

# Spectroscopy and shape change in neutron-rich Sr and Zr isotopes

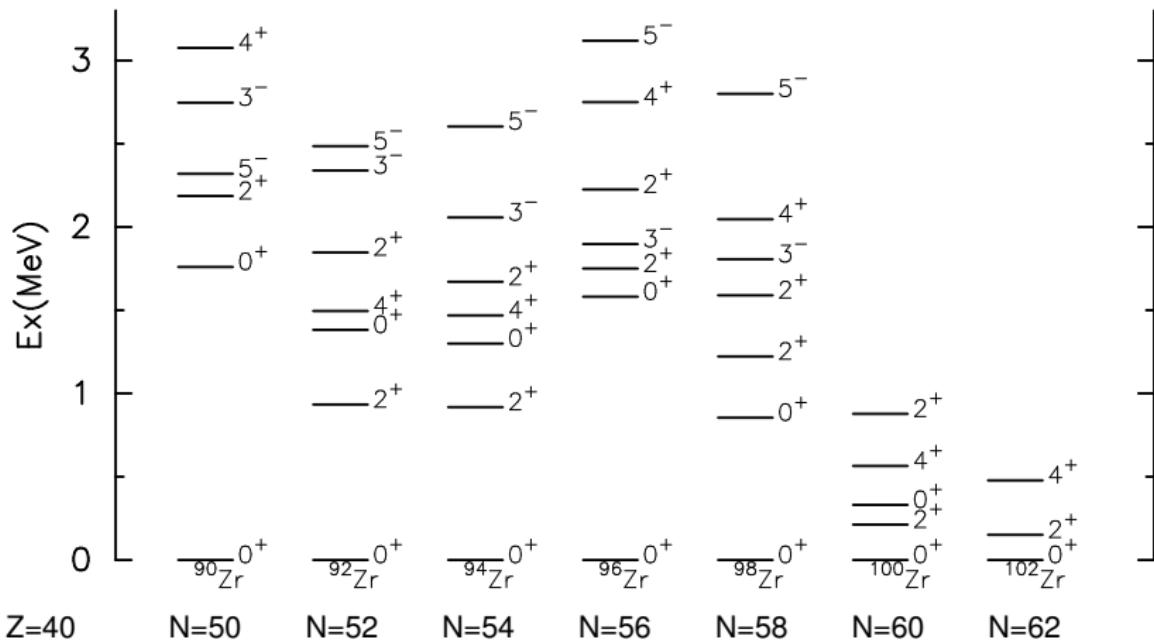
Kamila Sieja

*Institut Pluridisciplinaire Hubert Curien, Strasbourg*

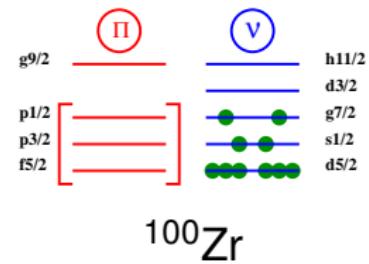
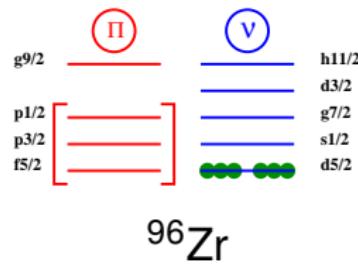
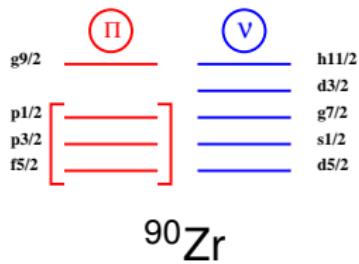
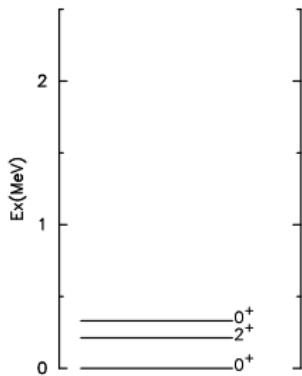
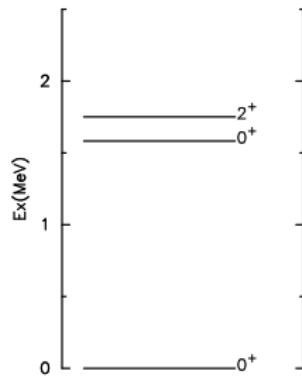
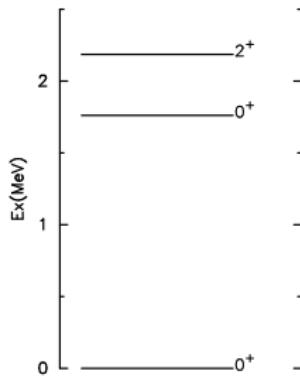


Warsaw, 27.05.2021

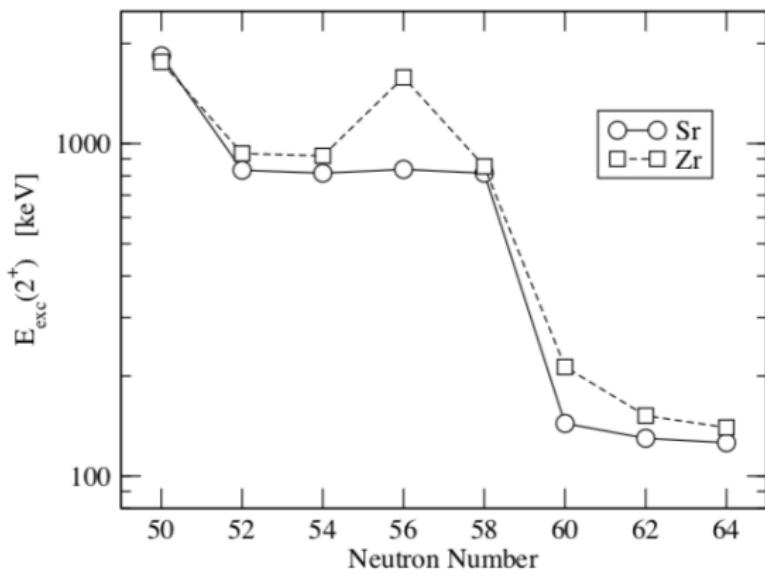
# Introduction: Zr chain



# Introduction: Zr chain



# Introduction: Zr and Sr isotopes



## The general many-body problem for fermions

The Schrödinger equation for a system of  $n$  particles each of mass  $m$  is:

$$\hat{H}|\Psi\rangle = E|\Psi\rangle, \quad (1)$$

$$H = \sum_{k=1}^n (T_k + U_k) + \left( \sum_{k < l} V_{kl} - \sum_{k=1}^n U_k \right) = H^0 + W \quad (2)$$

The standard solution of this problem is to solve first the simpler one

$$\hat{H}^0|\Phi_a\rangle = E_a^0|\Phi_a\rangle, \quad (3)$$

with

$$|\Phi_a\rangle = \prod_{k=1}^n |\alpha_k\rangle = \phi_{\alpha_1}(\vec{r}_1)\phi_{\alpha_2}(\vec{r}_2)\dots\phi_{\alpha_n}(\vec{r}_n) \quad (4)$$

$$E_a^0 = \sum_{k=1}^n \varepsilon_{\alpha_k}. \quad (5)$$

The  $|\alpha\rangle$  are solutions of the single-particle equation:

$$(T + U)|\alpha\rangle = \varepsilon_\alpha|\alpha\rangle \quad (6)$$

$a$ - set of quantum numbers  $\alpha_k$ , for example  $|n_r, l, j, m_j\rangle$  in a spherical basis.

# Shell-model approaches

- The wave-function of the ground state is expressed as a sum of the vacuum  $\Phi_0$  and particle-hole excitations build on this vacuum state

$$|\Psi_0\rangle = C_0|\Phi_0\rangle + \sum_{i\alpha} C_{i\alpha}|\Phi_{i\alpha}\rangle + C_{ij\alpha\beta}|\Phi_{ij\alpha\beta}\rangle + \dots \quad (7)$$

where greek and latin symbols refer to particle and hole states, respectively. In short notation:

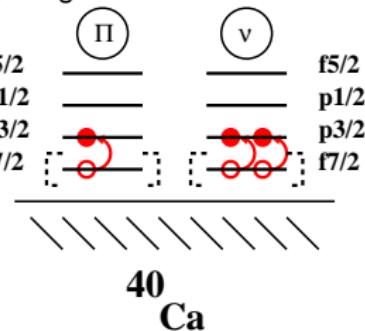
$$|\Psi_0\rangle = \sum_{ph} C_{ph}|\Phi_{ph}\rangle \quad (8)$$

- The equation for the energy reads

$$E = \langle \Psi_0 | \hat{H} | \Psi_0 \rangle = \sum_{pp'h'h'} C_{p'h'}^* \langle \Phi_{p'h'} | \hat{H} | \Phi_{ph} \rangle C_{ph} \quad (9)$$

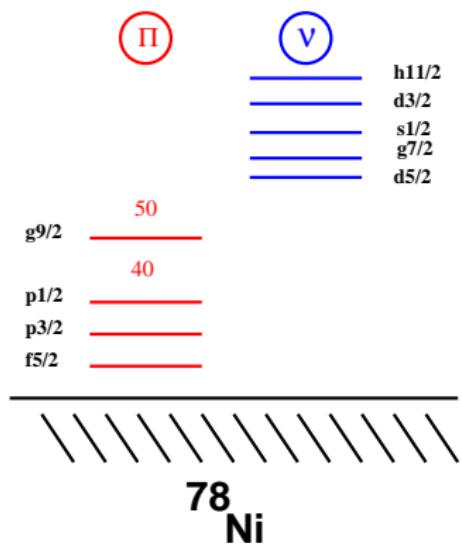
and it is solved by diagonalization.

- The vacuum for particle-hole excitations 1p-1h, 2p-2h,..., np-nh can be, e.g. the lowest-filling configuration (Slater determinant) outside a doubly-magic core.



- the problem solution is limited by computing capacities, i.e. the size of the matrix to diagonalize.

# Nuclei above $^{78}\text{Ni}$

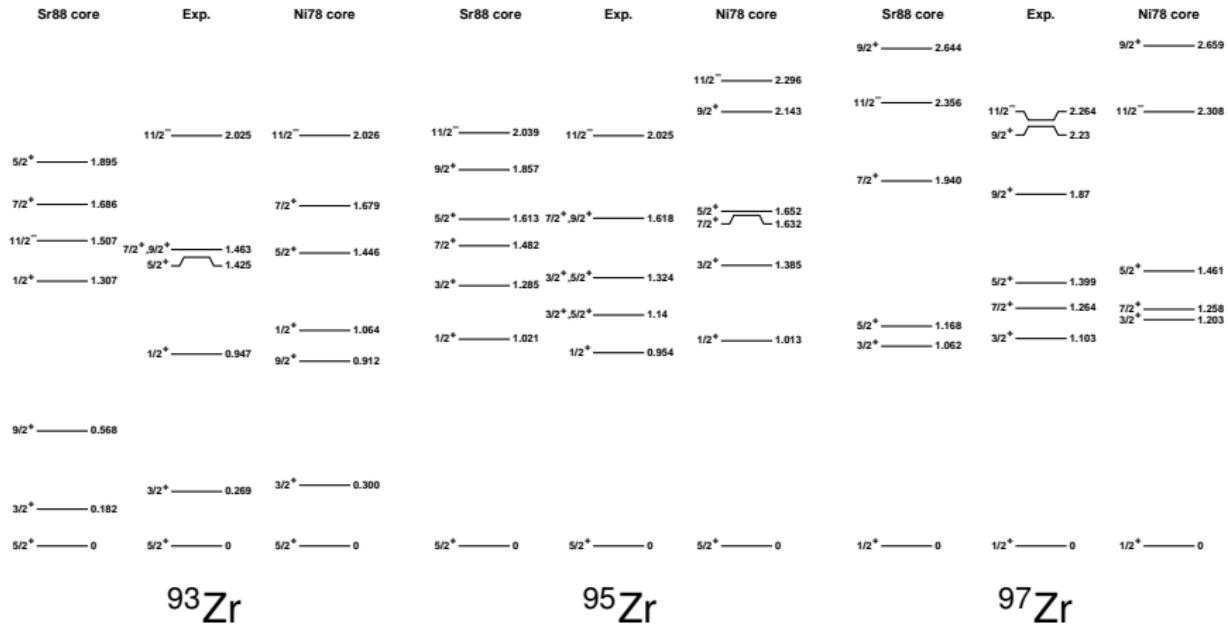


- Even-odd and odd-odd Br nuclei *PRC92 (2015) 014328, PRC94 (2016) 044328, PRC95 (2017) 024321, PRC100 (2019) 054331, PRC103 (2021) 034304*

**Interaction:** based on realistic TBME, monopoles corrected. Proven successful and predictive in a large number of applications:

- Structure, mixed symmetry states in Zr isotopes, shell evolution between  $^{91}\text{Zr}$  and  $^{101}\text{Sn}$  *PRC79 (2009) 064310*
- Collectivity of  $N = 52, 54$  nuclei *PRC88 (2013) 034327, PRC92 (2015) 034305, PRC92 (2015) 064322, PRC96 (2017) 011301R, PRC95 (2017) 051302R*
- Isomers and medium-spin structures of  $^{95}\text{Y}$ ,  $^{91-95}\text{Rb}$ ,  $^{92-96}\text{Sr}$  *PRC85 (2012) 014329, PRC79 (2009) 024319, PRC82 (2010) 024302, PRC79 (2009) 044304, PRC93 (2016) 034318*
- Collectivity and  $j-1$  anomaly of  $^{87}\text{Se}$  *PRC88 (2013) 034302*
- $\beta$ -decays of Ga nuclei and structure of  $N = 52, 54$  isotones *PRC88 (2013) 047301, PRC88 (2013) 044330, PRC88 (2013) 044314*
- Magnetic moments, MSS of  $^{86,88}\text{Kr}$ ,  $^{88}\text{Sr}$  *PRC 80 (2014) 064305, PRC94 (2016) 054323*
- Neutron-rich Cd isotopes *PRC79 (2009) 011301R, PRC82 (2010) 034323*

# Results: odd Zr isotopes N=51-57



PHYSICAL REVIEW C 79, 064310 (2009)

## Shell model description of zirconium isotopes

K. Sieja,<sup>1,2</sup> F. Nowacki,<sup>3</sup> K. Langanke,<sup>2,4</sup> and G. Martínez-Pinedo<sup>1</sup>

<sup>1</sup>GSI-Helmholtzzentrum für Schwerionenforschung mbH., Planckstrasse 1, D-64220 Darmstadt, Germany

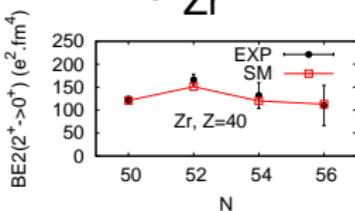
<sup>2</sup>Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany

<sup>3</sup>Instituto Superior de Astronomía, Universidad Nacional de La Plata, Casilla 67-3, 1900 La Plata, Argentina

# Results: even Zr isotopes N=50-58

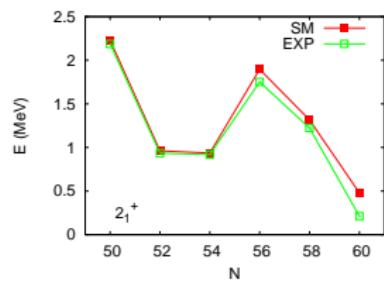
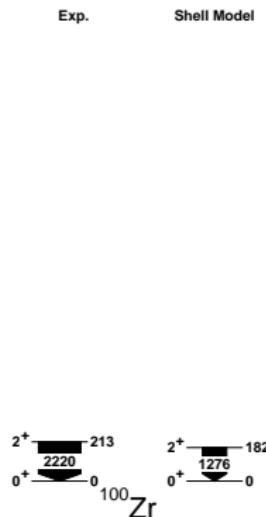
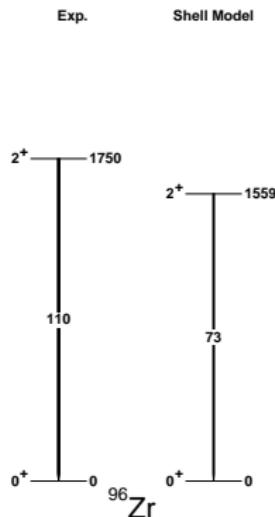
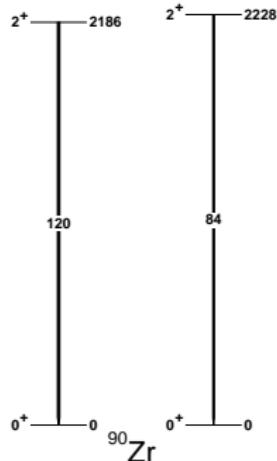
Sr88 core	Exp.	Ni78 core	Sr88 core	Exp.	Ni78 core	Sr88 core	Exp.	Ni78 core
$5^-$ ————— 2.486 $4^+$ ————— 2.398 $3^-$ ————— 2.340		$3^-$ ————— 2.565 $4^+$ ————— 2.536 $5^-$ ————— 2.316	$5^-$ ————— 2.945		$3^-$ ————— 2.601 $5^-$ ————— 2.460	$3^-$ ————— 3.578 $5^-$ ————— 3.478		$3^-$ ————— 3.292
$2^+$ ————— 1.921 $3^-$ ————— 1.904	$2^+$ ————— 1.847	$2^+$ ————— 1.862	$3^+$ ————— 2.223 $0^+$ ————— 2.213		$3^-$ ————— 2.058	$2^+$ ————— 2.619 $4^+$ ————— 2.449	$5^-$ ————— 2.800	$5^-$ ————— 2.859
$0^+$ ————— 1.693				$2^+$ ————— 1.764	$2^+$ ————— 1.783			
$4^+$ ————— 1.496 $5^-$ ————— 1.316	$0^+$ ————— 1.453 $4^+$ ————— 1.431		$2^+$ ————— 1.671	$4^+$ ————— 1.470	$0^+$ ————— 1.400 $0^+$ ————— 1.300	$2^+$ ————— 1.463	$4^+$ ————— 1.843 $3^-$ ————— 1.806 $2^+$ ————— 1.698	$4^+$ ————— 1.944
	$2^+$ ————— 0.935	$2^+$ ————— 0.961		$4^+$ ————— 0.817	$2^+$ ————— 0.919	$2^+$ ————— 0.899	$2^+$ ————— 1.223	$2^+$ ————— 1.293
$4^+$ ————— 0.823				$2^+$ ————— 0.520			$0^+$ ————— 0.854	$0^+$ ————— 0.848
$2^+$ ————— 0.581								
$0^+$ ————— 0	$0^+$ ————— 0	$0^+$ ————— 0	$0^+$ ————— 0	$0^+$ ————— 0	$0^+$ ————— 0	$0^+$ ————— 0	$0^+$ ————— 0	$0^+$ ————— 0

$^{92}\text{Zr}$

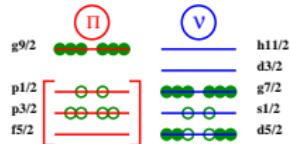


$^{94}\text{Zr}$

# Shape transition

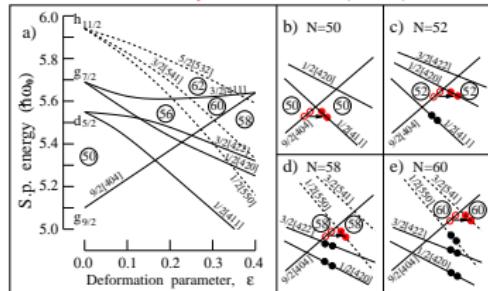


☞ prolate configuration  $\beta = 0.26$

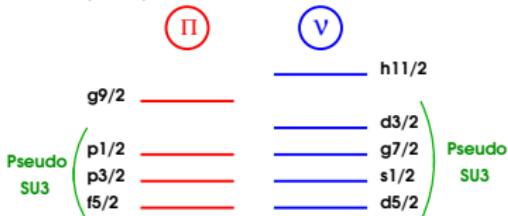


# Origin of deformation

- Spin-Orbit Partners mechanism: strong interaction between  $\pi g_{9/2}$  protons and  $\nu g_{7/2}$  neutrons  
*P. Federman and S. Pitel, Phys. Rev. C20 (1979) 820*
- Increased role of the neutrons from the extruder  $\nu g_{9/2}$  orbital  
*W. Urban et al., Phys. Rev. C102 (2020) 064321*



- increased role of the intruder orbitals  $\pi d_{5/2}$  and  $\nu f_{7/2}$   
*PRC79 (2009) 064310*



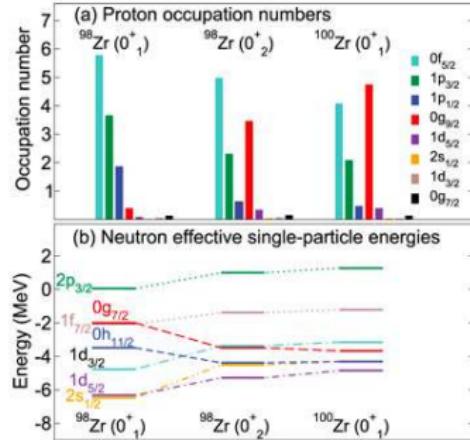
1. two pseudo-SU3 blocks:  
 $B(E2; 2^+ \rightarrow 0^+) = 770 e^2 \text{fm}^4$
2. adding  $\pi d_{5/2}$  orbital to form an extra symmetry block:  
 $B(E2; 2^+ \rightarrow 0^+) = 2200 e^2 \text{fm}^4$
3. adding also  $\nu f_{7/2}$  orbital to form another extra symmetry block:  
 $B(E2; 2^+ \rightarrow 0^+) = 3500 e^2 \text{fm}^4$

$$\text{EXP}=2220 e^2 \text{fm}^4$$

extension of the model space necessary

# MCSM in Zr and Sr isotopes

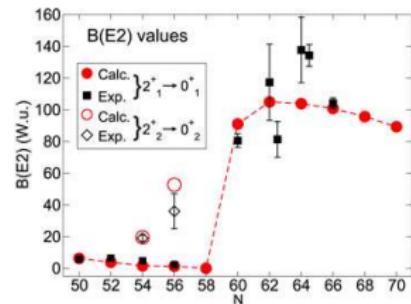
with MC and variational basis optimization procedure, problem equivalent to diagonalization of  $10^{23}$  SD (current diagonalization limit  $10^{11}$ )



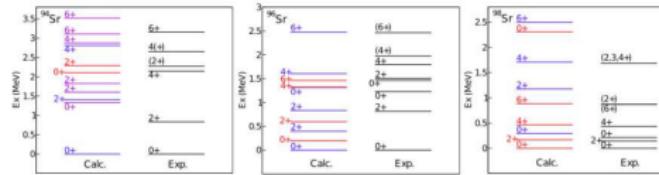
consistent with earlier SM suggestions

T. Togashi et al., PRL117 (2016) 172502

## E2 transitions in Zr isotopes



## Spectra of Sr isotopes



# Sr isotopes

A lot of new experimental studies were performed in recent years in Sr isotopes:

- Coulex of  $^{96-98}\text{Sr}$  at REX-ISOLDE  
*E. Clement et al. Phys. Rev. Lett. 116 (2016) 022701*  
-5DCH calculations with Gogny forces
- (d,p) reactions on  $^{94-96}\text{Sr}$  from TRIUMF  
*S. Cruz et al., Phys. Lett. B786 (2018) 94; PRC100 (2019) 054321; PRC102 (2020) 024335*  
-restricted SM calculations
- Lifetime measurements in  $^{94-98}\text{Sr}$   
*J.M. Regis et al., PRC95 (2017) 054319*  
-MCSM calculations

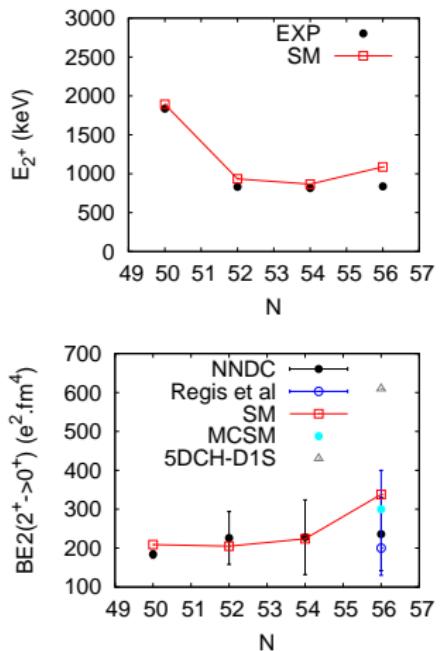
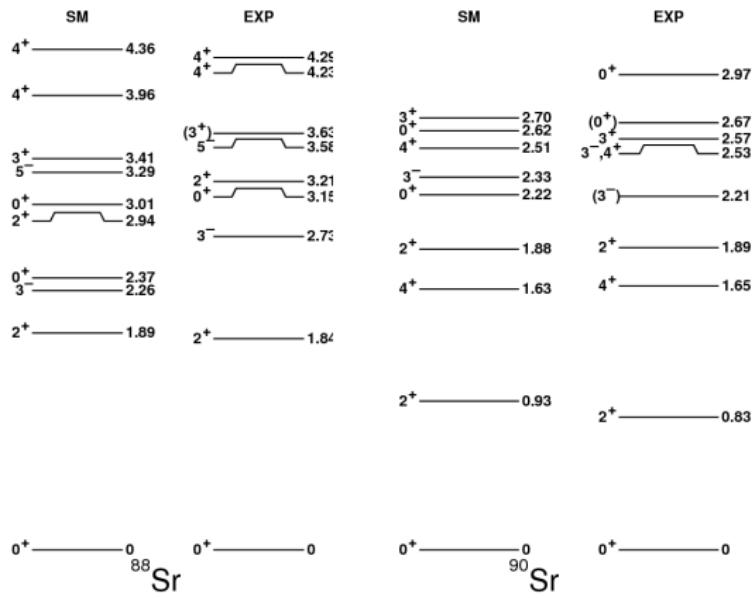
New data from the EXILL campaign:

- $^{90,92,94,96}\text{Sr}$ , 23 new levels, 30 new decays, 57 parity/spin assignments
- reliable assignments of spin/parity
- identification of the key collective excitations

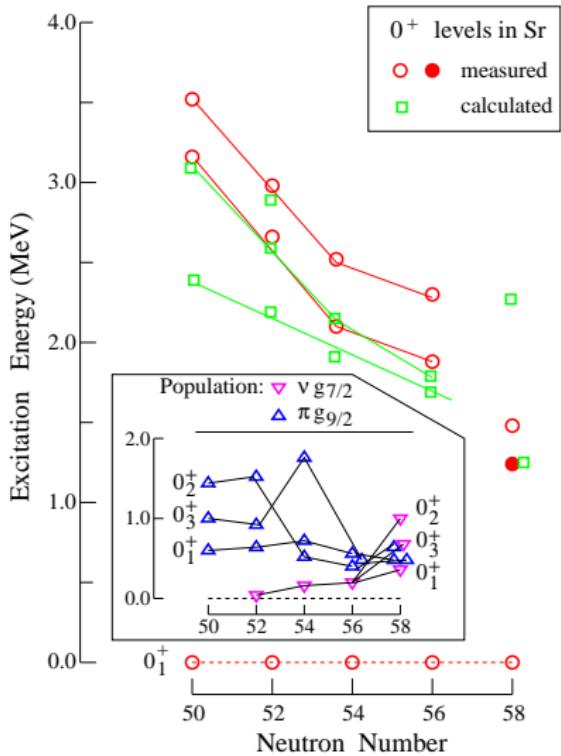
W. Urban, KS et al., *Structure of even-even Sr isotopes with  $50 \leq N \leq 58$ , in preparation.*

KS, *Single-particle and collective structures in neutron-rich Sr isotopes towards the  $N = 60$  shape transition, in preparation.*

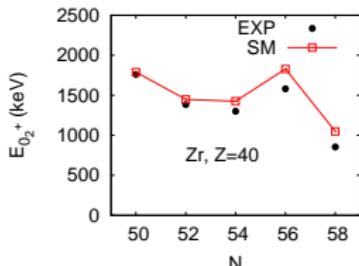
# Light Sr isotopes



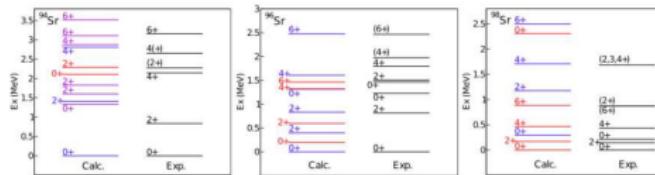
# Systematics of the $0^+$ excitations



First excited  $0^+$  in zirconium isotopes



Spectra of Sr isotopes from MCSM

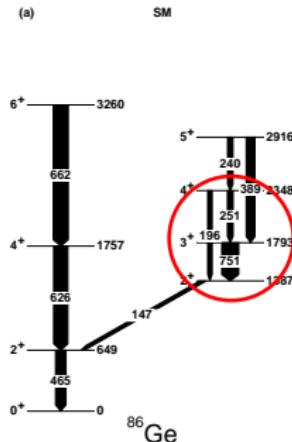


Is it experiment or theory?

# Non-axiality around $^{78}\text{Ni}$ core

Suggestion from SM and GCM-Gogny models:  
triaxial bands close to the core in N=52,54 Se  
and Ge

K. Sieja, T.R. Rodriguez, K. Kolos and D. Verney, Phys. Rev. C88  
(2013) 034327



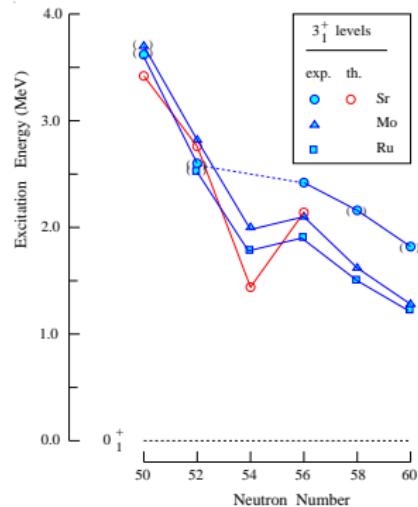
confirmed in recent experiments

Phys. Rev. C92 (2015) 034305;

Phys. Rev. C92 (2015) 064322;

Phys. Rev. C96 (2017) 011301R

## What about Sr isotopes?



A  $\gamma$  band ( $K = 2$ ), apart of a characteristic level sequence, has  $Q(2_2^+) = -Q(2_1^+)$  and  $Q(3^+) \sim 0$ .

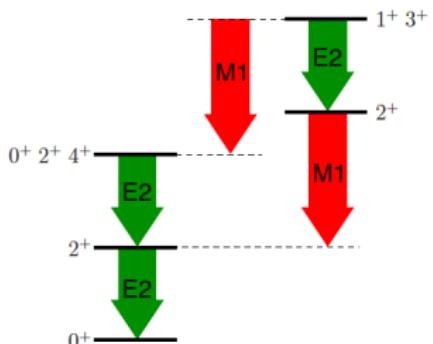
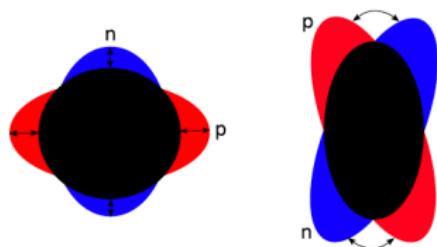
This is not true for  $^{88-90,94}\text{Sr}$ . But in  $^{92}\text{Sr}$ :

$$Q(2_2^+) = 24.8 e^2 \cdot \text{fm}^2$$

$$Q(2_1^+) = -16.5 e^2 \cdot \text{fm}^2$$

$$Q(3^+) = 1.89 e^2 \cdot \text{fm}^2$$

# Mixed-symmetry states



F-spin: concept of isospin extended to proton and neutron bosons  
 maximum F : state is fully symmetric (FS)

nonmaximum F: state is said to have a mixed-symmetry (MS)

signature: strong  $M1$  decay between  $2_1^+$  and MS  $\langle 2_{MS}^+ | T(M1) | 2_1^+ \rangle \sim 1 \mu_N$

*K. Sieja et al., Description of proton-neutron mixed-symmetry states near  $^{132}\text{Sn}$  within a realistic LSSM, PRC80 (2009) 054311*

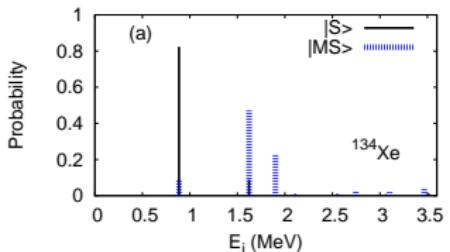
Q-phonon scheme:

$$|2_1^+\rangle = Q_S |0_1^+\rangle$$

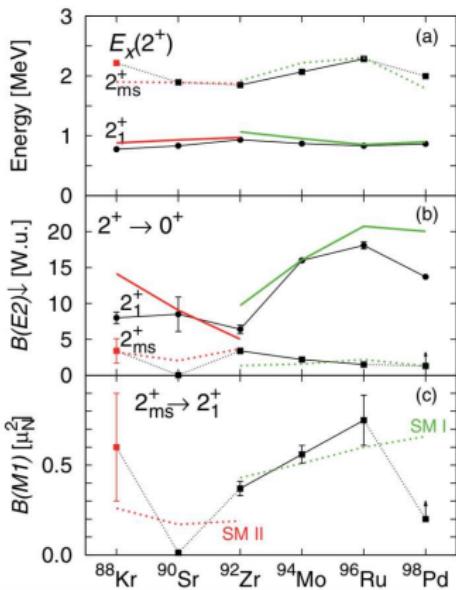
$$|2_{MS}^+\rangle = Q_{MS} |0_1^+\rangle$$

$$Q_S = Q_p + Q_n$$

$$Q_{MS} = Q_p - \alpha Q_n$$

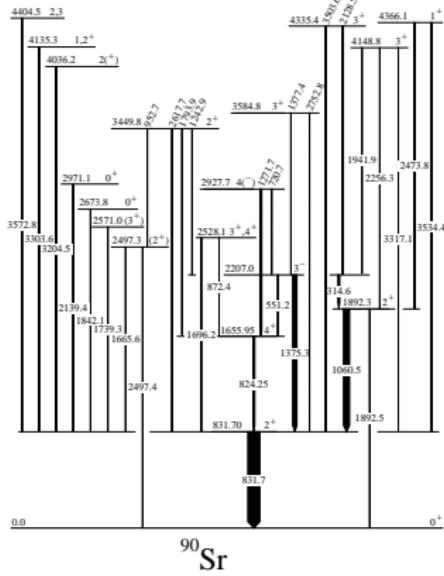


## MS states in N=52 isotones



K. Moschner et al., PRC94 (2016) 054323

B(XL) in $^{92}\text{Zr}$			
N	Transition	exp	SM
52	$B(E2; 2_2^+ \rightarrow 0_1^+)$	$164\text{e}^2\text{fm}^4$	$151\text{e}^2\text{fm}^4$
	$B(E2; 2_1^+ \rightarrow 0_2^+)$	$71\text{e}^2\text{fm}^4$	$50\text{e}^2\text{fm}^4$
	$B(E2; 2_2^+ \rightarrow 0_1^+)$	$84\text{e}^2\text{fm}^4$	$100\text{e}^2\text{fm}^4$
	$B(M1; 2_2^+ \rightarrow 2_1^+)$	$0.37(4)\mu_N^2$	$0.21\mu_N^2$
	$B(M1; 2_3^+ \rightarrow 2_1^+)$	$\leq 0.024\mu_N^2$	$0.06\mu_N^2$

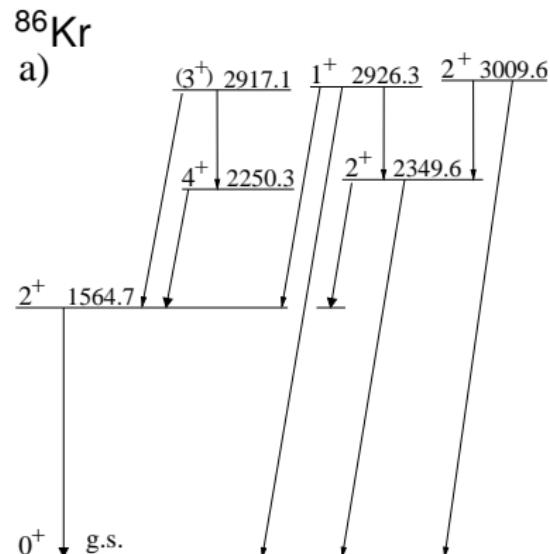
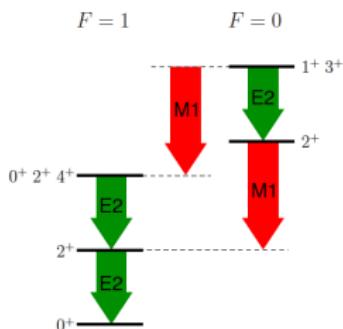


# What become MS states at N=50?

B(XL) in  $^{86}\text{Kr}$

N	Transition	SM
50	$\text{B}(\text{E}2; 2_1^+ \rightarrow 0_1^+)$	$190\text{e}^2\text{fm}^4$
	$\text{B}(\text{E}2; 2_2^+ \rightarrow 0_1^+)$	$18\text{e}^2\text{fm}^4$
	$\text{B}(\text{M}1; 2_2^+ \rightarrow 2_1^+)$	$0.25\mu_N^2$

- Strong  $M1$  transitions between  $f_{5/2}$  and  $p_{3/2}$  orbitals



W. Urban et al. PRC94 (2016) 044328

# What become MS states at N=50?

$^{88}\text{Sr}$

a)  $0^+$  44484.8

$4^+$  4299.5

$2^+$  4299.5

$2^+$  3522.8

$(3^+)$  3635.1

$2^+$  3218.5

$(0^+)$  3992.4

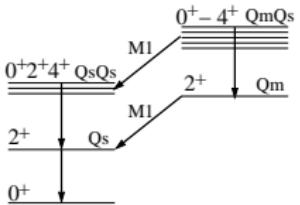
$1^+$  3486.6

$2^+$  1836.1

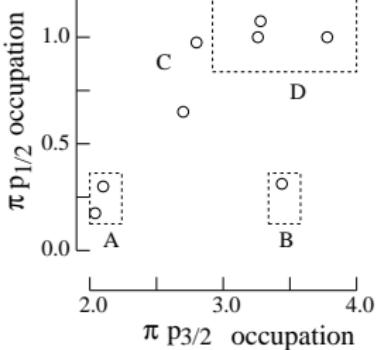
$0^+$

g.s.

b)



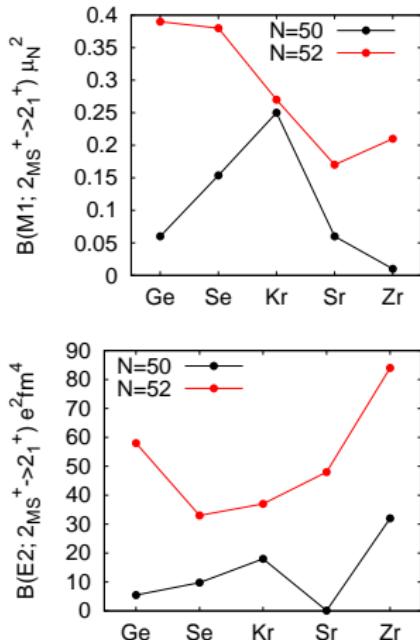
c)



# Systematics at N=50,52

## B(XL) in $^{88,90}\text{Sr}$

N	Transition	SM
50	$B(E2; 2_1^+ \rightarrow 0_1^+)$	$209 e^2 \text{fm}^4$
	$B(E2; 2_2^+ \rightarrow 0_1^+)$	$0.1 e^2 \text{fm}^4$
	$B(M1; 2_2^+ \rightarrow 2_1^+)$	$0.06 \mu_N^2$
52	$B(E2; 2_1^+ \rightarrow 0_1^+)$	$205 e^2 \text{fm}^4$
	$B(E2; 2_2^+ \rightarrow 0_1^+)$	$48 e^2 \text{fm}^4$
	$B(M1; 2_2^+ \rightarrow 2_1^+)$	$0.17 \mu_N^2$



# Conclusions

- Understanding of the nature of low-energy excitations around  $^{78}\text{Ni}$
- Mixed-symmetry states: what they really are?
- Tracking evolution of single-particle excitations in the region
- Need experimental data: less exotic nuclei, better probes

# Advances in Nuclear Physics

The image shows the front cover of a journal issue. At the top left is the logo for "universe" with a stylized blue and white geometric design. To the right of the logo is the word "universe". Below the logo, it says "an Open Access Journal by MDPI". In the center, the title "Advances in Nuclear Physics" is displayed. On the right side of the cover, there is a yellow circular badge with the text "IMPACT FACTOR 1.752". At the bottom left, it says "Guest Editors Dr. Kamila Sieja, Dr. Johan Ljungvall". Below that, the word "Deadline" and the date "31 July 2021" are listed. At the very bottom left is the URL "mdpi.com/si/56471". On the right side, the words "Special Issue" are written vertically, and below them is a small blue bar with the text "Invitation to submit".

- open access (30% – 100% waiver for special issues)
- review, regular, communication
- deadline end of 2021 (progressive publication before the deadline)