrt{7}x(3y^2+3z^2-2x^2)}{4\sqrt{\pi}r^3} \sim F_{0x}$$

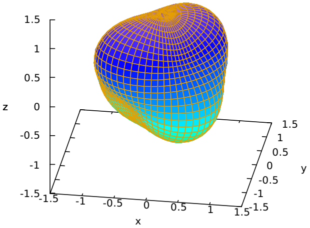
$f_z = 0.4$



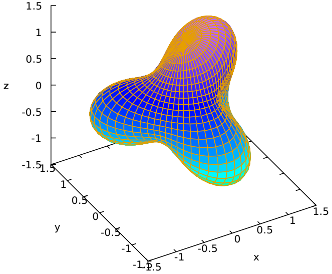
$g_z = 0.4$



$b = 0.4$


z
1.5
1
0.5
0
-0.5
-1
-1.5
x
-1.5
-1
-0.5
0
0.5
1
1.5
y
1.5
1
0.5
0
-0.5
-1
-1.5

$b = f_s = g_s = 0.3$


z
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-1
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0
0.5
1
1.5

$$A_2^- \oplus F_1^- \oplus F_2^-$$
$$\alpha_\mu \to (a_m, b_m) \to (b, f_s, g_s)$$

Układ wewnętrzny dla oktupoli, dwa warianty:

$$\alpha_\mu \to \begin{cases} \mathscr{F}: & (b, f_s, \boldsymbol{\omega}), \quad \boldsymbol{\omega} \leftarrow g_s = 0 \\ \mathscr{G}: & (b, g_s, \boldsymbol{\omega}), \quad \boldsymbol{\omega} \leftarrow f_s = 0 \end{cases}$$

Dla $\mathscr{F}$

$$\alpha_\mu = A_\mu(\boldsymbol{\omega})\, b + \sum_{s=x,y,z} F_{\mu s}(\boldsymbol{\omega}) f_s$$

Czy warianty $\mathscr{F}$, $\mathscr{G}$ są równoważne?

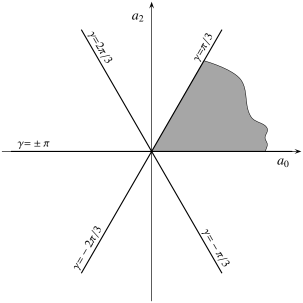
„Przejście do układu wewnętrznego" — zamiana zmiennych

Wybrane gólne własności zmiennych wewnętrznych

- Moment pędu — funkcja $\boldsymbol{\omega}, \partial_{\boldsymbol{\omega}}$
- Energia potencjalna nie zależy od $\boldsymbol{\omega}$

Jakobian, jednoznaczność, minimalny zakres

$$|\det Z_{QC}| = 2|(a_{22}^2 - 3a_{20}^2)a_{22}|\sin\theta = 8\beta^3 \sin 3\gamma \sin\theta$$

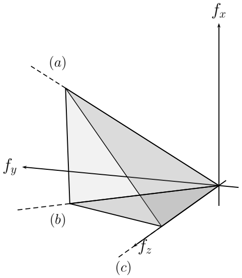


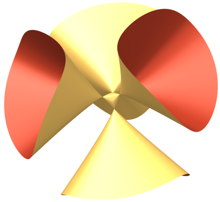
Jakobian

$$|\det J_{\mathscr{F}}| = 8\left(b^3 - 15b(f_x^2 + f_y^2 + f_z^2)/16 + 15\sqrt{15}f_xf_yf_z/32\right)\sin\theta = d_f \sin\theta$$

$$|\det J_{\mathscr{G}}| = 15\sqrt{15}g_xg_yg_z \sin\theta/4 = d_g \sin\theta$$

Jednoznaczność, minimalny zakres zmiennych b, f_s, g_s





$\det J_{\mathscr{F}} = 0, b = 0.5$

Ogólnie $[\partial_{q_i}], \quad \tilde{q}_j = \tilde{q}_j(\boldsymbol{q}) \quad \partial_{\tilde{q}_j} = \sum_k \frac{\partial q_k}{\partial \tilde{q}_j} \partial_{q_k}$

$$\sum c_{ij}(\boldsymbol{q}) \partial_{q_j} \stackrel{?}{=} \partial_{\tilde{q}_i}$$

Oktupole $[\partial_b, \partial_s, \partial_{\boldsymbol{\omega}}] \to [\partial_b, \partial_s, S_k]$, $\quad \partial_s = \partial_{f_s}$ lub ∂_{g_s}

$$S_x = -\frac{\cos\psi}{\sin\theta}\partial_\phi + \sin\psi\,\partial_\theta + \cos\psi\,\mathrm{ctg}\,\theta\,\partial_\psi$$
$$S_y = \frac{\sin\psi}{\sin\theta}\partial_\phi + \cos\psi\,\partial_\theta - \sin\psi\,\mathrm{ctg}\,\theta\,\partial_\psi$$
$$S_z = \partial_\psi$$

S_k — generatory prawej regularnej reprezentacji grupy SO(3), $(T_R f)(S) = f(SR)$

Operatory fizyczne $S_k^{(\mathrm{ph})} = iS_k$

Zmienne (a_m, b_m)

$$g_R = I_7 \quad \to \quad \sum_m \dot{a}_m^2 + \sum_m \dot{b}_m^2$$

Zmienne α_μ

$$(g_C)_{\mu\nu} = (-1)^\mu \delta_{\mu,-\nu} \quad \to \quad \sum_\mu \dot{\alpha}_\mu \dot{\alpha}_\mu^*$$

Przypadek $\mathscr{F}$, $(\dot{b}, \dot{f}_s, \boldsymbol{\eta})$

$$g_{\mathscr{F}} = \begin{pmatrix} I_4 & M_{12} \\ M_{12}^T & M_{22} \end{pmatrix}$$

$$M_{12} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -3f_z/2 & 3f_y/2 \\ 3f_z/2 & 0 & 3f_x/2 \\ -3f_y/2 & 3f_x/2 & 0 \end{pmatrix}$$

Kwadrupole $(\dot{a}_{20}, \dot{a}_{22}, \boldsymbol{\eta})$

$$g_{\mathrm{quad}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & (a_{22}+\sqrt{3}a_{20})^2 & 0 & 0 \\ 0 & 0 & 0 & (a_{22}-\sqrt{3}a_{20})^2 & 0 \\ 0 & 0 & 0 & 0 & 4a_{22}^2 \end{pmatrix}$$

Nowa baza w przestrzeni stycznej

$$W_{k,f} = S_k + \Lambda_{k,f}$$

$$\Lambda_{1,f} = 3(f_y \partial_{f_z} - f_z \partial_{f_y})/2$$
$$\Lambda_{2,f} = 3(f_z \partial_{f_x} - f_x \partial_{f_z})/2$$
$$\Lambda_{3,f} = 3(f_x \partial_{f_y} - f_y \partial_{f_x})/2$$

Teraz

$$g_{\mathscr{F}} = \begin{pmatrix} I_4 & M_{12} \\ M_{12}^T & M_{22} \end{pmatrix} \quad \to \quad g'_{\mathscr{F}} = \begin{pmatrix} I_4 & 0 \\ 0 & T_f \end{pmatrix}$$

$$T_f = \frac{1}{4}\begin{pmatrix} 16b^2 + 15{f_y}^2 + 15{f_z}^2 & 15f_x f_y + 8\sqrt{15}bf_z & 8\sqrt{15}bf_y + 15f_x f_z \\ 15f_x f_y + 8\sqrt{15}bf_z & 16b^2 + 15{f_x}^2 + 15{f_z}^2 & 8\sqrt{15}bf_x + 15f_y f_z \\ 8\sqrt{15}bf_y + 15f_x f_z & 8\sqrt{15}bf_x + 15f_y f_z & 16b^2 + 15{f_x}^2 + 15{f_y}^2 \end{pmatrix}$$

Przypadek $\mathscr{F}$

$$\alpha_\mu = A_\mu(\boldsymbol{\omega})\, b + \sum_{s=x,y,z} F_{\mu s}(\boldsymbol{\omega}) f_s$$

$$\sum_\mu G^*_{\mu s}(\boldsymbol{\omega})\,\alpha_\mu = 0 \qquad (g_s)$$
$$b = \sum_\mu A^*_\mu(\boldsymbol{\omega})\,\alpha_\mu$$
$$f_s = \sum_\mu F^*_{\mu s}(\boldsymbol{\omega})\,\alpha_\mu$$

$$\frac{\partial}{\partial \alpha_\mu} = A^*_\mu \frac{\partial}{\partial b} + \sum_s F^*_{\mu s} \frac{\partial}{\partial f_s} + \frac{1}{d_f} \sum_k x^*_{k,f} W_{k,f}$$

$$x_{1,f} = G_{\mu x}\left(\frac{15 f_x^2}{4} - 4b^2\right) + G_{\mu y}\left(\sqrt{15}\, b f_z - \frac{f_x f_y}{4}\right) + G_{\mu z}\left(\sqrt{15}\, b f_y - \frac{f_x f_z}{4}\right)$$

...

Operator Laplace'a (-Beltramiego)

W bazie współrzędnościowej

$$\Delta W = \frac{1}{\sqrt{\det g}} \sum_{ij} \frac{\partial}{\partial q_i} \sqrt{\det g}\, g^{ij} \frac{\partial W}{\partial q_j}$$

$$\sum_{\mu} \frac{\partial}{\partial \alpha_\mu} \frac{\partial}{\partial \alpha_\mu^*} = \frac{\partial^2}{\partial a_0^2} + \sum_{m=1,2,3} \left(\frac{\partial^2}{\partial a_m^2} + \frac{\partial^2}{\partial b_m^2} \right)$$

$$\Delta f = \operatorname{div} \operatorname{grad} f$$

$$\operatorname{div} Y = \sum_k (X_k Y^k + \sum_j \Gamma^j_{kj} Y^k) = \sum_k (X_k + \Omega_k) Y^k$$

X_k — wektory bazowe w przestrzeni stycznej, $\Omega_k = \sum_j \Gamma^j_{kj}$

Dla baz współrzędnościowych $X_k = \partial_{q_k}$ i $\Omega_k = \partial_{q_k} \ln \sqrt{\det g}$

Ogólniej

$$\Gamma_{ijk} = \frac{1}{2}(g_{ij,k} + g_{ik,j} - g_{jk,i} + c_{ijk} + c_{ikj} - c_{jki})$$

$$g_{ij,k} = X_k g_{ij}$$

$$[X_i, X_j] = \sum_m c^m_{ij} X_m, \qquad c_{ijk} = \sum_m g_{km} c^m_{ij}$$

$$\Gamma^m_{kj} = (g^{-1})^{mi} \Gamma_{ikj}$$

$$\sum_\mu \frac{\partial}{\partial \alpha_\mu} \frac{\partial}{\partial \alpha_\mu^*} = \frac{1}{d_f} \frac{\partial}{\partial b} d_f \frac{\partial}{\partial b} + \sum_s \frac{1}{d_f} \frac{\partial}{\partial f_s} d_f \frac{\partial}{\partial f_s} + \frac{1}{d_f} \sum_{k,j} W_{k,f} M_{kj}^{(f)} d_f W_{j,f}$$

$$d_f = 8\left(b^3 - 15b(f_x^2 + f_y^2 + f_z^2)/16 + 15\sqrt{15} f_x f_y f_z/32\right)$$

$$W_{k,f} = S_k + \Lambda_{k,f}$$

$$\Lambda_{1,f} = 3(f_y \partial_{f_z} - f_z \partial_{f_y})/2$$

...

$$M^{(f)} = T_f^{-1}$$

$$L_{1\kappa}(\boldsymbol{\alpha},\boldsymbol{\partial\alpha}) = -2\sqrt{7}\sum_{\mu\nu}(3\mu 3\nu|1\kappa)\alpha_\mu \frac{\partial}{\partial\alpha^*_\nu}$$

Zmienne wewnętrzne

$$L_{1\kappa} = \mathscr{D}^1_{\kappa 0}(\boldsymbol{\omega})S^{(\mathrm{ph})}_z + \mathscr{D}^1_{\kappa 1}(\boldsymbol{\omega})\left(-S^{(\mathrm{ph})}_x + iS^{(\mathrm{ph})}_y\right)/\sqrt{2} + \mathscr{D}^1_{\kappa,-1}(\boldsymbol{\omega})\left(S^{(\mathrm{ph})}_x + iS^{(\mathrm{ph})}_y\right)/\sqrt{2}$$

$S^{(\mathrm{ph})}_{x,y,z}$ zależą tylko od $(\boldsymbol{\omega},\partial\boldsymbol{\omega})$

To nie przypadek

$$\alpha_\mu = \sum_\nu \mathscr{D}_{\mu\nu}(\Omega^{-1})\tilde{\alpha}_\nu$$

$$b = \sum_\mu A^*_\mu(\boldsymbol{\omega})\alpha_\mu = \sum_{\mu,\nu} A^*_\mu(\boldsymbol{\omega})\mathscr{D}_{\mu\nu}(\Omega^{-1})\tilde{\alpha}_\nu = \sum_\nu A^*_\nu(\boldsymbol{\omega}\circ\Omega)\tilde{\alpha}_\nu$$

$$f_s = \sum_\mu F^*_{\mu s}(\boldsymbol{\omega})\alpha_\mu = \ldots = \sum_\nu F^*_{\nu s}(\boldsymbol{\omega}\circ\Omega)\tilde{\alpha}_\nu$$

$$0 = \sum_\mu G^*_{\mu s}(\boldsymbol{\omega})\alpha_\mu = \ldots = \sum_\nu G^*_{\nu s}(\boldsymbol{\omega}\circ\Omega)\tilde{\alpha}_\nu \quad \longrightarrow \quad (b,f_s,\boldsymbol{\omega}) \to (b,f_s,\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega}\circ\Omega)$$

Elementarne tensory

$$\boldsymbol{t}_l^{(n)} = [\underbrace{\boldsymbol{\alpha} \times \ \ldots \ \times \boldsymbol{\alpha}}_{n}]_l$$

QC. Elementarne skalary $l = 0$, $n = 2, 3$

$$[\boldsymbol{\alpha}_2 \times \boldsymbol{\alpha}_2]_0 \sim \beta^2, \qquad [[\boldsymbol{\alpha}_2 \times \boldsymbol{\alpha}_2]_2 \times \boldsymbol{\alpha}_2]_0 \sim \beta^3 \cos 3\gamma$$

Oktupole. Elementarne skalary, $l = 0$, $n = 2, 4, 6, 10, 15$

$$\boldsymbol{t}_0^{(2)} = -\frac{1}{\sqrt{7}}(b^2 + \sigma_2)$$

$$\boldsymbol{t}_0^{(4)} = \frac{1}{84\sqrt{5}}\left(16\sigma_4 - 13\sigma_{42} + 80b^2\sigma_2 + 24\sqrt{15}b\sigma_3\right)$$

$$\sigma_2 = f_x^2 + f_y^2 + f_z^2, \quad \sigma_4 = f_x^4 + f_y^4 + f_z^4, \quad \sigma_6 = f_x^6 + f_y^6 + f_z^6$$

$$\sigma_3 = f_x f_y f_z, \quad \sigma_{42} = f_x^2 f_y^2 + f_y^2 f_z^2 + f_x^2 f_z^2$$

$$H(\boldsymbol{\alpha},\boldsymbol{\partial\alpha}) = -\frac{1}{2W(\boldsymbol{\alpha})}\sum_{\mu,\nu}\frac{\partial}{\partial\alpha_\mu}W(\boldsymbol{\alpha})B^{-1}_{\mu\nu}(\boldsymbol{\alpha})\frac{\partial}{\partial\alpha_\nu} + V(\boldsymbol{\alpha}),$$

zwykle $W(\boldsymbol{\alpha}) = \sqrt{\det(B_{\mu\nu}(\boldsymbol{\alpha}))}$.

$B_{\mu\nu}(\boldsymbol{\alpha})$

- symetryczny i dodatnio okrślony (w zmiennych rzeczywistych)
- można go rozłożyć na tensory

$$B_{\mu\nu}(\boldsymbol{\alpha}) = \sum_{\lambda=0,2,4,6}(3\mu 3\nu|\lambda\kappa)\tau_{\lambda\kappa}(\boldsymbol{\alpha})$$

- 28 niezależnych składowych (nawet w zmiennych wewnętrznych)

- Kwadrupol plus oktupol
- Proste, nietrywialne przypadki
- Obliczenia mikroskopowe

- Nature, 497 (2013) 199, *Studies of pear-shaped nuclei using accelerated radioactive beams*

Going pear-shaped