# Nucleon structure from Lattice Quantum Chromodynamics

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in collaboration with:

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Fernanda Steffens (Univ. of Bonn)

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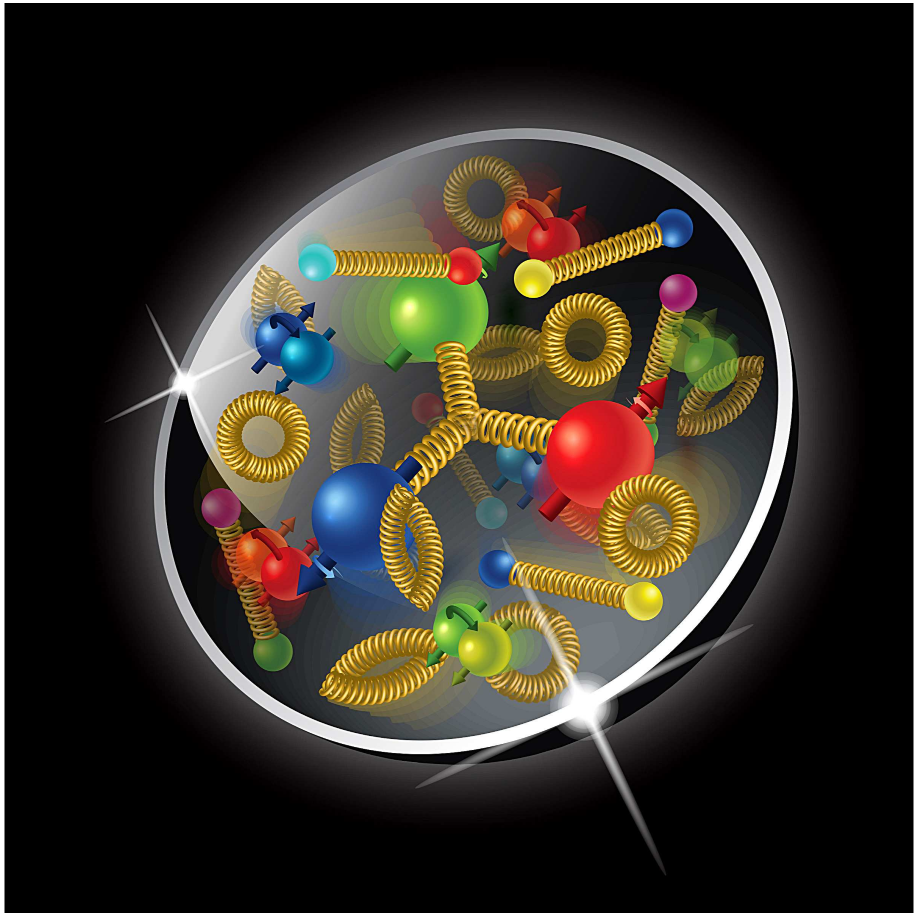
# Outline of the talk

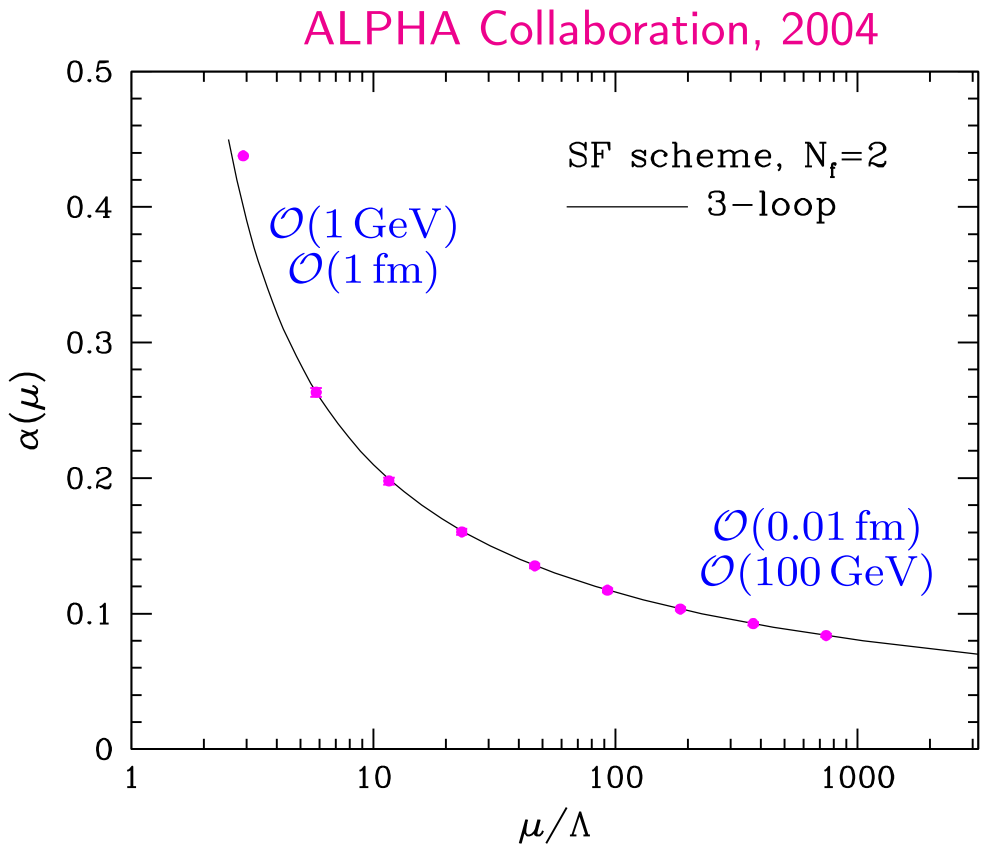
1. Lattice QCD
   - Why do we need this?
   - Lattice formulation of QCD
   - QCD simulations
   - What can we compute?
2. Nucleon structure
   - Basics
   - Parton distribution functions from the lattice
   - Results
3. Conclusions and prospects

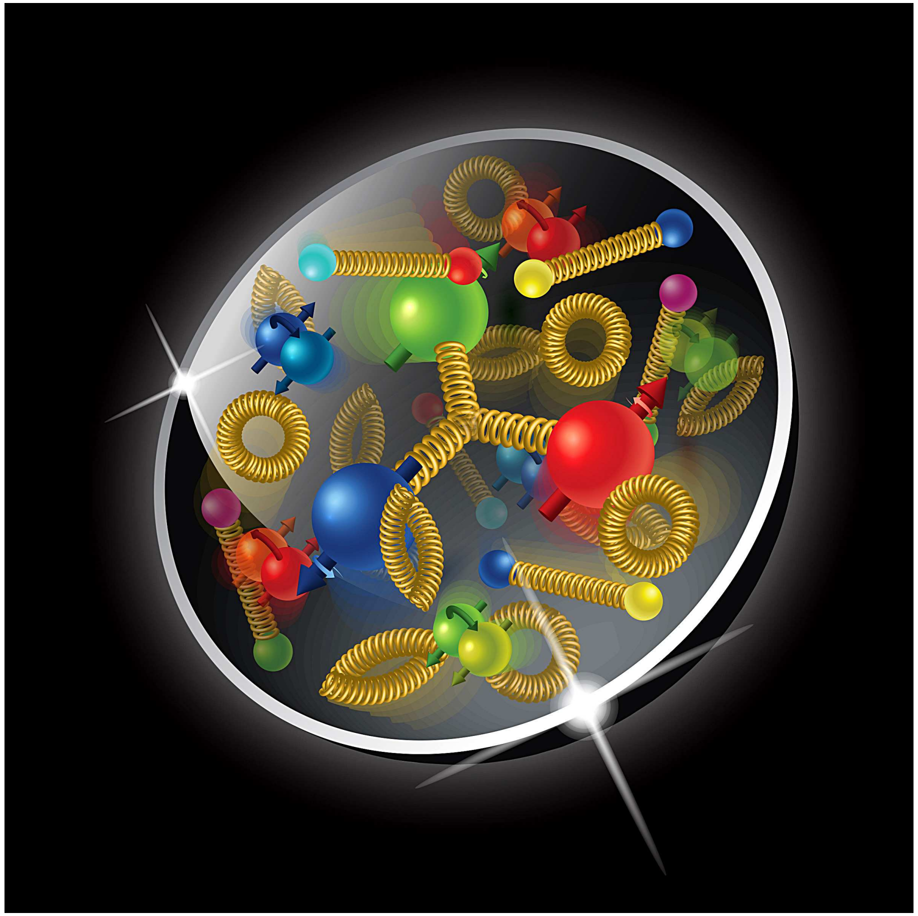
Results based on:

- **C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Reconstruction of light-cone parton distribution functions from lattice QCD simulations at the physical point", Phys. Rev. Lett. 121 (2018) 112001**
- **C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Transversity parton distribution functions from lattice QCD", Phys. Rev. D (Rapid Communications), in press, arXiv: 1807.00232 [hep-lat]**
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (invited Frontiers Article)
- K. Cichy, M. Constantinou, "A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results", invited review article for a special issue of Advances in High Energy Physics, arXiv: 1811.07248 [hep-lat]

# Lattice QCD




ALPHA Collaboration, 2004
SF scheme, $N_f$=2
3−loop
$\mathcal{O}(1\,\mathrm{GeV})$
$\mathcal{O}(1\,\mathrm{fm})$
$\mathcal{O}(0.01\,\mathrm{fm})$
$\mathcal{O}(100\,\mathrm{GeV})$
$\alpha(\mu)$
$\mu/\Lambda$
0.5
0.4
0.3
0.2
0.1
0
1
10
100
1000



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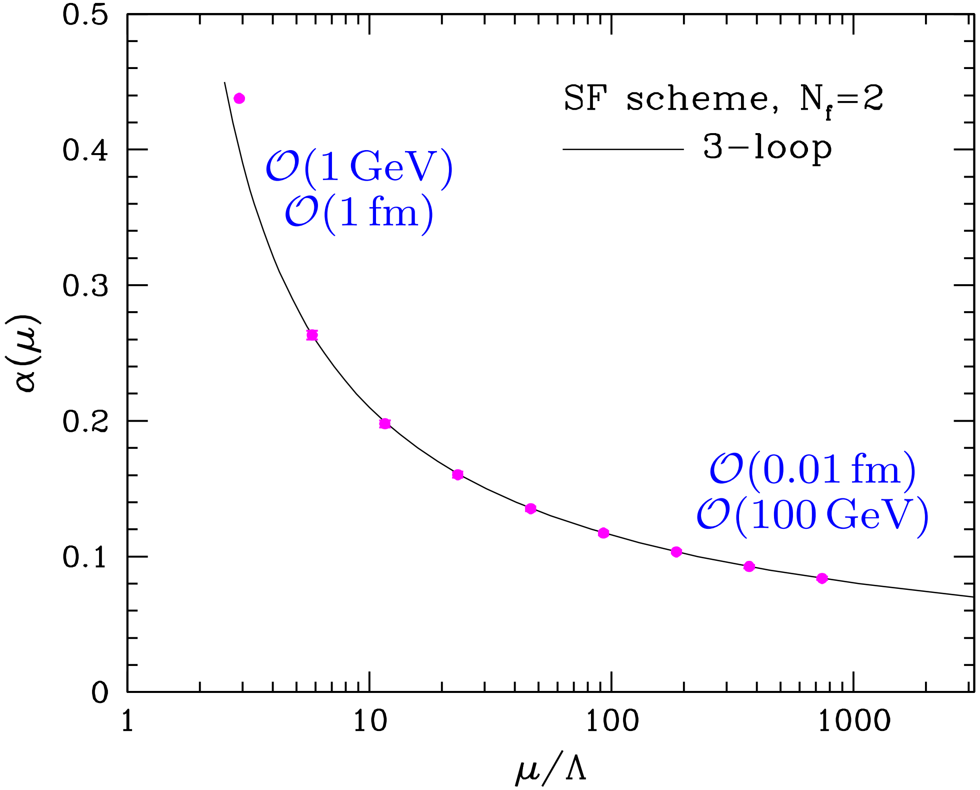


NON-PERTURBATIVE REGIME

$\Rightarrow$

quantitative study needs LATTICE

Lagrangian of QCD:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \sum_{(f)=1}^{N_f} \bar{\psi}_{(f)} \left( i\gamma^\mu D_\mu - m_{(f)} \right) \psi_{(f)}$$

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- Minkowski path integral can not be used in practice – the phase factor $e^{iS}$ would lead to oscillatory behaviour.
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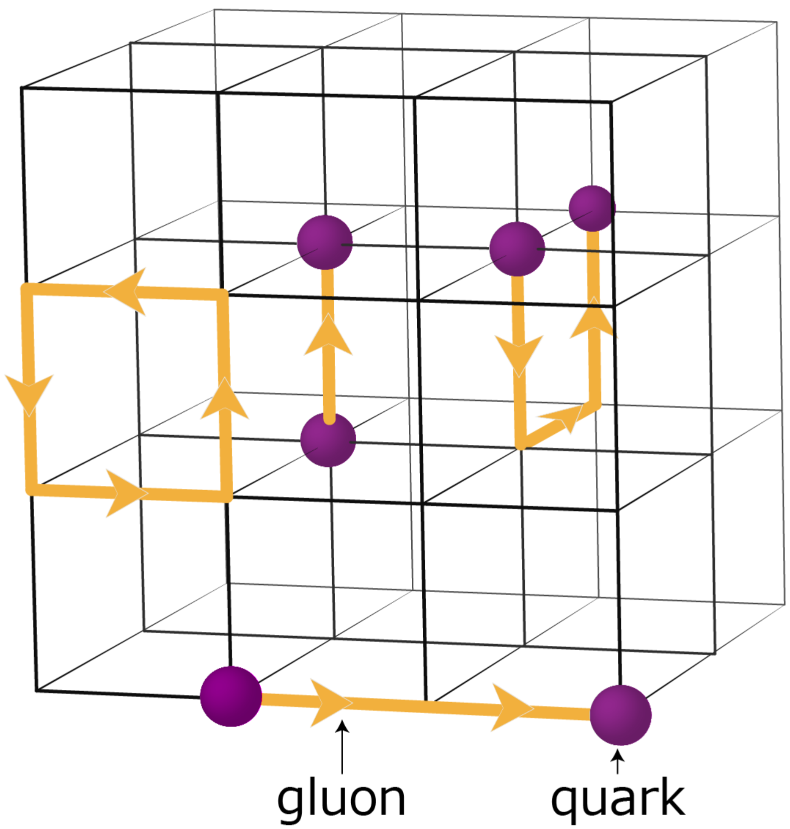
# Lattice formulation




Source: JICFuS, Tsukuba

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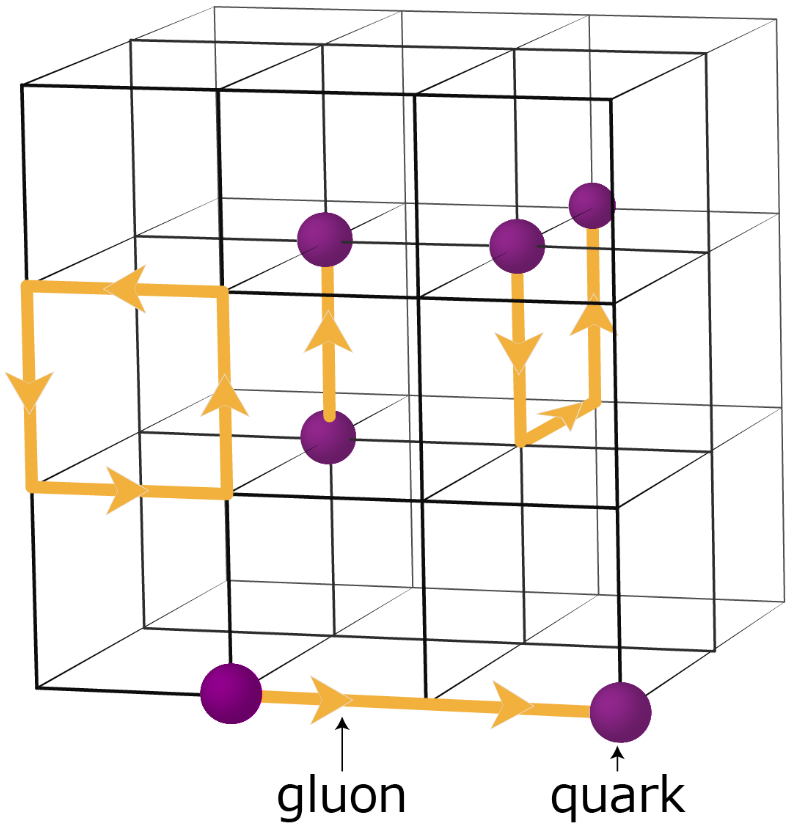


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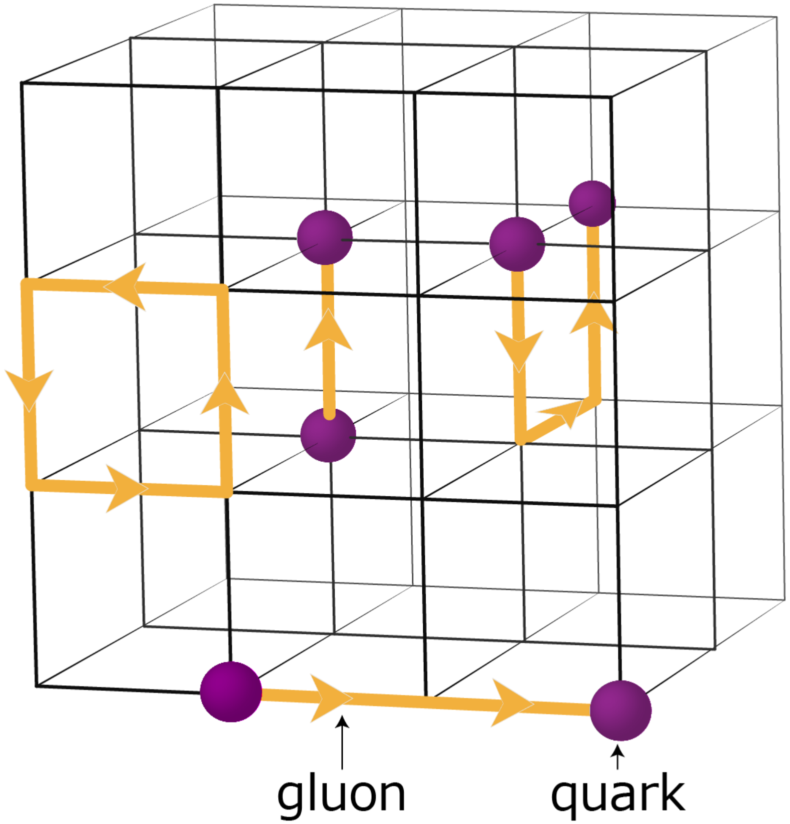


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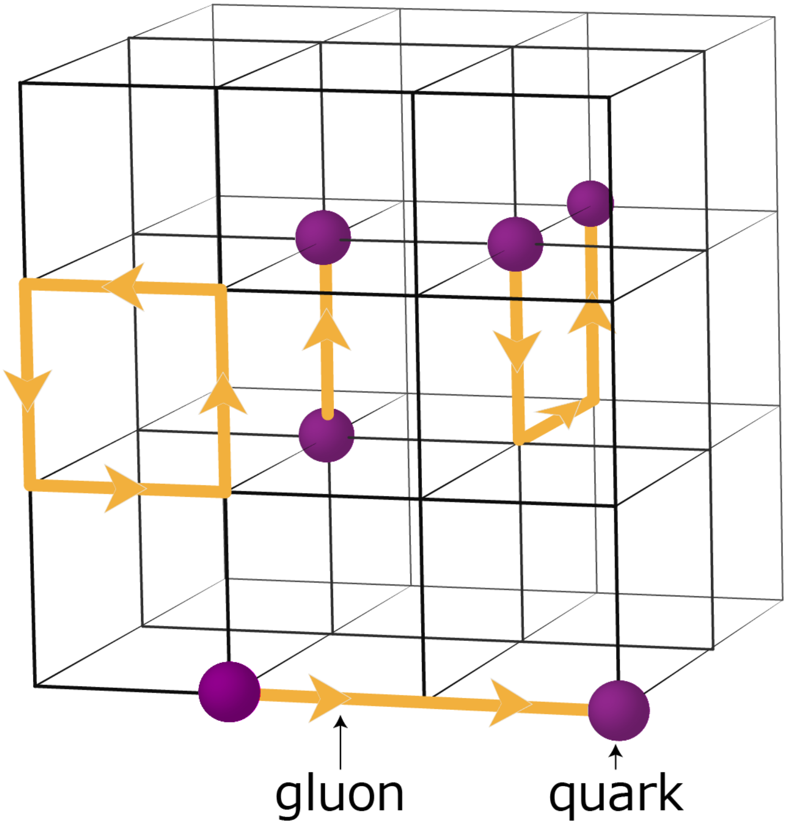


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- Remove the regulator:
  - ⋆ continuum limit $a \to 0$,
  - ⋆ infinite volume limit $L \to \infty$.

# Discretization of the action

- gluonic part – "easy" – gauge action constructed from Wilson loops of size 1x1 (plaquettes) and 1x2 (rectangles):

$$S_G[U] = \frac{\beta}{3}\sum_x \Big( b_0 \sum_{\mu,\nu=1} \text{Re Tr}\big(1 - P^{1\times1}_{x;\mu,\nu}\big) + b_1 \sum_{\mu\neq\nu} \text{Re Tr}\big(1 - P^{1\times2}_{x;\mu,\nu}\big)\Big),$$

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  - ⋆ discretizations used in practice:
    - ◇ clover fermions,
    - ◇ twisted mass (TM) fermions,
    - ◇ overlap fermions,
    - ◇ domain wall fermions,
    - ◇ staggered fermions,
    - ◇ other less popular.

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- Hence, huge computational resources needed!
- **QCD was one of the first branches of science that "asked" for such computational resources and thus inspired the development of supercomputers.**

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# Lattice QCD



Some of the aspects of QCD that can be studied on the lattice:

- QCD parameters: $\alpha_s$, $\Lambda_{QCD}$, quark masses etc.
- hadron spectrum: meson and baryon masses, exotic hadrons
- hadron structure: nucleon charges, EM form factors, parton distribution functions, GPDs, nucleon spin content
- QCD thermodynamics: QCD phase diagram, deconfinement, chiral symmetry restoration
- Standard Model parameters: CKM matrix
- constraints on effective theories: $\chi$PT, HQET

Some collaborations in LQCD:

Alpha, BMW, CLS, CP-PACS, ETMC, HALQCD, hotQCD, JLQCD, LHC, LSD, Mainz, MILC, NME, NPLQCD, QCDSF, PNDME, RBC, RQCD, SWME, tmFT, TWQCD, UKQCD, USQCD, WHOT-QCD

in total $\approx 500-600$ physicists

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- Two major strategies:
  - ⋆ "**brute force**" – up to $\mathcal{O}(12)$ nucleons,
  - ⋆ **matching to an EFT** – refine effective nuclear (many-body) forces through appropriate finite-volume matching calculations.

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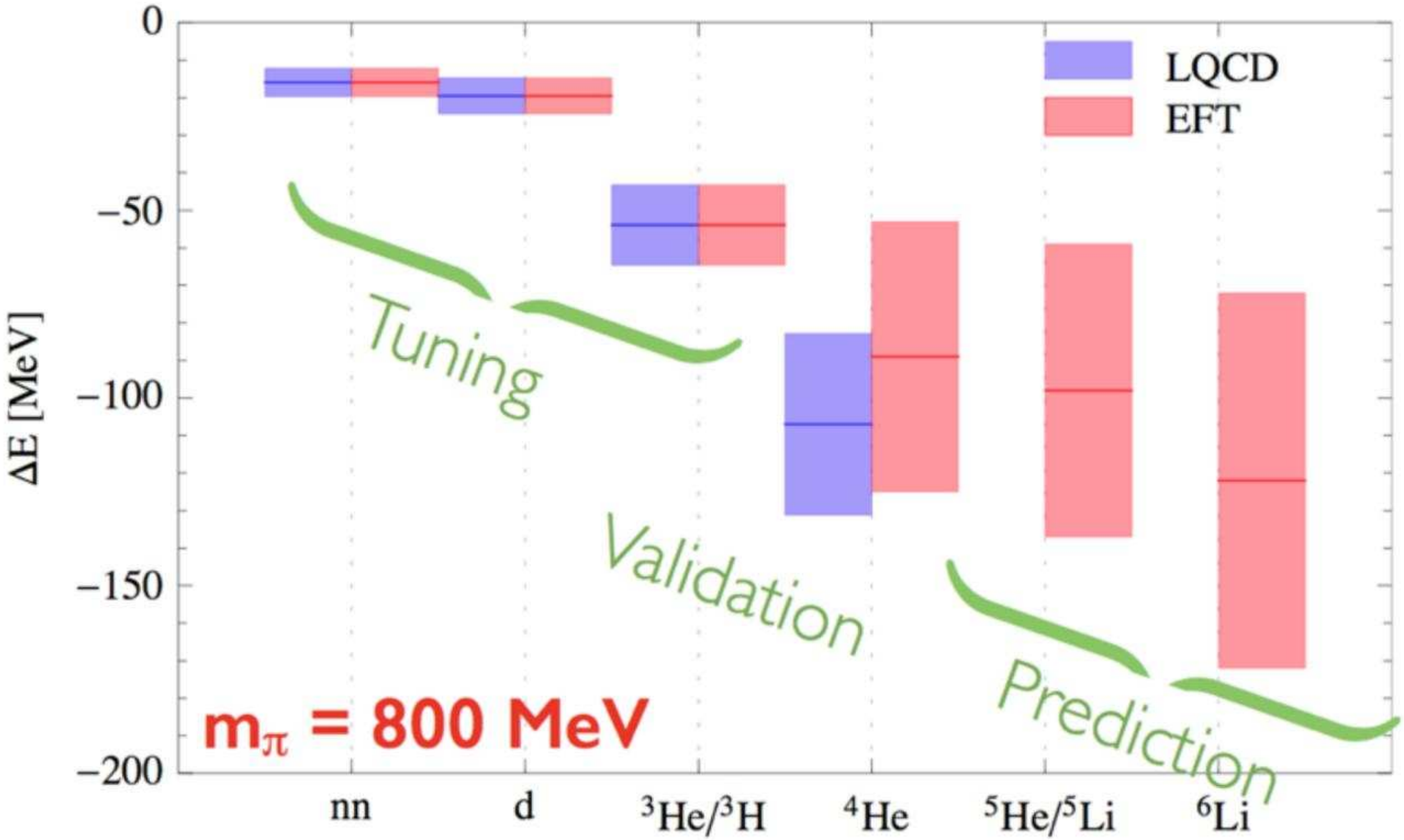



Source: M. Savage, 1611.02078

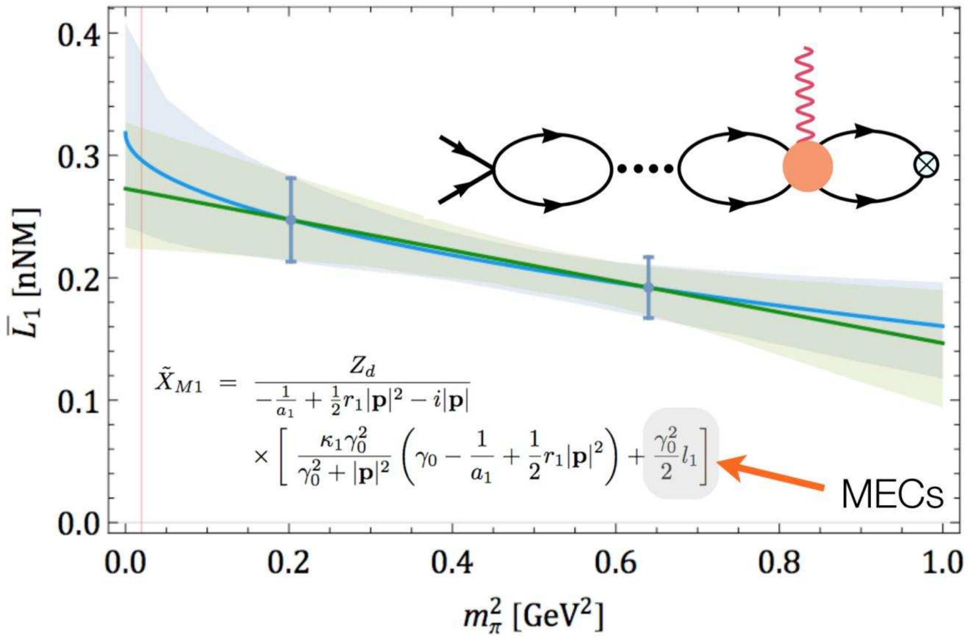
Based on: N. Barnea et al., Phys. Rev. Lett. 114, 052501 (2015), 1311.4966, J. Kirscher, Int. J. Mod. Phys. E25, 1641001 (2016), 1509.07697.

LQCD calculation of an inelastic nuclear reaction cross section $np \to d\gamma$

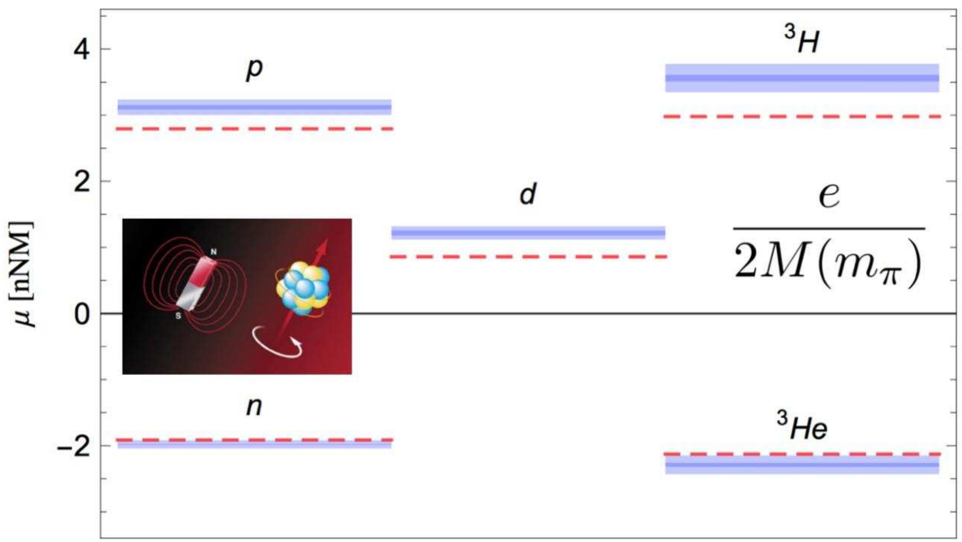
2 pion masses, extrapolated to the physical pion mass

Lattice: $\sigma = 334.9(5.3)$ mb (incident neutron speed 2200 m/s)

vs. experiment $\sigma = 334.2(0.5)$ mb




Magnetic moments @ $m_\pi = 805$ MeV

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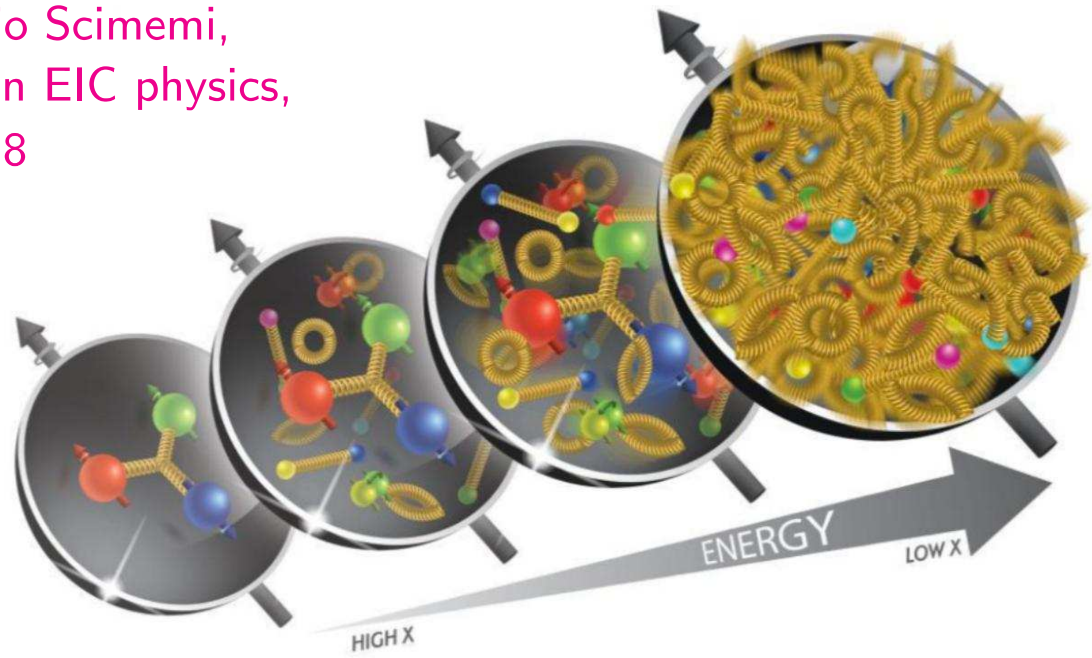
Based on: S. Beane et al. (NPLQCD), Phys. Rev. Lett. 115, 132001 (2015), 1505.02422,
S. Beane et al. (NPLQCD), Phys. Rev. Lett. 113, 252001 (2015), 1409.3566

# Nucleon structure
# Parton distribution functions (PDFs)

Source: Ignazio Scimemi, review talk on EIC physics, Cracow 2018



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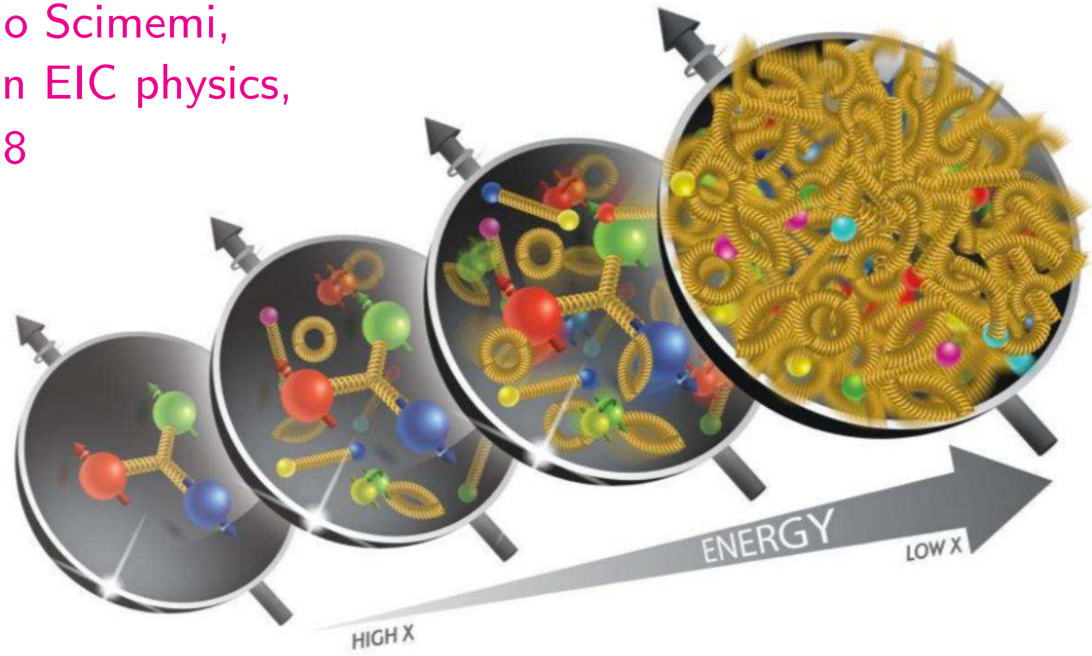


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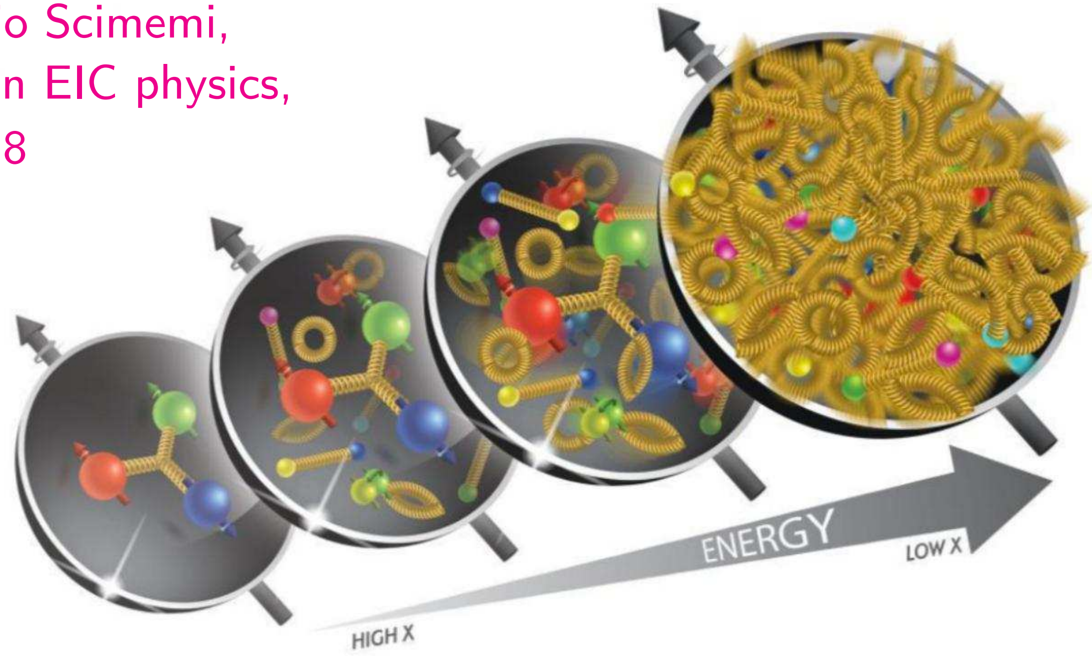


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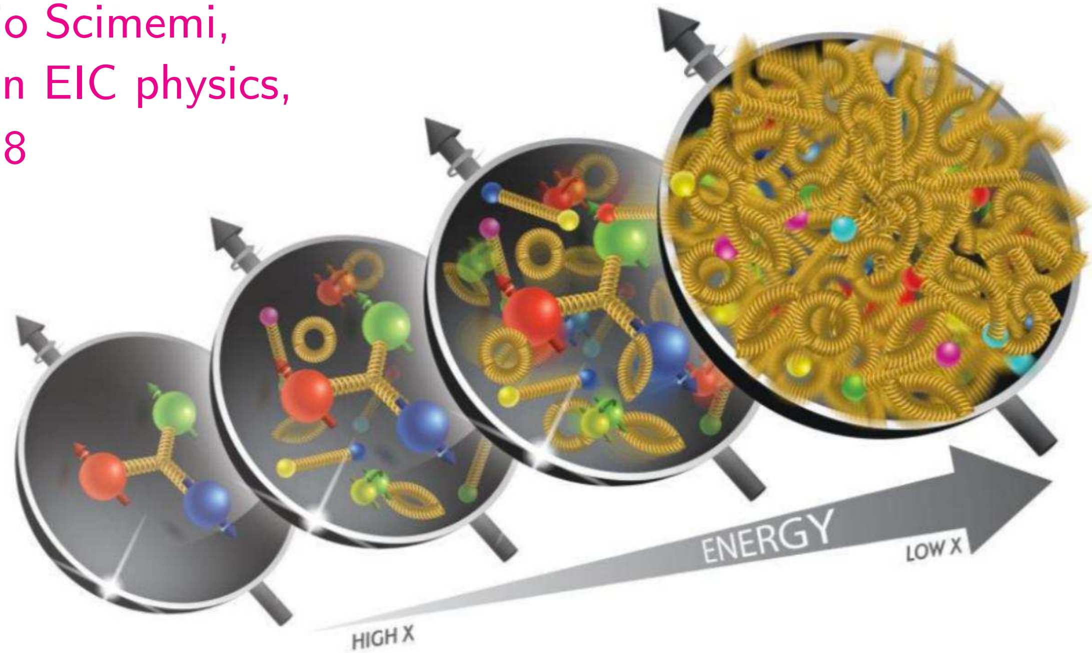


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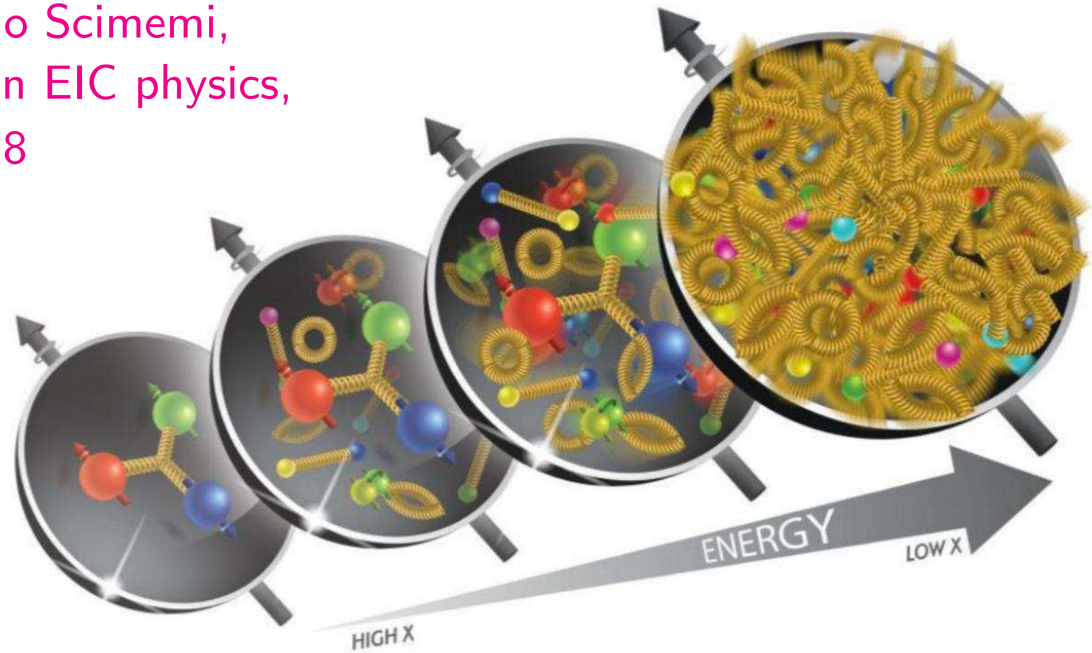


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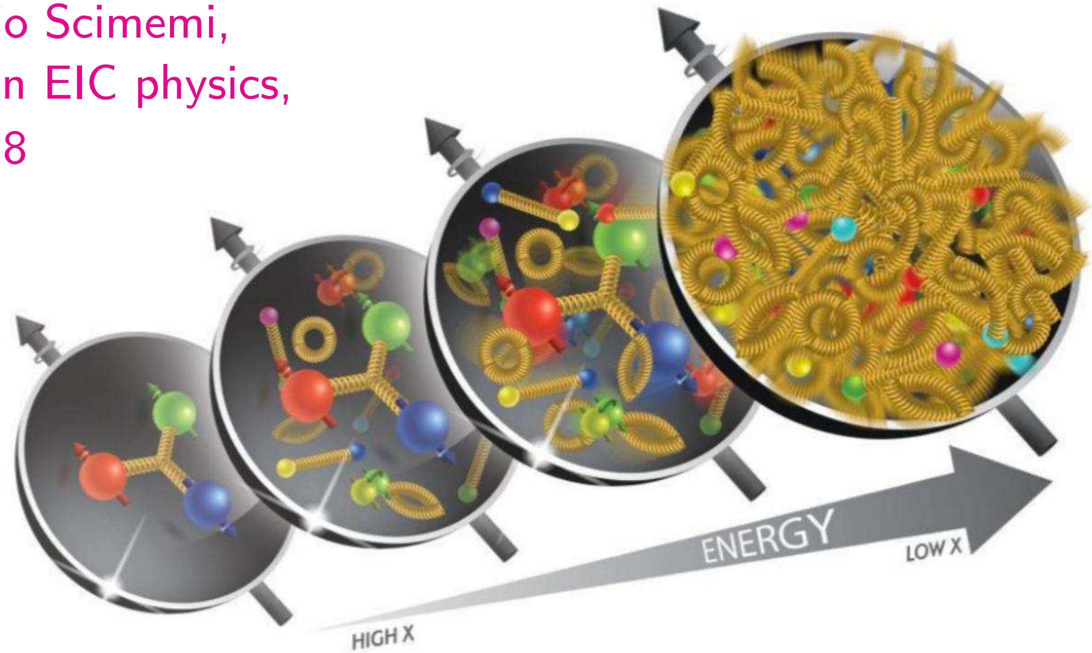


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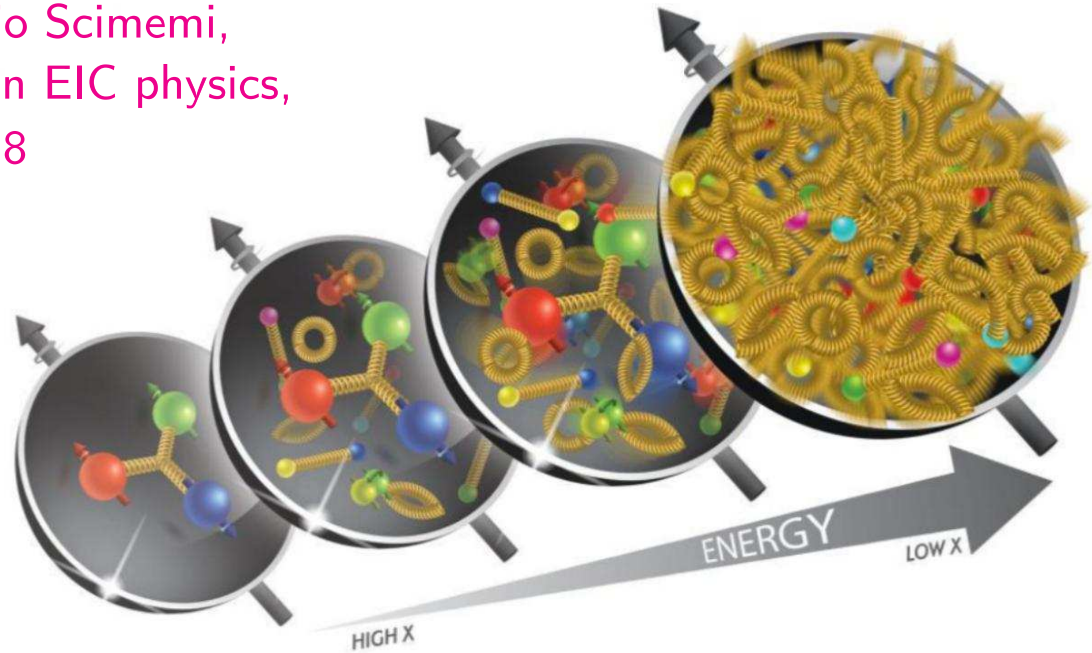


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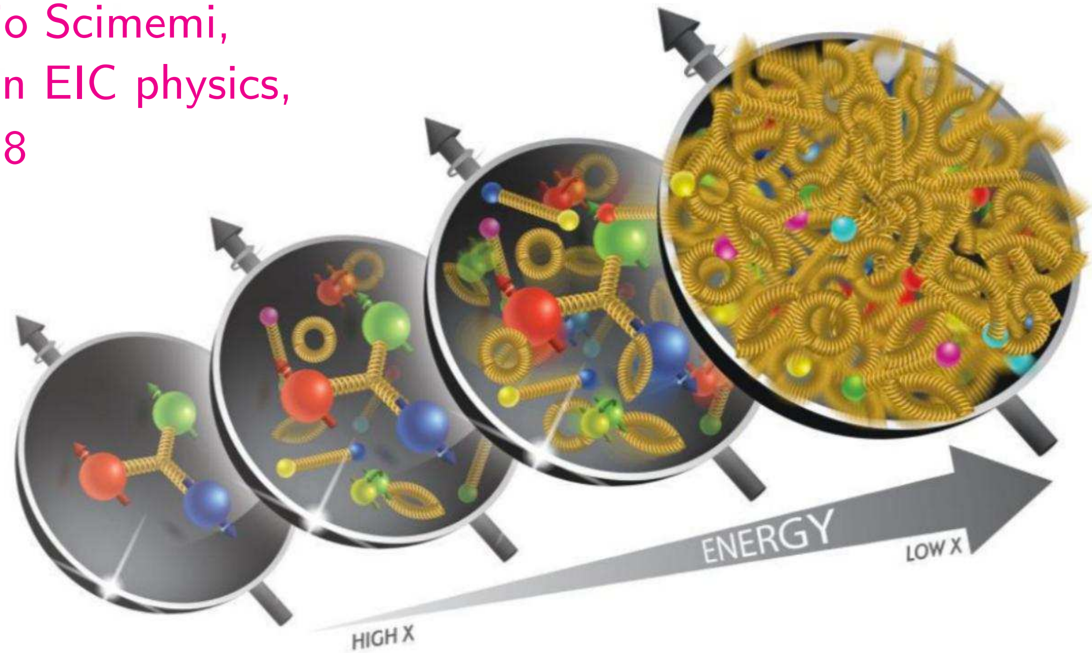


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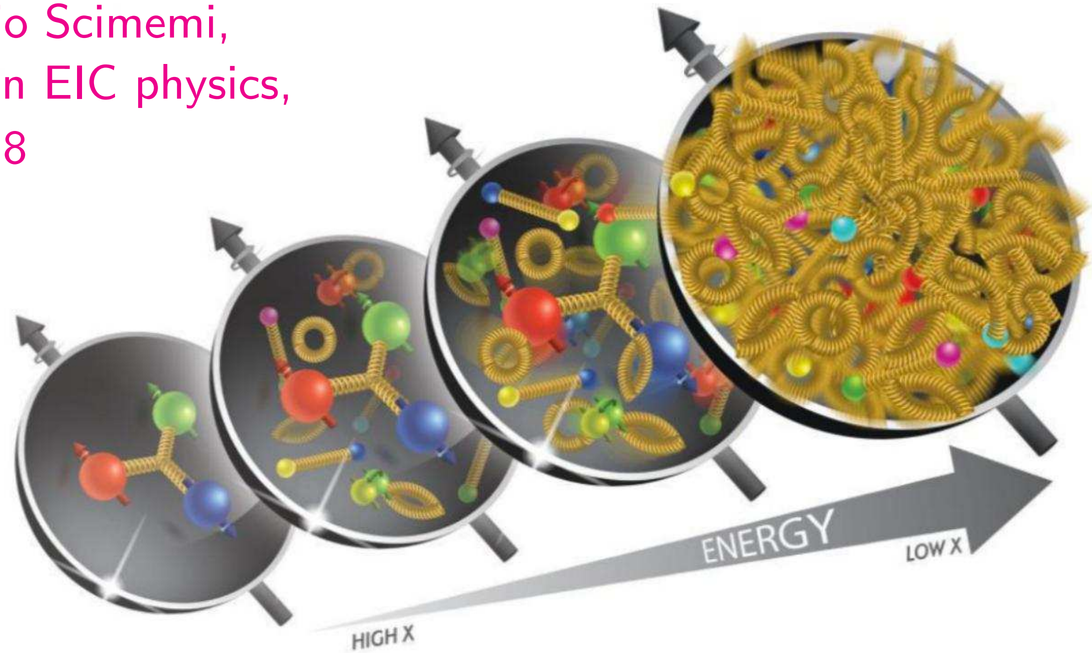


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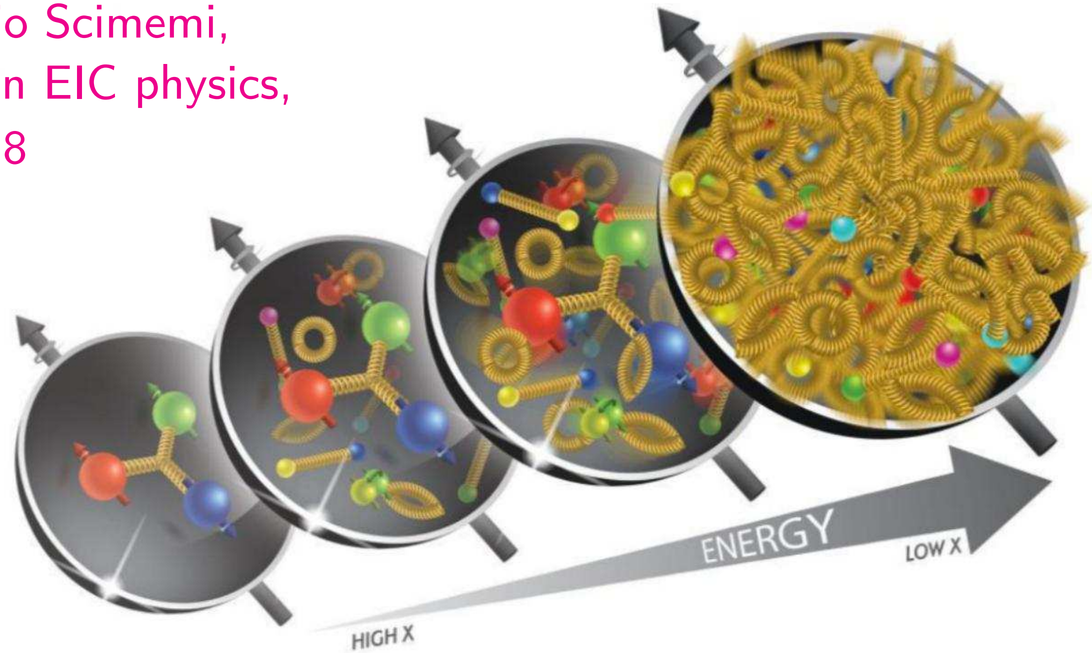


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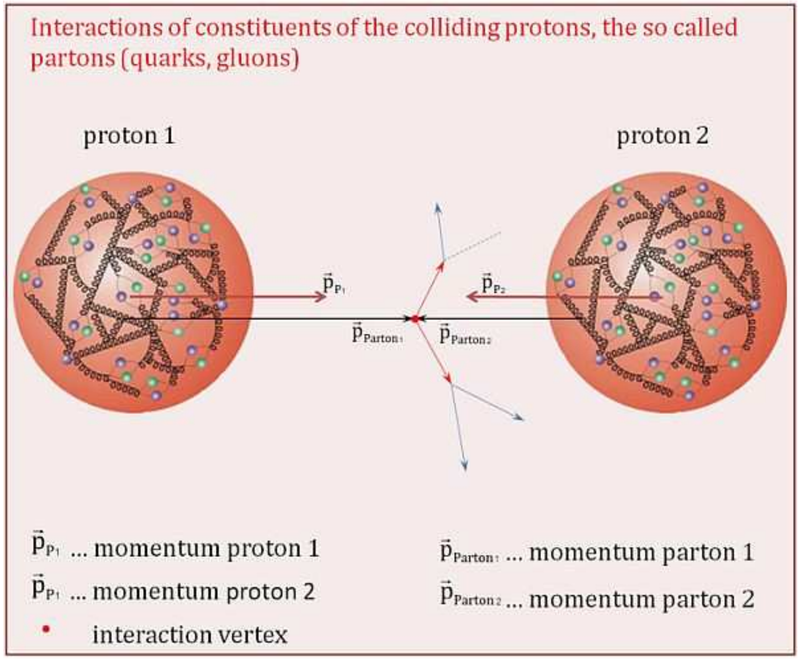
# Quantifying nucleon structure

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Different functions characterizing the behavior of partons:

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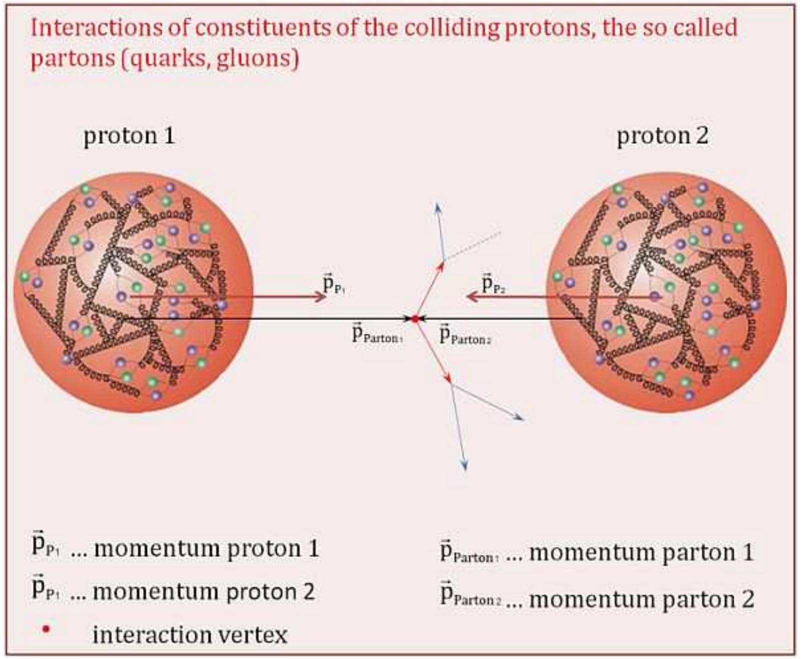


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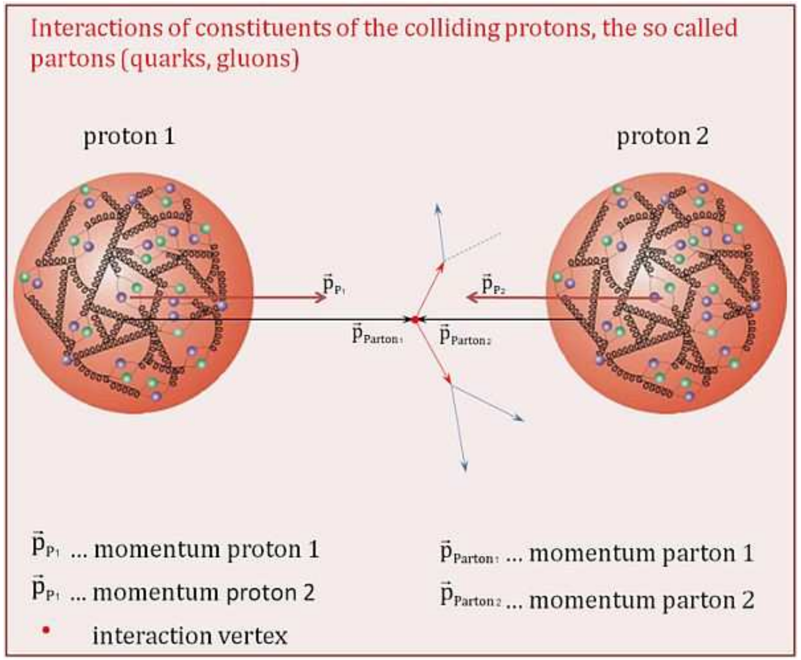
- **generalized parton distributions (GPDs)** – probe the three-dimensional structure,
- **transverse momentum dependent parton distribution functions (TMDs)** – complement the 3D picture.

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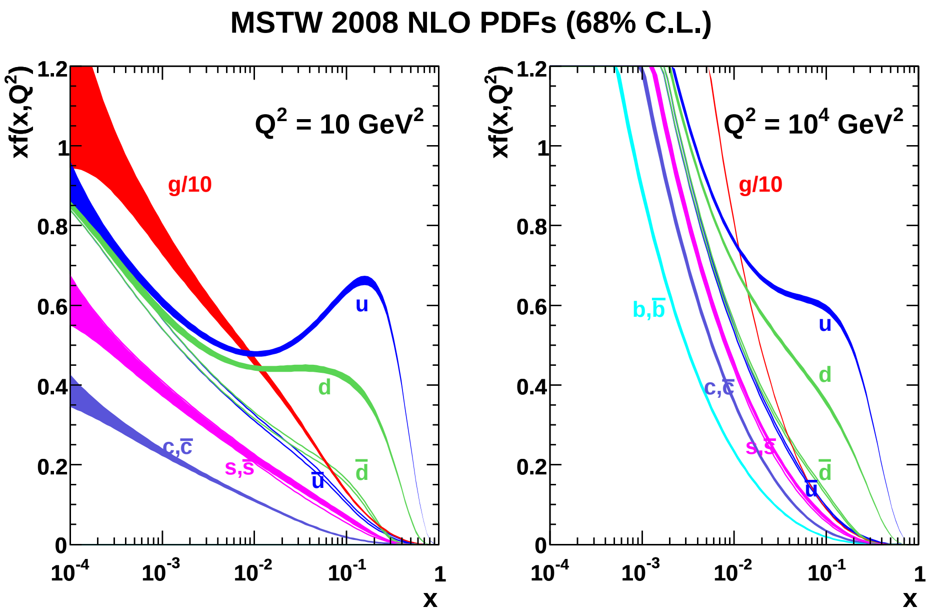


MSTW2008, Eur. Phys. J. C63, 189

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- Accessible on the lattice – moments of the distributions, but ...

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  X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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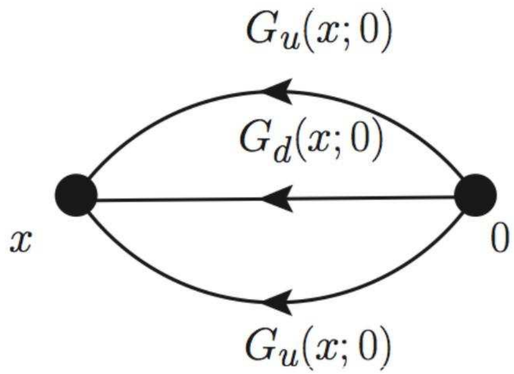


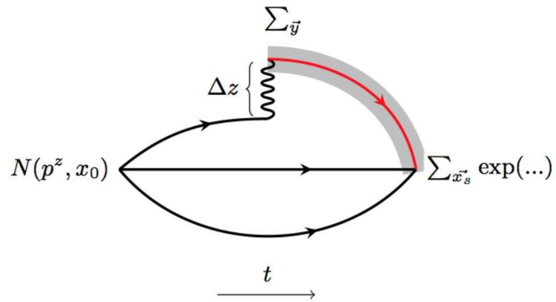

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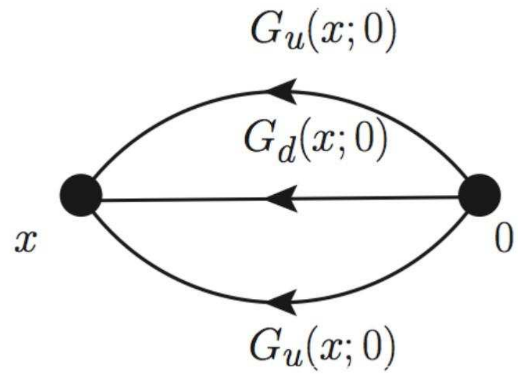


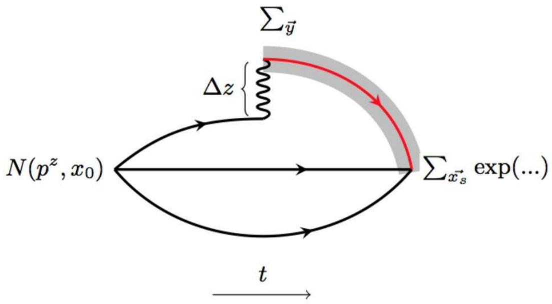

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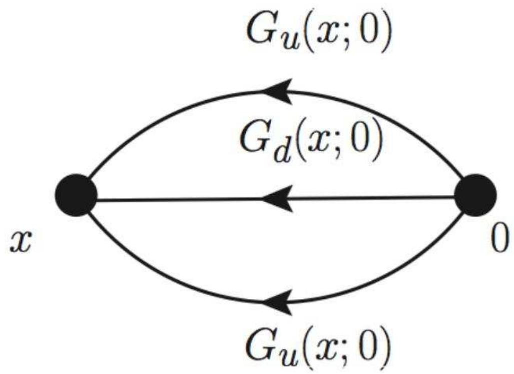
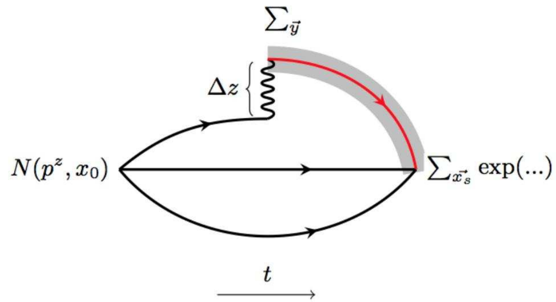
- The highly non-trivial aspect:
  how to relate $\tilde{q}(x,\mu^2,P_3)$ to the light-front PDF $q(x,\mu^2)$ (infinite momentum frame)
  $\Rightarrow$ **Large Momentum Effective Theory (LaMET)**



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Non-perturbative renormalization scheme: RI'-MOM.

G. Martinelli et al., Nucl. Phys. B445 (1995) 81

## Matching of quasi-PDFs and PDFs

To relate the quasi-PDFs to the usual PDFs, one uses the fact that the IR region of the distributions is untouched when going from a finite to an infinite momentum. In other words, if $q(x,\mu)$ is the usual PDF defined through light-cone correlations, then one should have:

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$$q(x,\mu) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi} Z_F(\mu)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 q^{(1)}(x/y,\mu) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2),$$

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Explicit formulae for 1-loop perturbative matching:

- transverse momentum cutoff scheme to $\overline{\mathrm{MS}}$ matching
  X. Xiong et al., PRD **90** (2014) 014051
- $\overline{\mathrm{MS}}$ to $\overline{\mathrm{MS}}$ matching W. Wang, S. Zhao, R. Zhu, arXiv:1708.02458 [hep-ph]
- RI to $\overline{\mathrm{MS}}$ matching I.W. Stewart, Y. Zhao, arXiv:1709.04933 [hep-ph]
- treatment of the UV log divergence in wave function corrections T. Izubuchi et al., arXiv:1801.03917 [hep-ph], C. Alexandrou et al., arXiv:1803.02685, 1807.00232 [hep-lat]

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5. Apply nucleon mass corrections to eliminate residual $m_N/P_3$ effects.

# Lattice setup



- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$

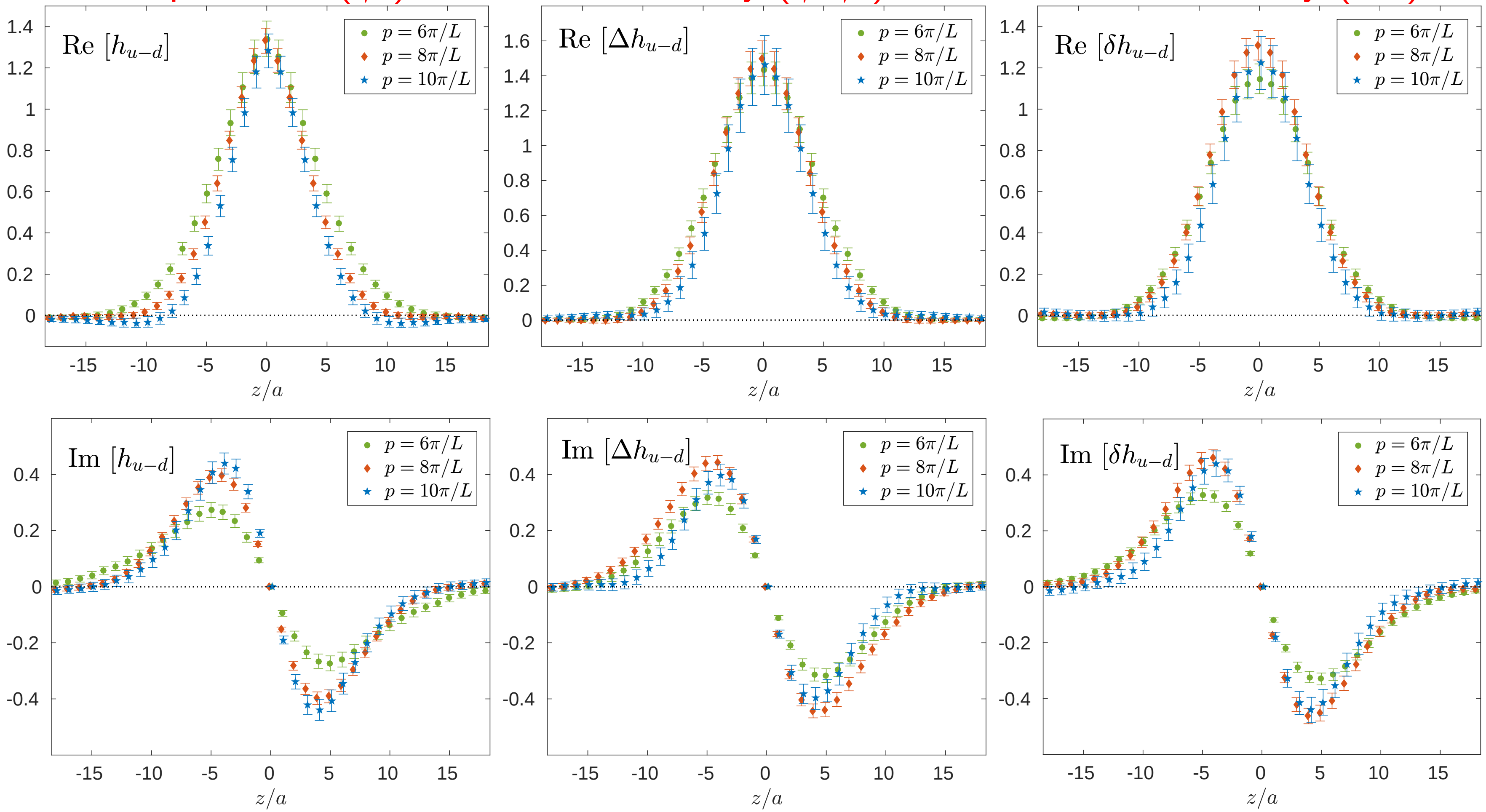
| | | |
|---|---|---|
| $\beta$=2.10, | $c_{\rm SW}$=1.57751, | $a$=0.0938(3)(2) fm |
| $48^3 \times 96$ | $a\mu = 0.0009$ | $m_N = 0.932(4)$ GeV |
| $L = 4.5$ fm | $m_\pi = 0.1304(4)$ GeV | $m_\pi L = 2.98(1)$ |

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001
C. Alexandrou et al., arXiv: 1807.00232 [hep-lat]

# Results

# Bare matrix elements at $t_s = 12a$

unpolarized $(\gamma_0)$ | helicity $(\gamma_5\gamma_3)$ | transversity $(\sigma_{3i})$

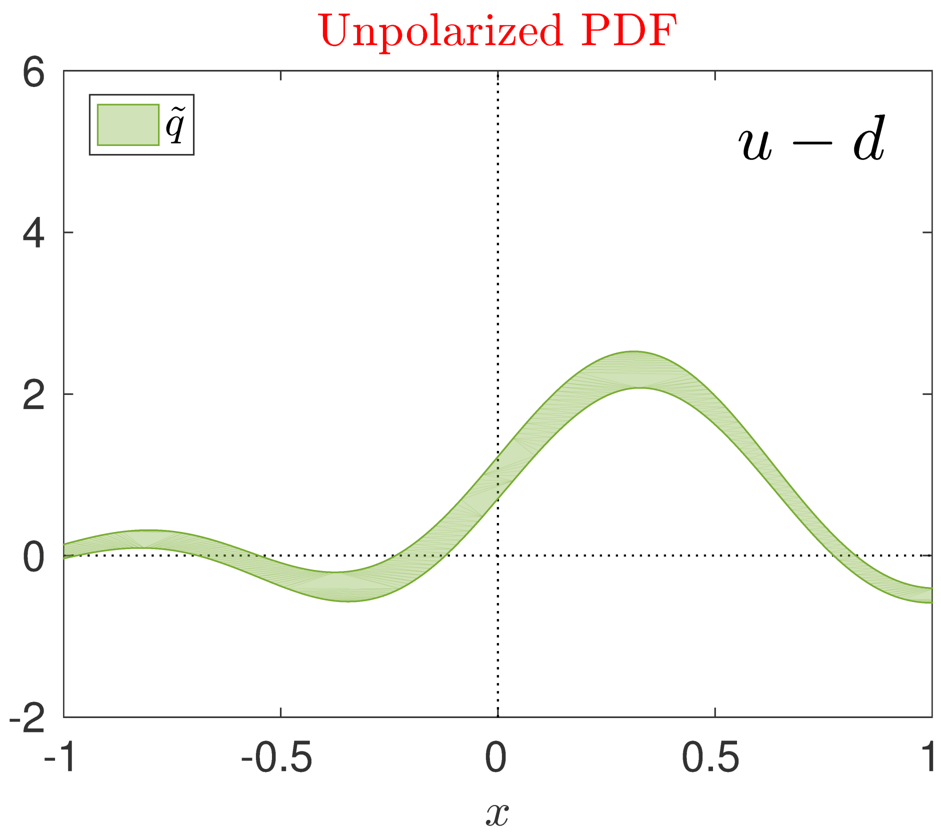



C. Alexandrou et al.: Phys. Rev. Lett. 121 (2018) 112001 and 1807.00232 [hep-lat]

STATISTICS:

| | | |
|---|---|---|
| $P_3 = \frac{6\pi}{L}$ – 4800 meas. | $P_3 = \frac{6\pi}{L}$ – 6240 meas. | $P_3 = \frac{6\pi}{L}$ – 9600 meas. |
| $P_3 = \frac{8\pi}{L}$ – 38250 meas. | $P_3 = \frac{8\pi}{L}$ – 38250 meas. | $P_3 = \frac{8\pi}{L}$ – 38250 meas. |
| $P_3 = \frac{10\pi}{L}$ – 72990 meas. | $P_3 = \frac{10\pi}{L}$ – 72990 meas. | $P_3 = \frac{10\pi}{L}$ – 72990 meas. |

Nucleon momentum $\frac{10\pi}{48}$




C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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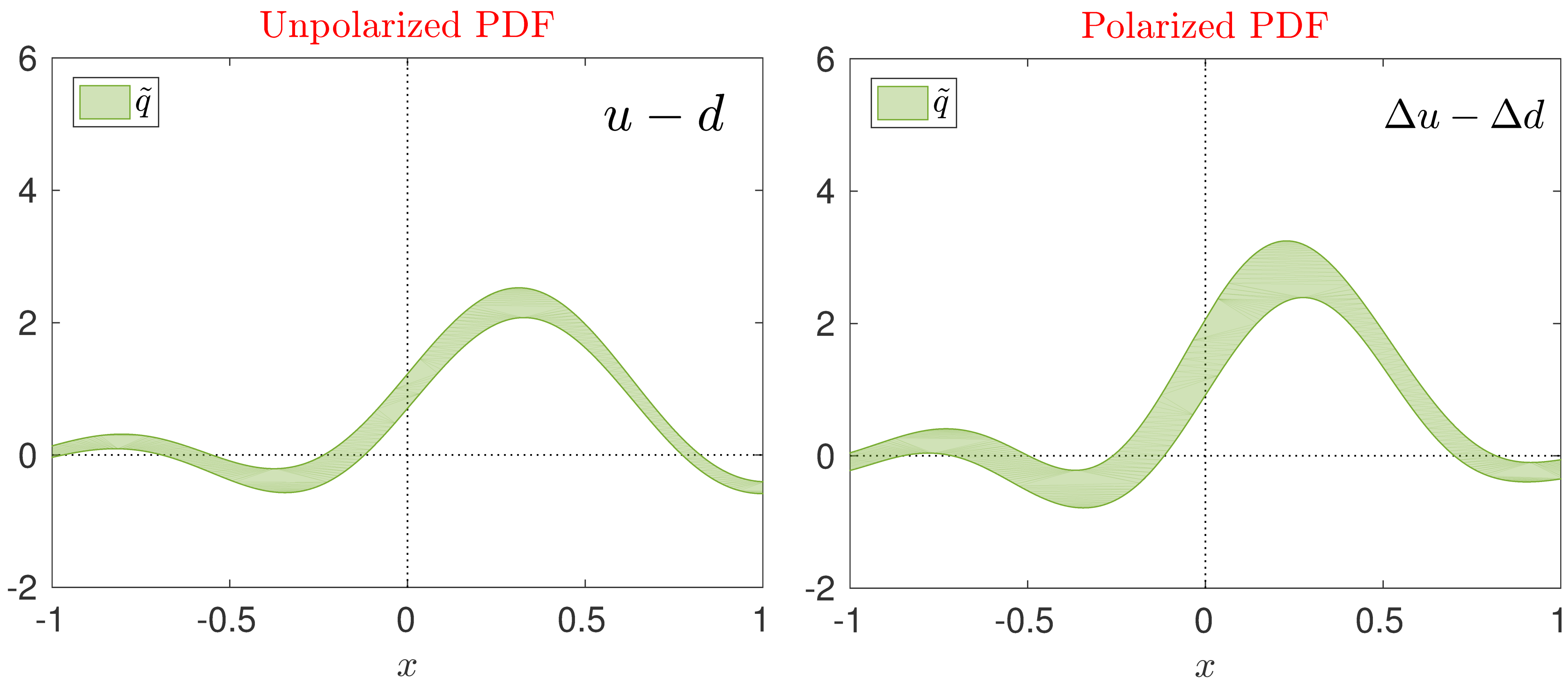
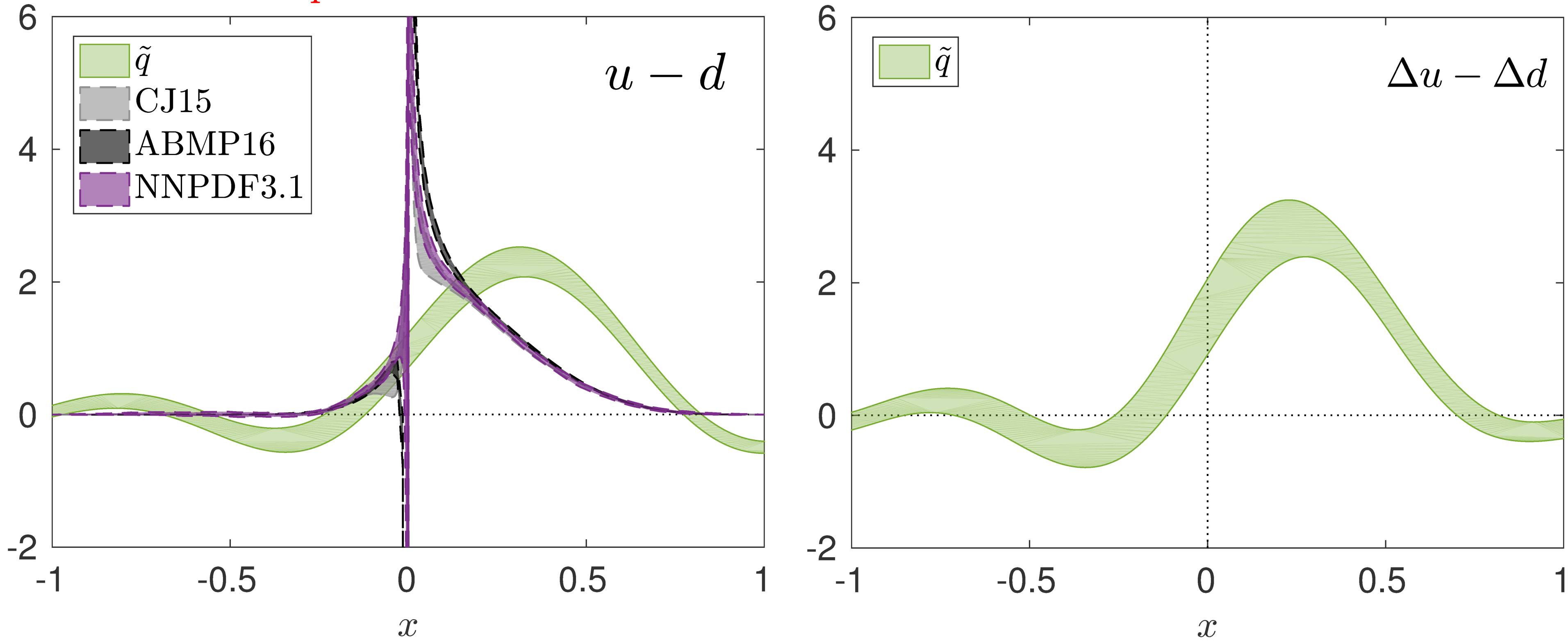



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Nucleon momentum $\frac{10\pi}{48}$

Unpolarized PDF | Polarized PDF

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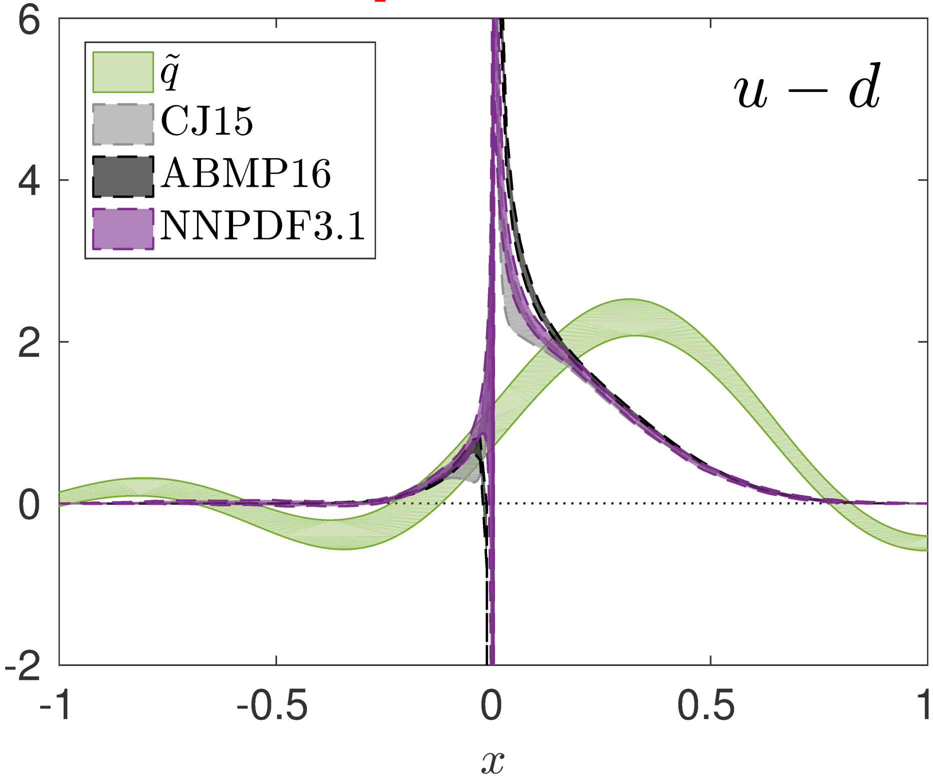
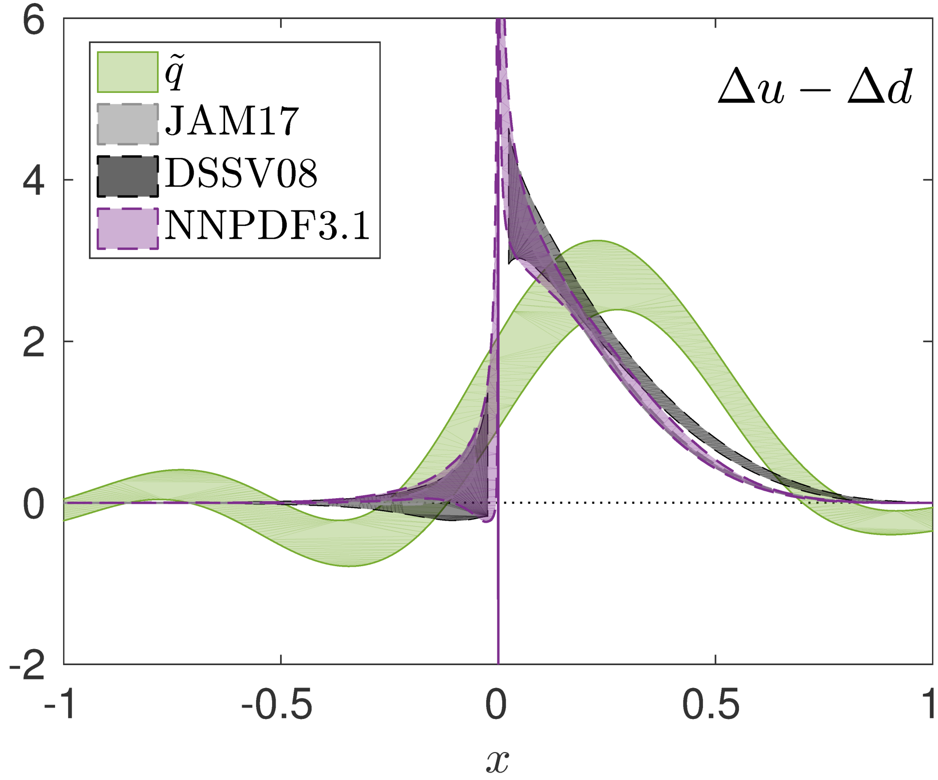



Polarized PDF




C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

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$C$ – matching kernel:

$$C\left(\xi,\frac{\xi\mu}{xP_3}\right)=\delta(1-\xi)+\frac{\alpha_s}{2\pi}C_F\begin{cases}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1+\frac{3}{2\xi}\right]_+ & \xi>1,\\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{x^2P_3^2}{\xi^2\mu^2}\left(4\xi(1-\xi)\right)-\frac{\xi(1+\xi)}{1-\xi}+2\iota(1-\xi)\right]_+ & 0<\xi<1,\\ \left[-\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}-1+\frac{3}{2(1-\xi)}\right]_+ & \xi<0,\end{cases}$$

[T. Izubuchi et al., arXiv:1801.03917 [hep-ph], C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

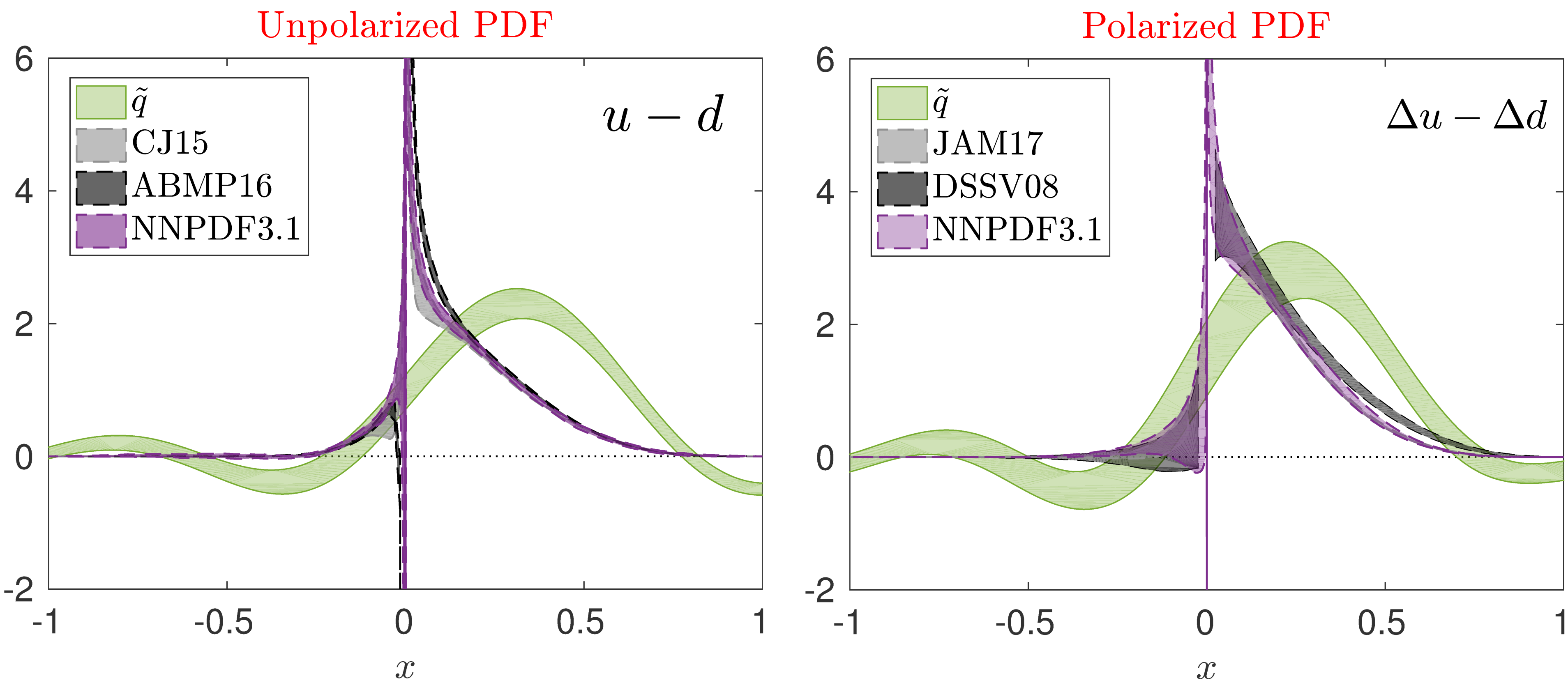
$\iota=0$ for $\gamma_0$ and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

Plus prescription at $\xi=1$:

$$\int\frac{d\xi}{|\xi|}\left[C\left(\xi,\frac{\xi\mu}{xP_3}\right)\right]_+\tilde{q}\left(\frac{x}{\xi}\right)=\int\frac{d\xi}{|\xi|}C\left(\xi,\frac{\xi\mu}{xP_3}\right)\tilde{q}\left(\frac{x}{\xi}\right)-\tilde{q}\left(x\right)\int d\xi\,C\left(\xi,\frac{\mu}{xP_3}\right).$$

# Matched PDFs

Nucleon momentum $\frac{10\pi}{48}$




C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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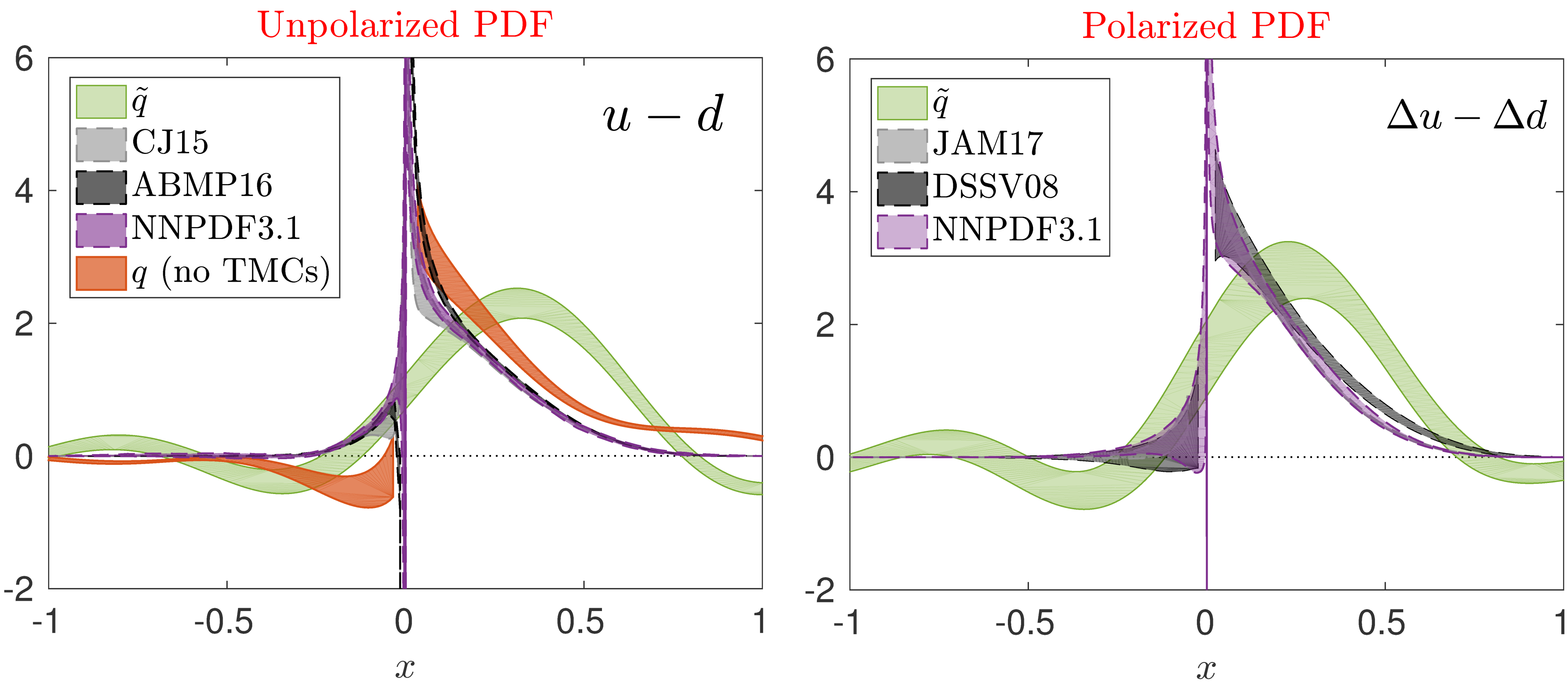
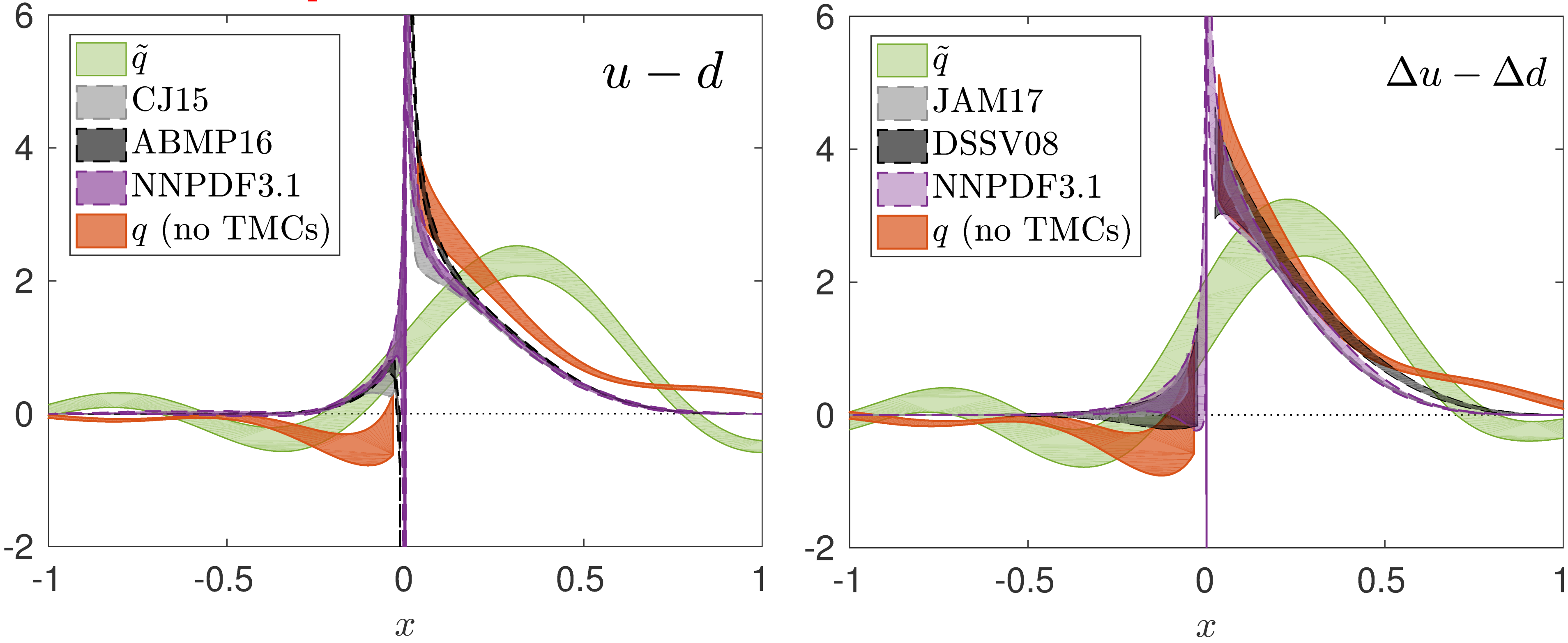



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

# Matched PDFs

Nucleon momentum $\frac{10\pi}{48}$

Unpolarized PDF | Polarized PDF




C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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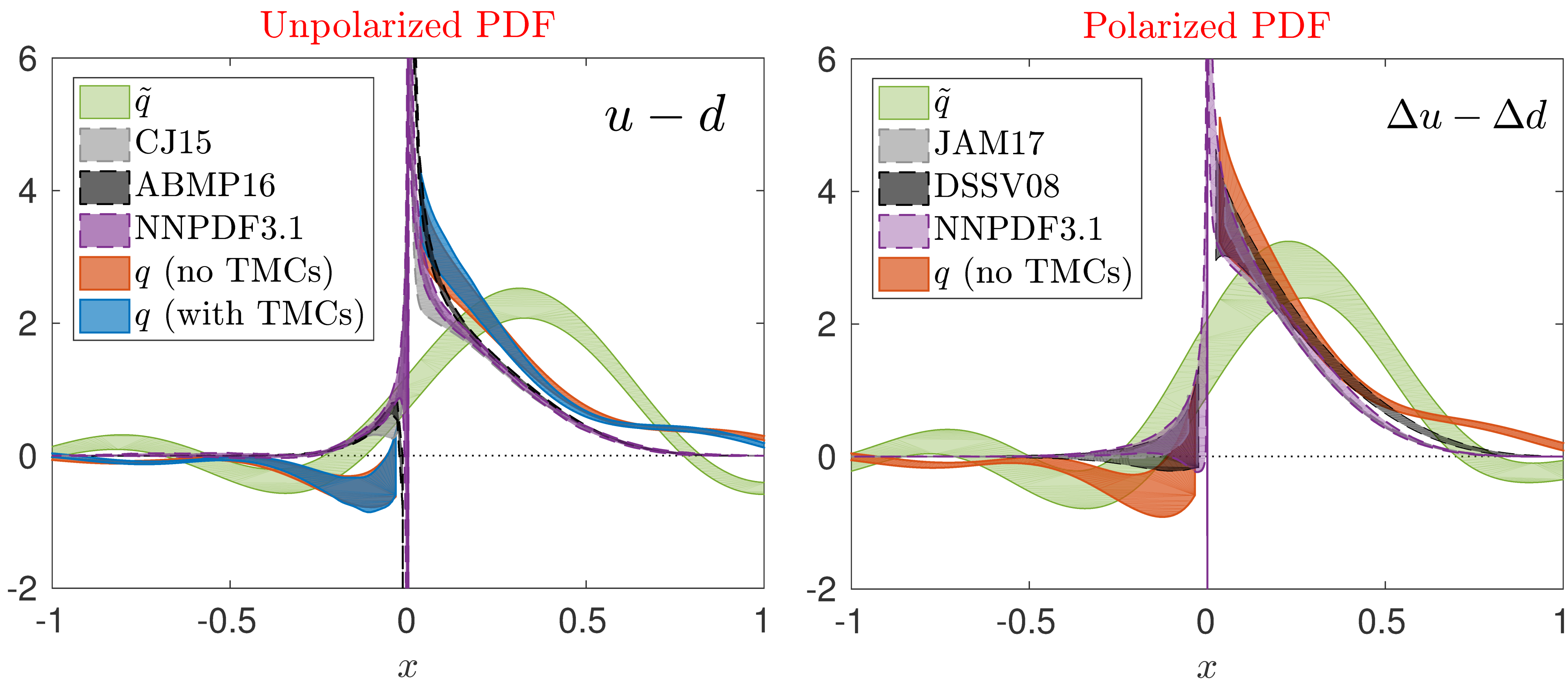
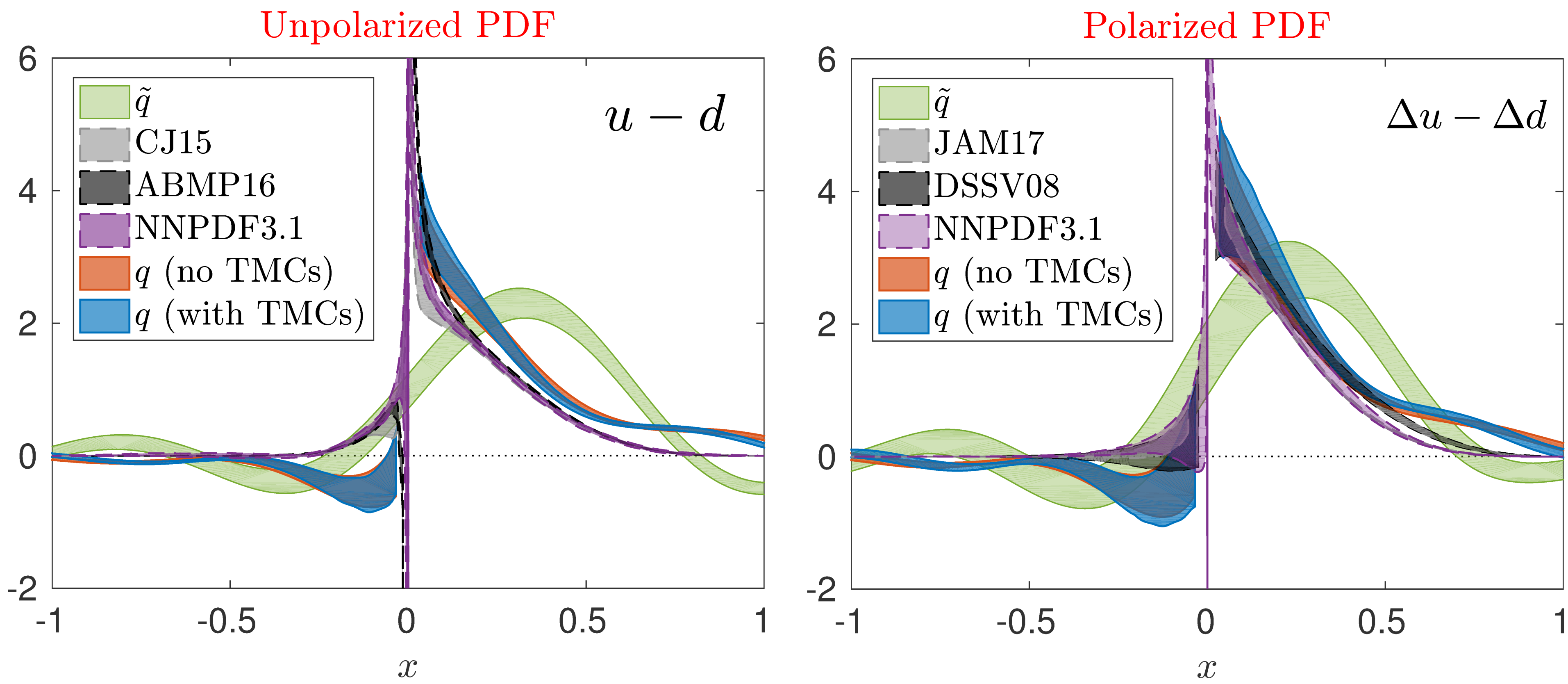



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

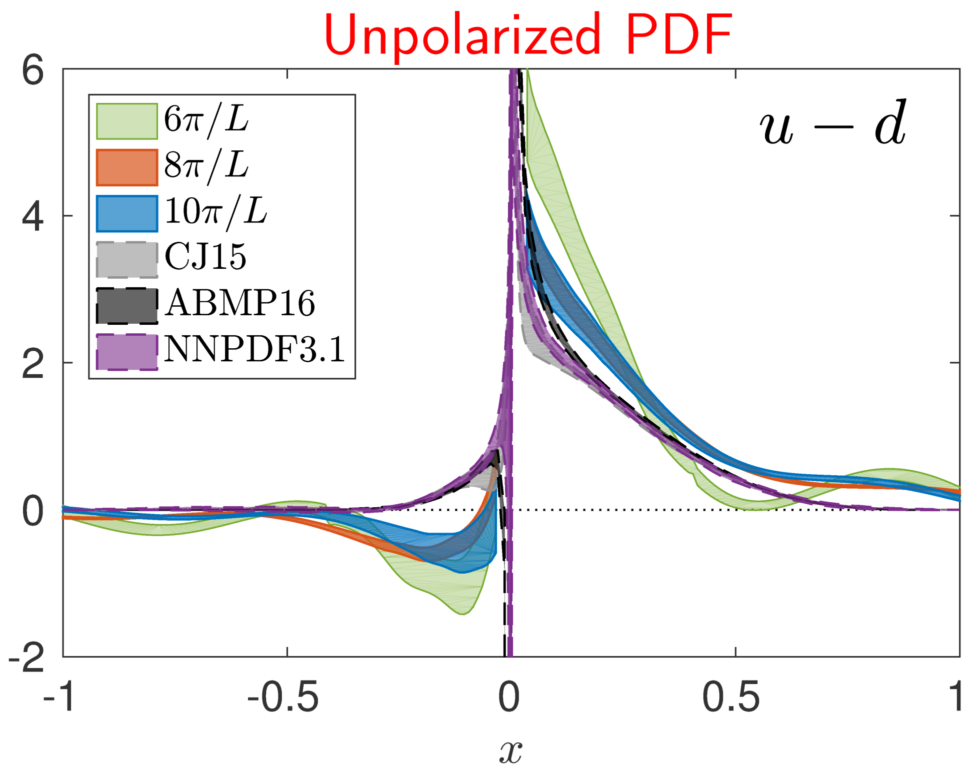
# Matched PDF + TMCs

Nucleon momentum $\frac{10\pi}{48}$




C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Nucleon momenta $\frac{\{6,8,10\}\pi}{48}$

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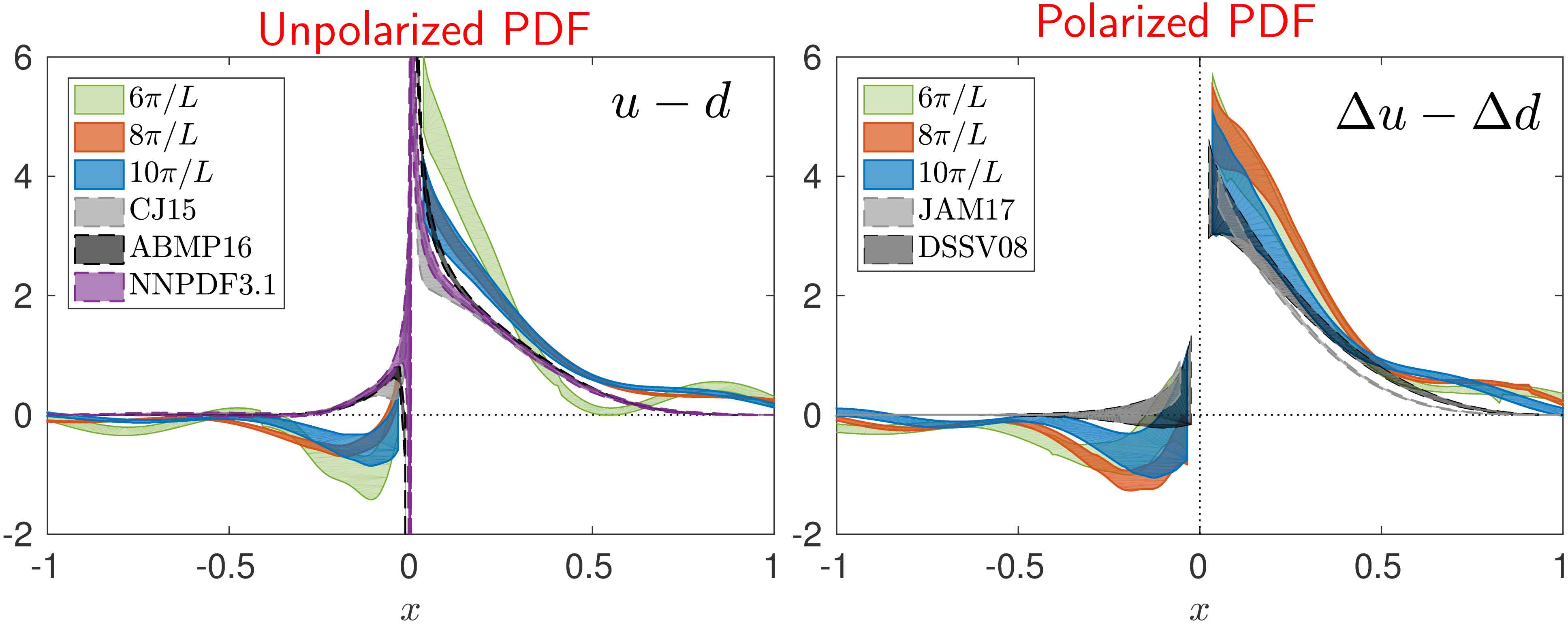
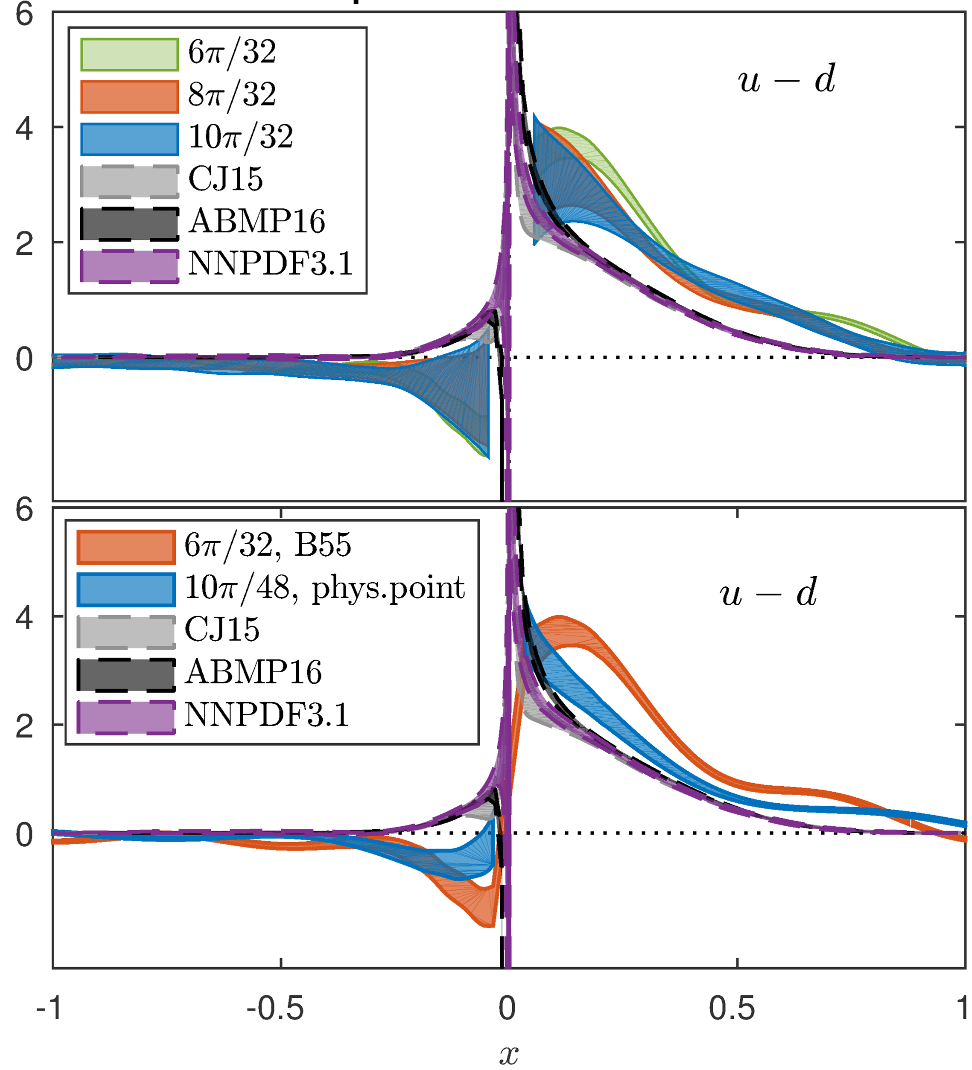
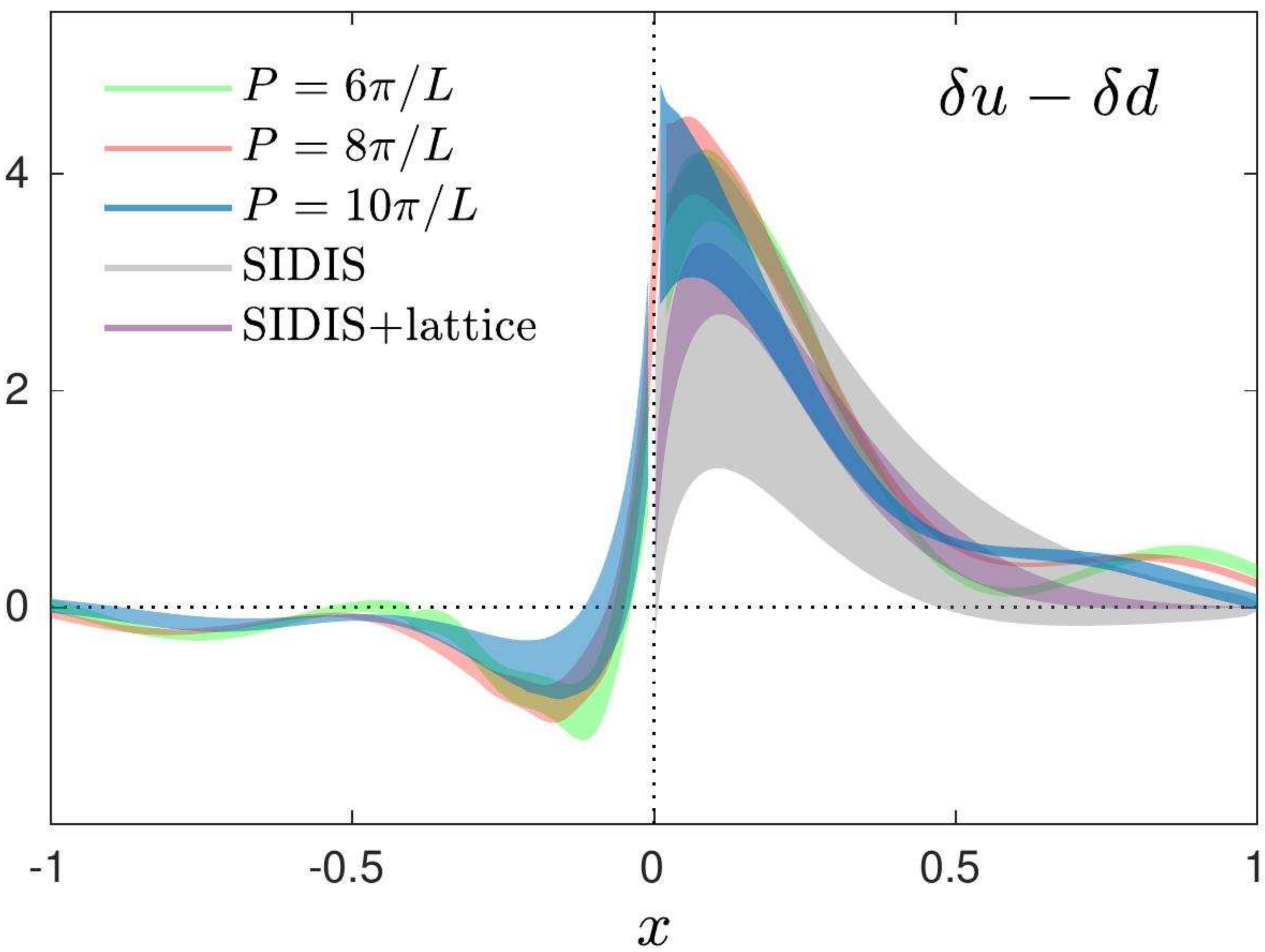



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF




C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001




C. Alexandrou et al., Phys. Rev. D (Rapid Communications), in press, arXiv: 1807.00232 [hep-lat]

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Biggest challenge:
**Reach large momenta at large source-sink separations**

## A guide to light-cone PDFs from lattice QCD: an overview of approaches, techniques and results

Krzysztof Cichy[1], Martha Constantinou[2] [a]

[1] *Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*

[2] *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

- **97 pages, arXiv: 1811.07248 [hep-lat]**
- discusses in detail quasi-distributions:
  nucleon: **non-singlet quark qPDFs**, qGPDs, qTMDs, singlet qPDFs, gluon qPDFs; pion: qPDFs, qDAs
- reviews also other approaches:
  hadronic tensor, auxiliary scalar quark, auxiliary heavy quark, auxiliary light quark, pseudo-distributions, "OPE without OPE", lattice cross sections

# Conclusions and prospects

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001
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Thank you for your attention!