Three-dimensional structure of the nucleon from Lattice QCD

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NATIONAL SCIENCE CENTRE POLAND

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Outline:

Introduction
GPDs from lattice:
– how to access
– reference frames
– results
Prospects/conclusion

Many thanks to my Collaborators for work presented here:

C. Alexandrou, S. Bhattacharya, M. Constantinou, J. Dodson, X. Gao, K. Hadjiyiannakou, K. Jansen, A. Metz, J. Miller, S. Mukherjee, P. Petreczky, A. Scapellato, F. Steffens, Y. Zhao

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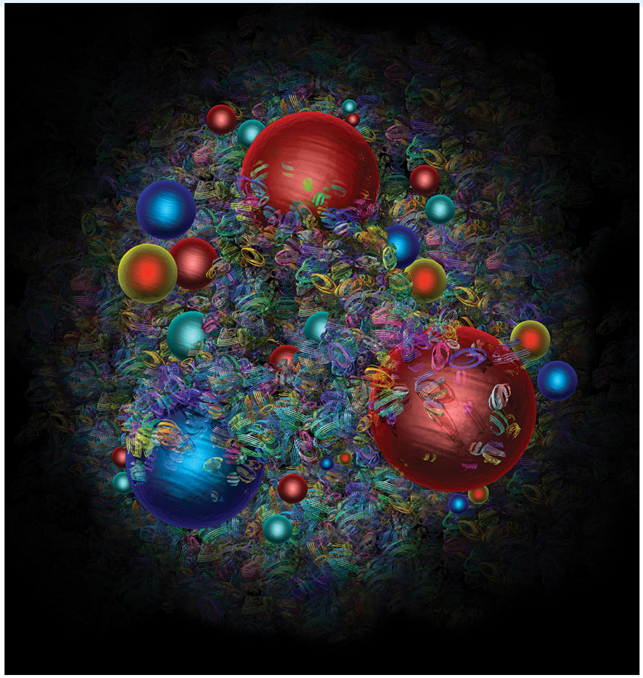

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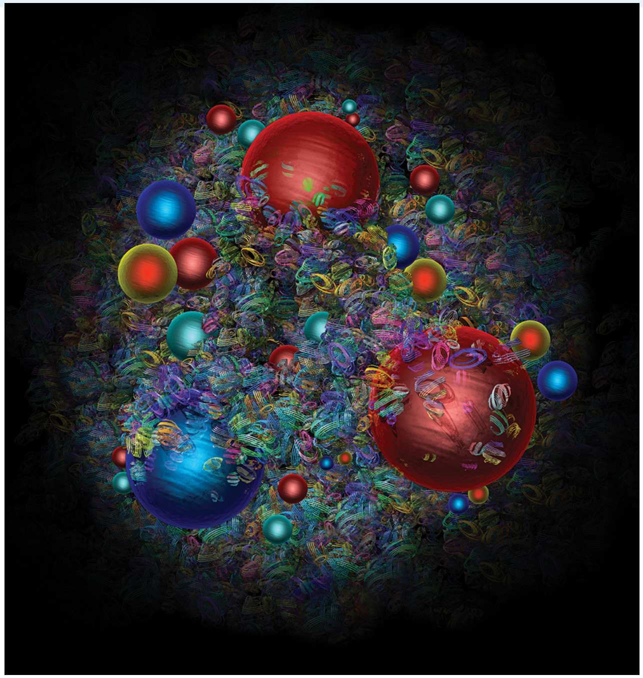

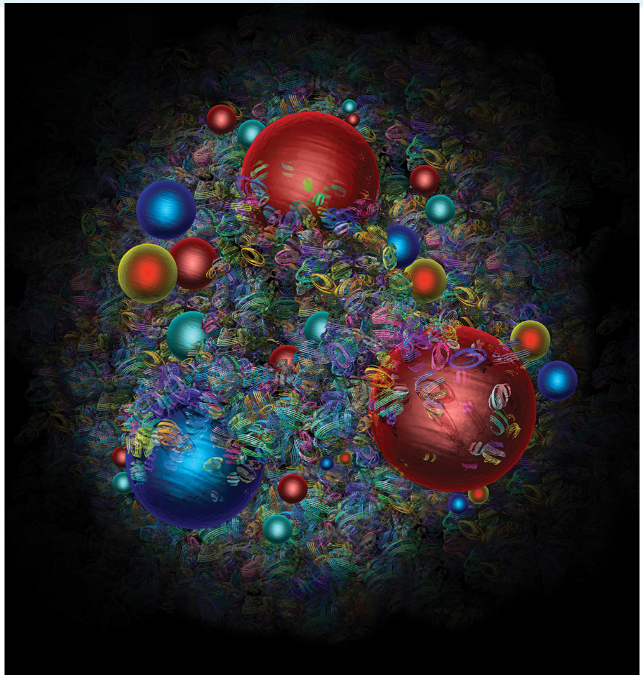

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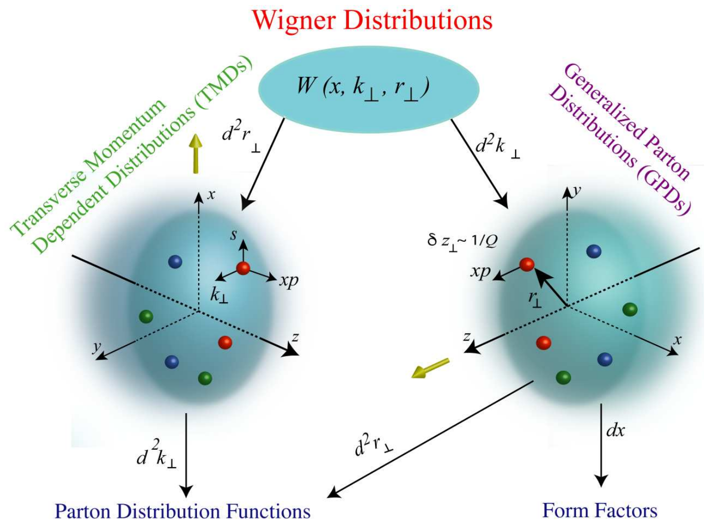

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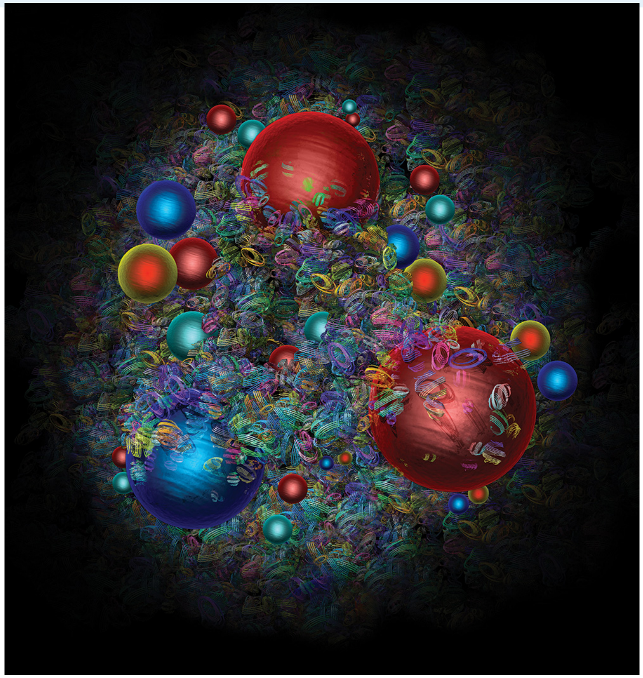

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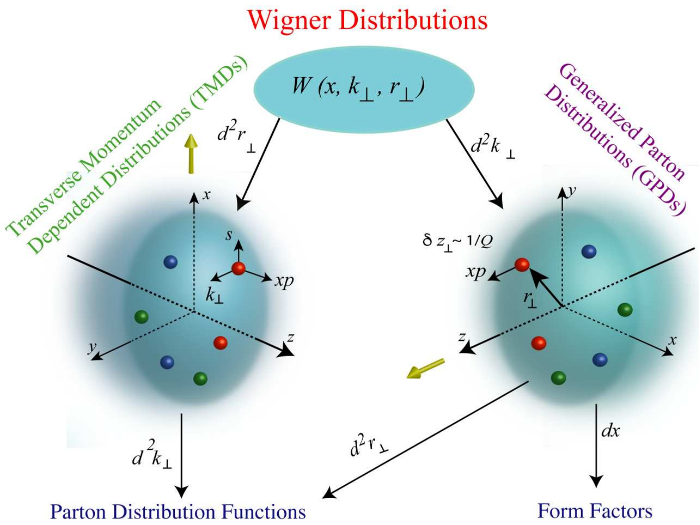

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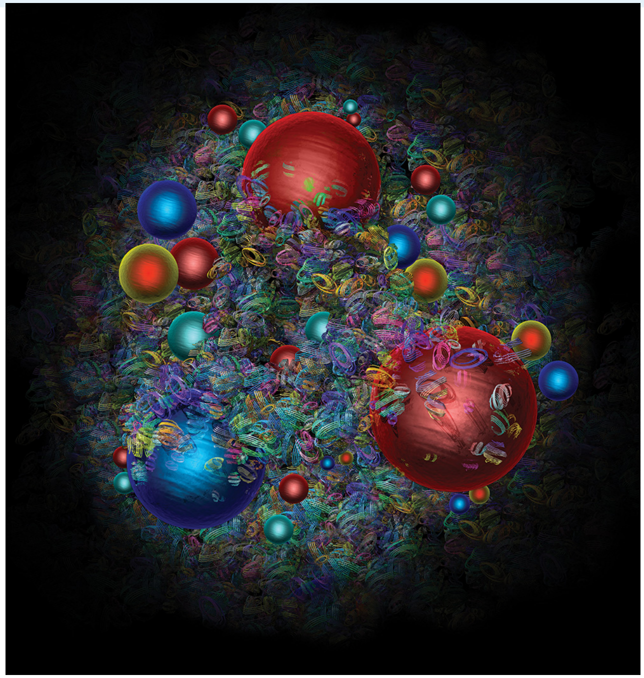

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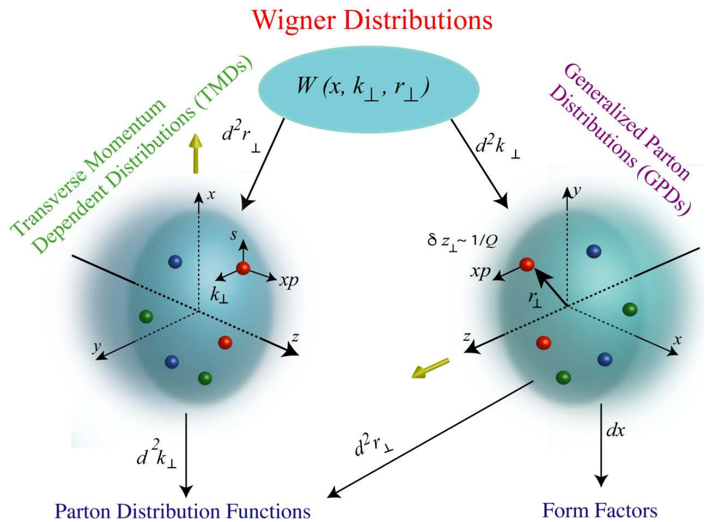

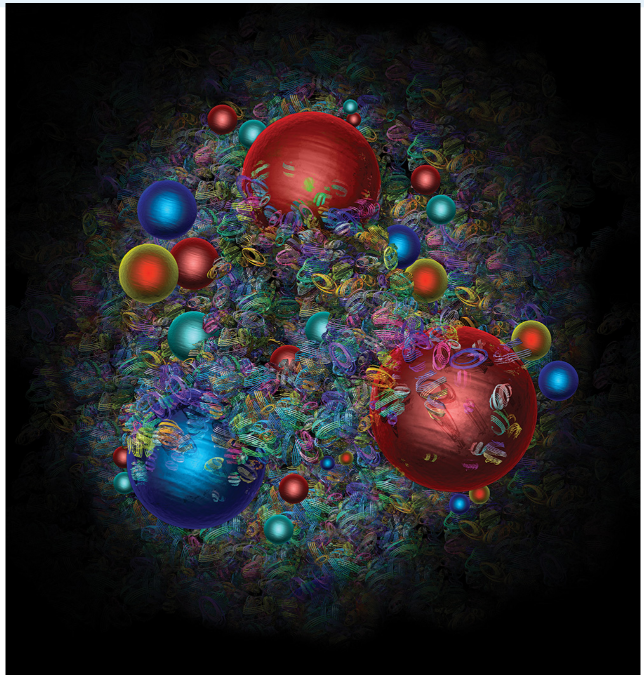

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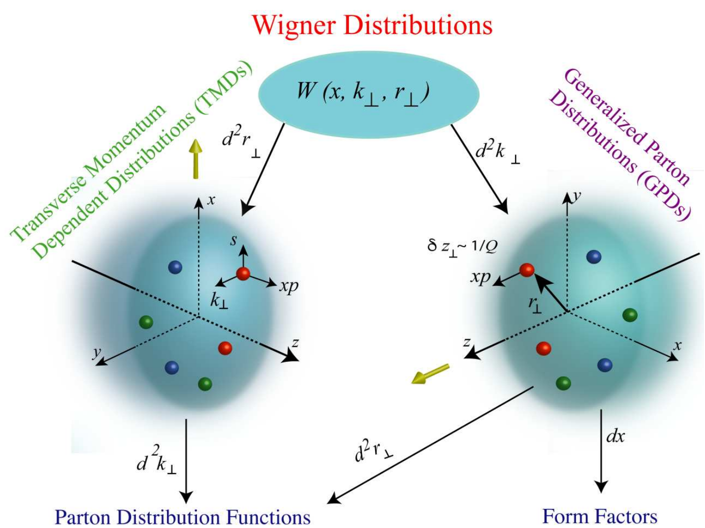

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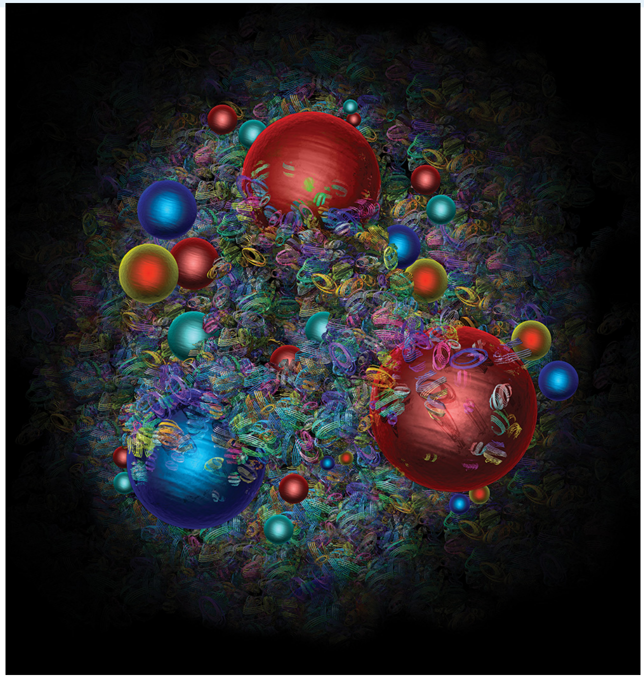

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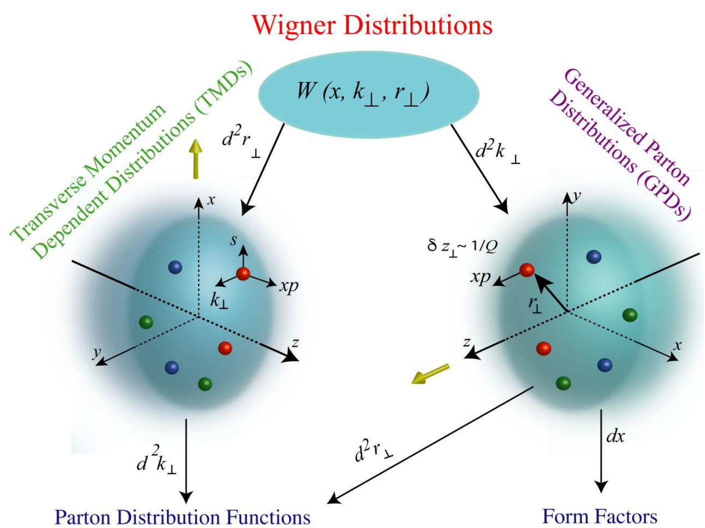
- reduce to PDFs in the forward limit, e.g. $H(x,0,0) = q(x)$,
- their moments are form factors, e.g. $\int dx\, H(x,\xi,t) = F_1(t)$.



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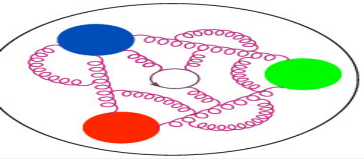

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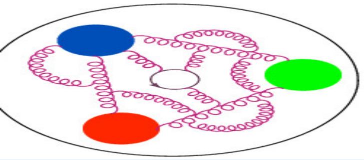

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Recently: new direct approaches to get x-dependence.

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- Examples:
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

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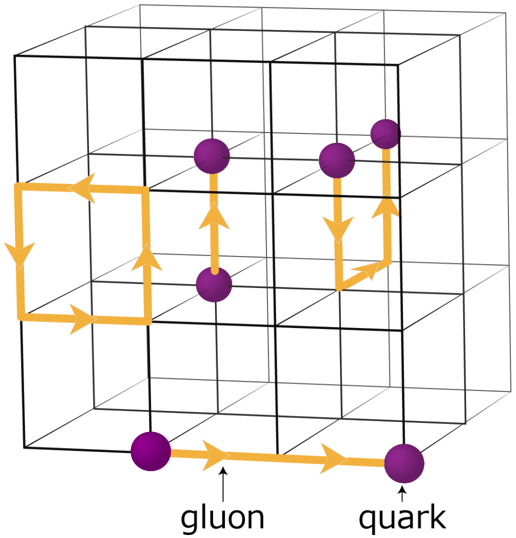

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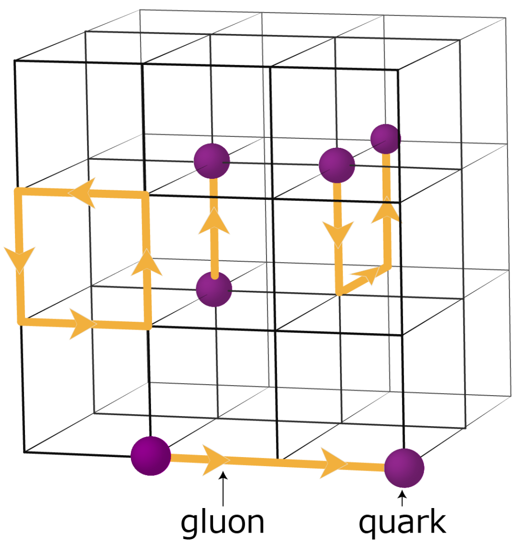

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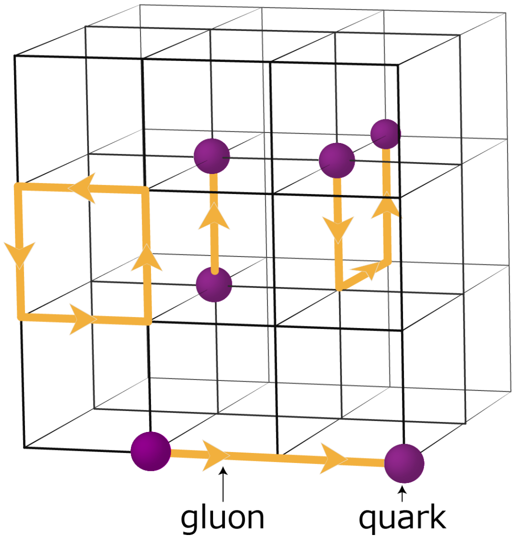

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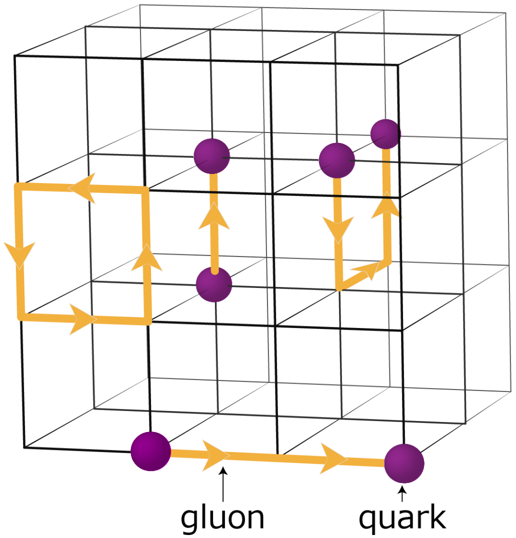

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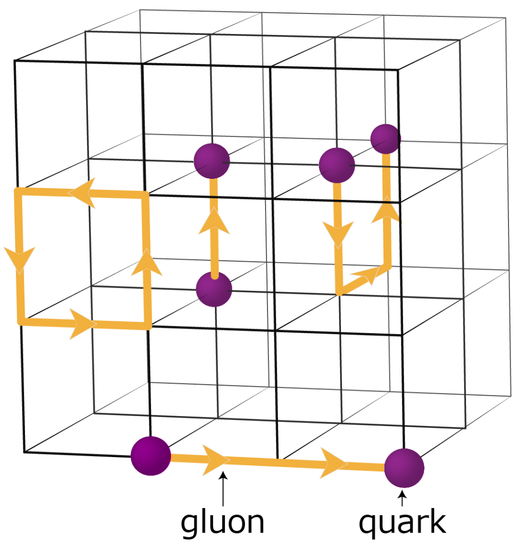

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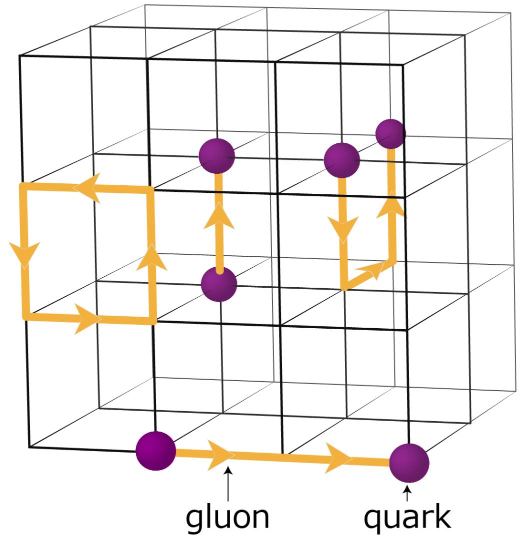

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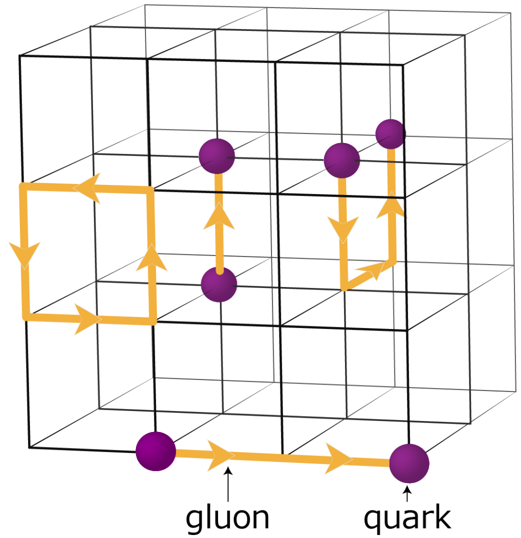

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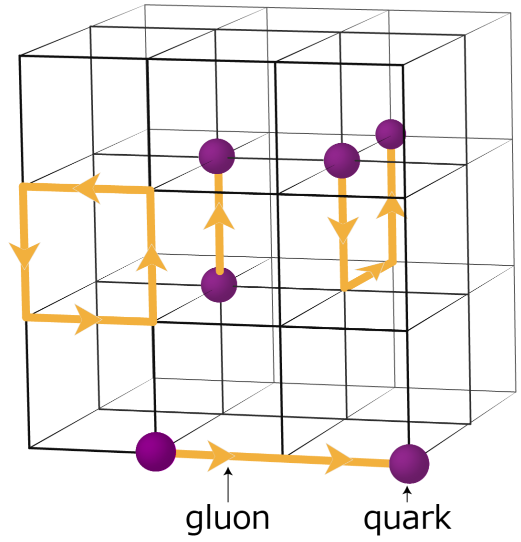

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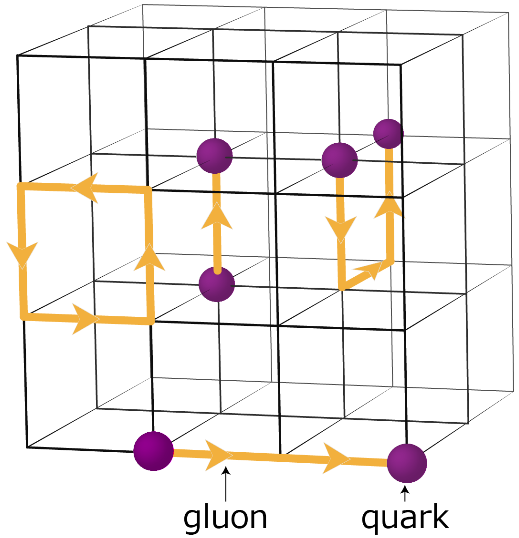
- prior to regulator removal – (non-perturbative) renormalization



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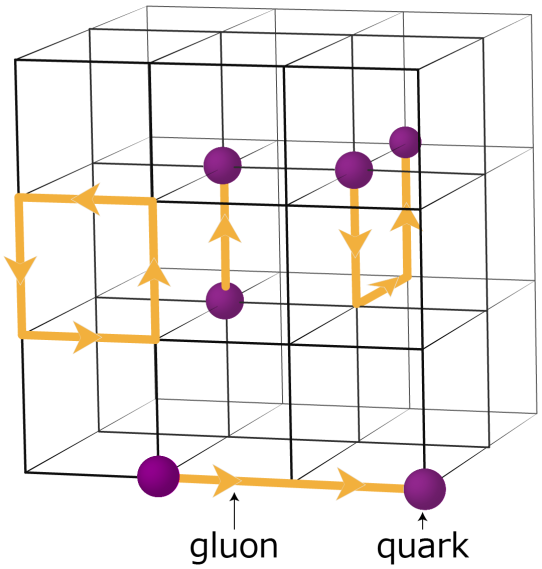

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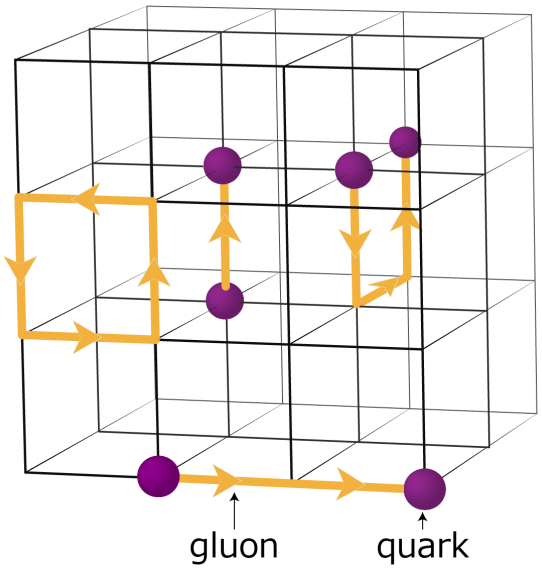
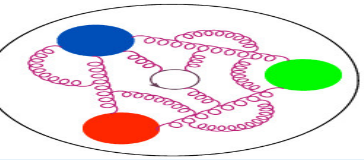
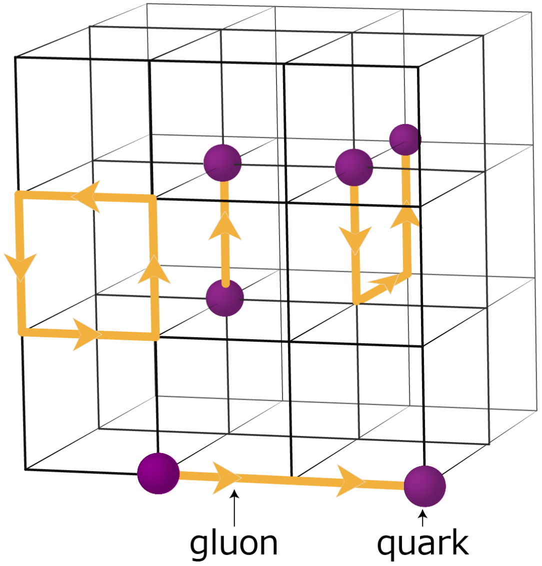
- For many aspects, already precision results with percent/per mille total uncertainty.
- However, many aspects (the difficult ones!) with only exploratory studies.





Lattice QCD – what one should keep in mind

- Lattice QCD offers a way for a careful *ab initio* study of non-perturbative aspects of QCD.
- Its huge strength: possibility to control all systematic effects: *cut-off effects, finite volume effects, quark mass effects, isospin breaking, excited states,* ...
- For many aspects, already precision results with percent/per mille total uncertainty.
- However, many aspects (the difficult ones!) with only exploratory studies.
- Difficult problems need time to:
  - ⋆ find the proper way to address
  - ⋆ prove computational feasibility
  - ⋆ optimize the computational method
  - ⋆ acquire all data (long computations...)
  - ⋆ analyze all systematics



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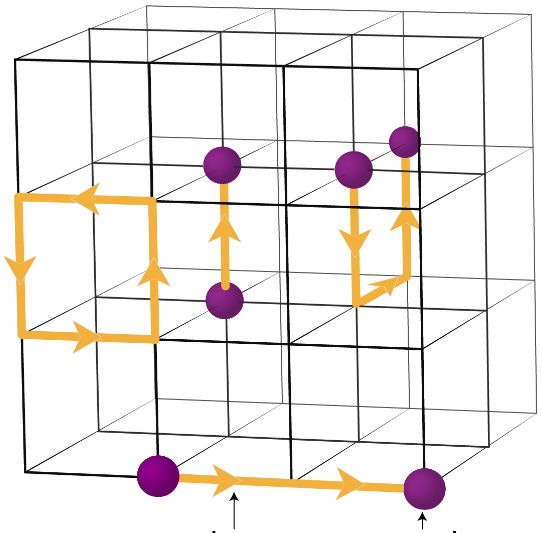

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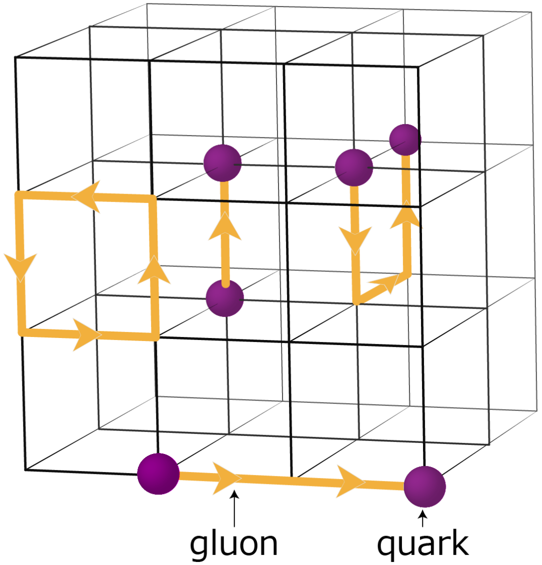

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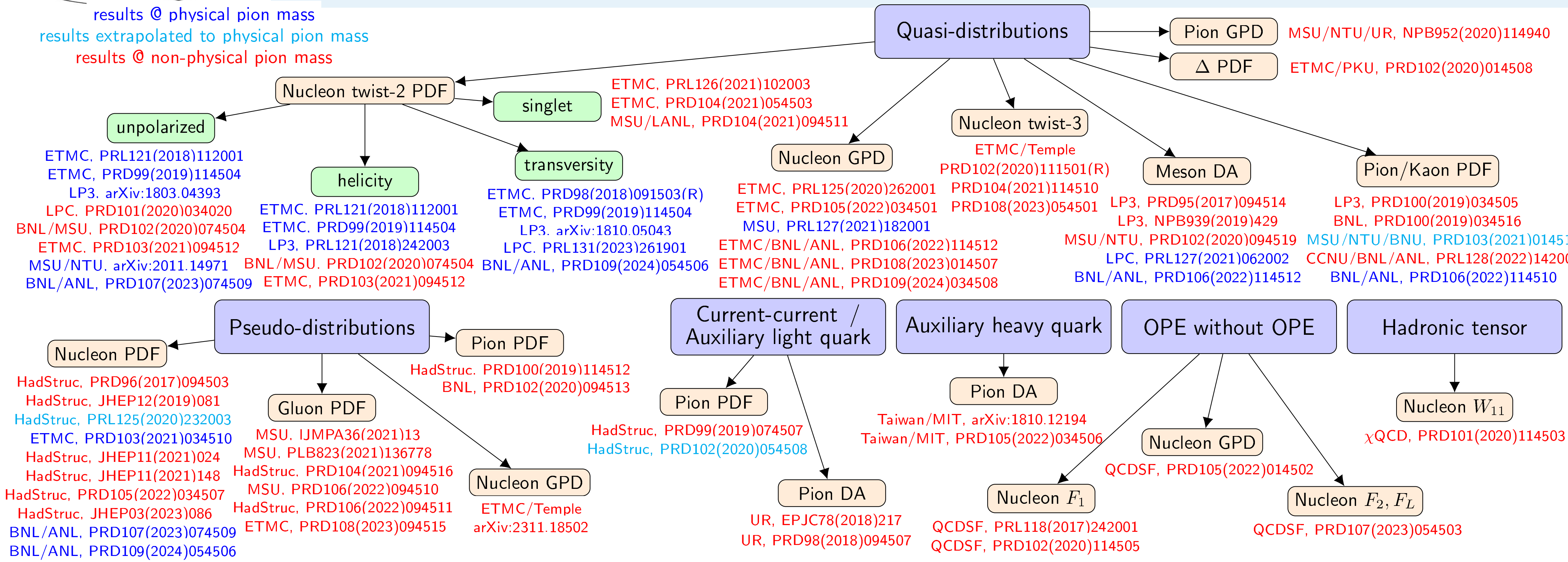
- Thus, do not expect miracles.
- Overall, **expect complementary role of lattice**.
- Robust quantitative statements: *low moments, form factors.*
- x-dependence: breakthrough in recent years, but a long way to go to solid quantitative statements.

Lattice PDFs/GPDs: dynamical progress



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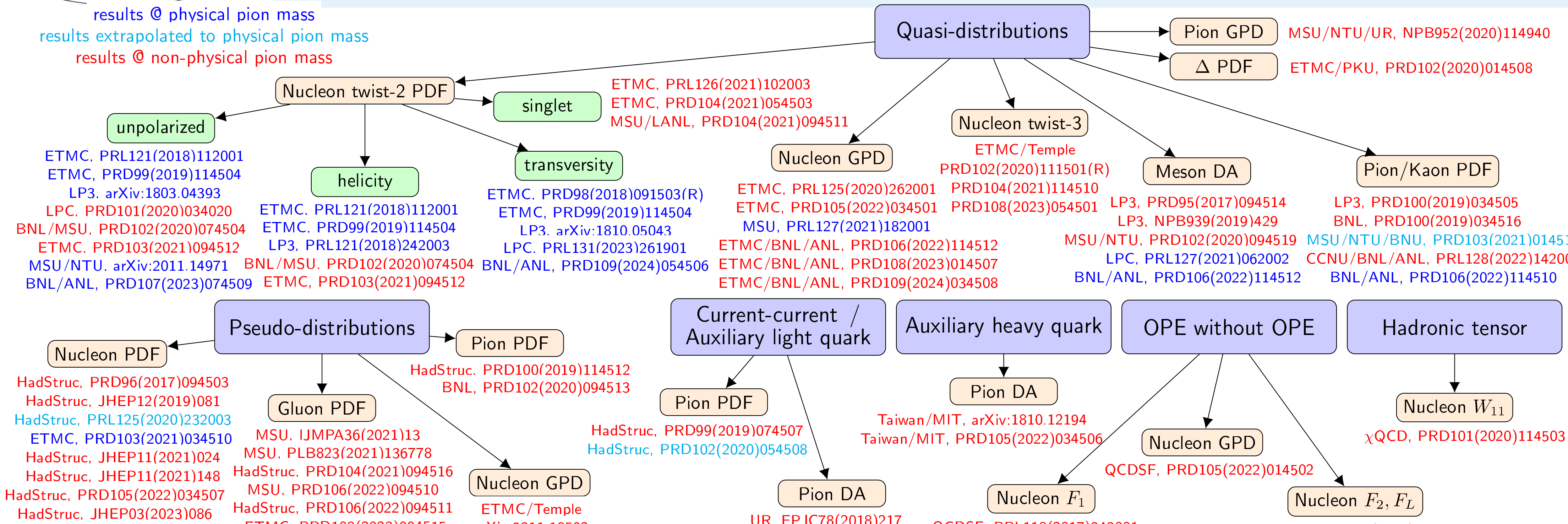

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K. Cichy, *Overview of lattice calculations of the x-dependence of PDFs, GPDs and TMDs*, plenary talk of Virtual Tribute to Quark Confinement 2021, 2111.04552

K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, invited review for a special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248

M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD* (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445

X. Ji et al., *Large-Momentum Effective Theory*, Rev. Mod. Phys. 93 (2021) 035005

M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, PPNP 121 (2021) 103908

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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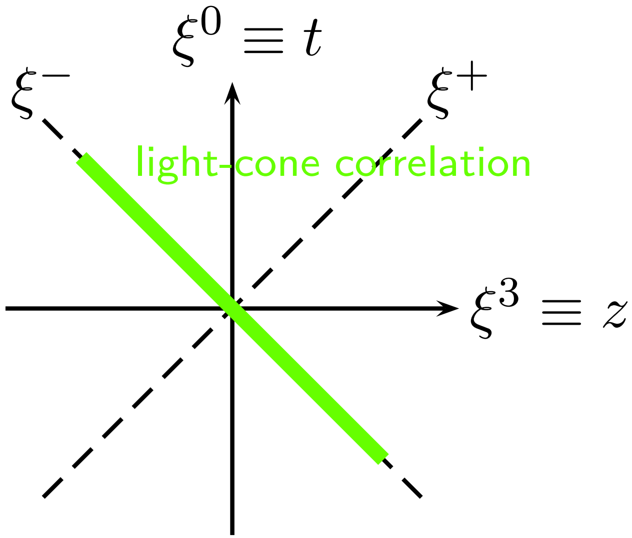

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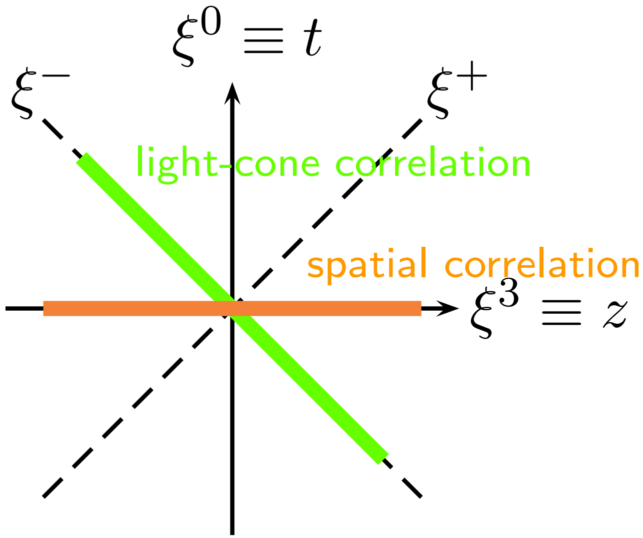

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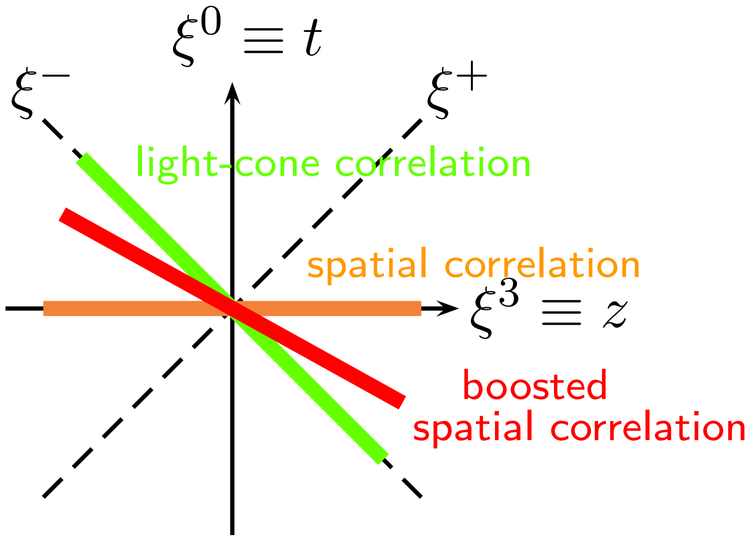

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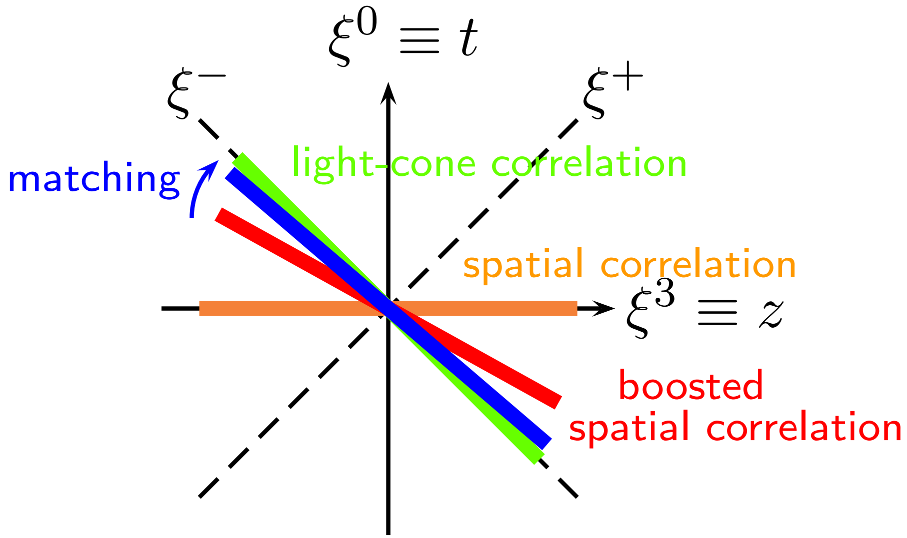

Quasi-distributions

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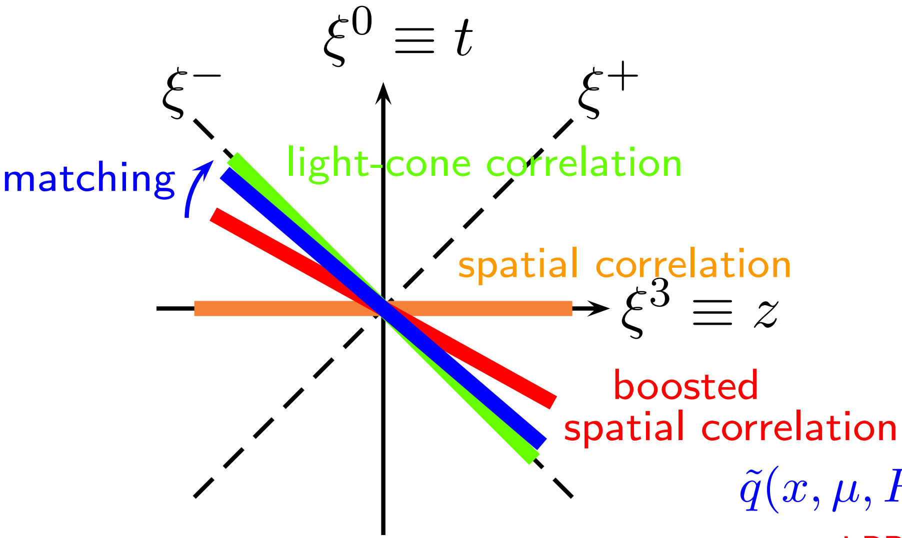

Euclidean matrix element:

$\langle P_f|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P_i\rangle$

Its Fourier transform (quasi-distribution)
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$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

quasi-PDF; pert.kernel; PDF; higher-twist effects

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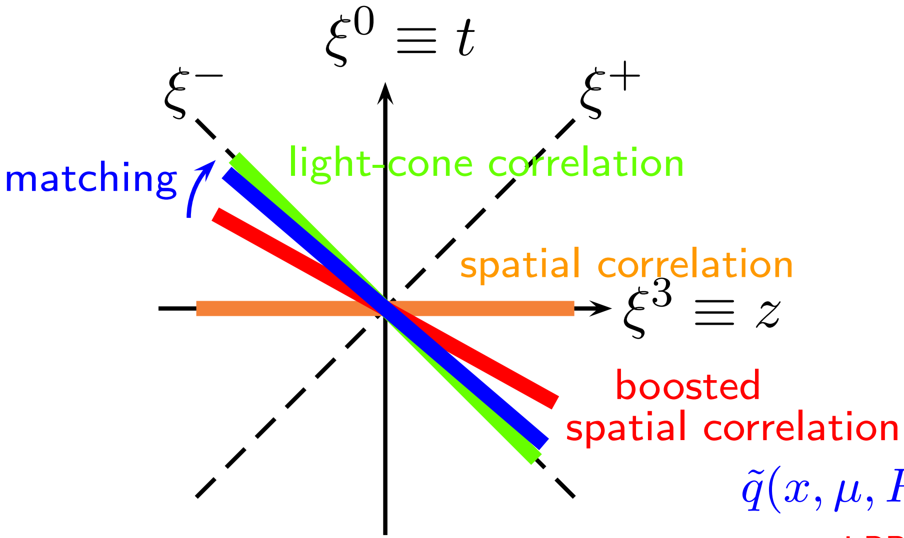

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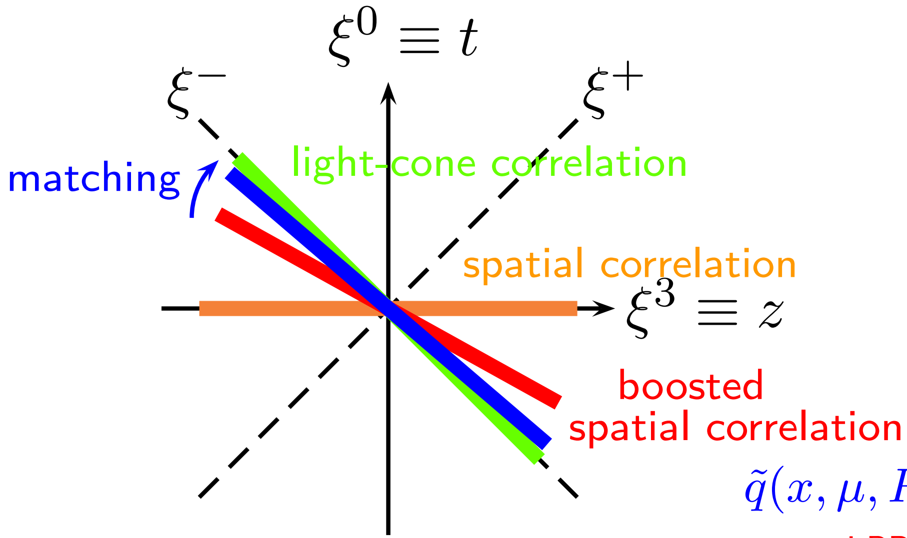

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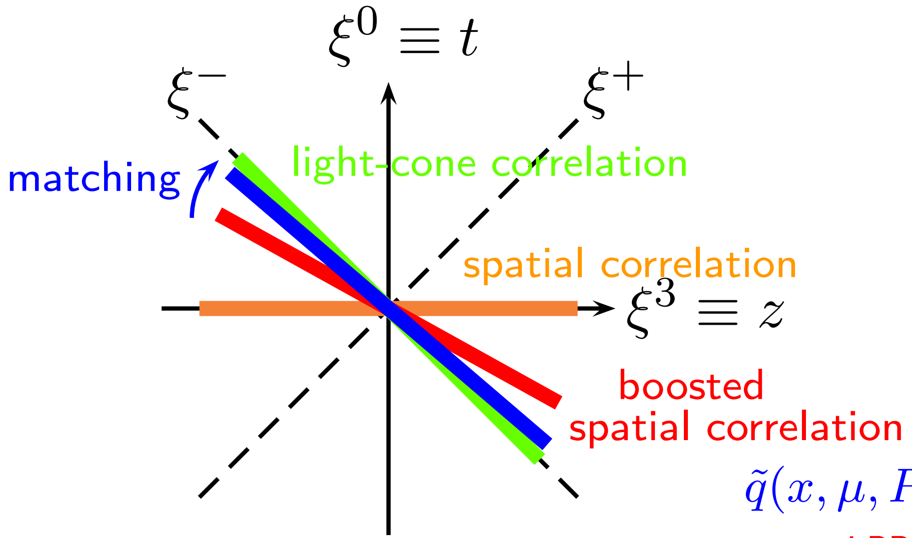

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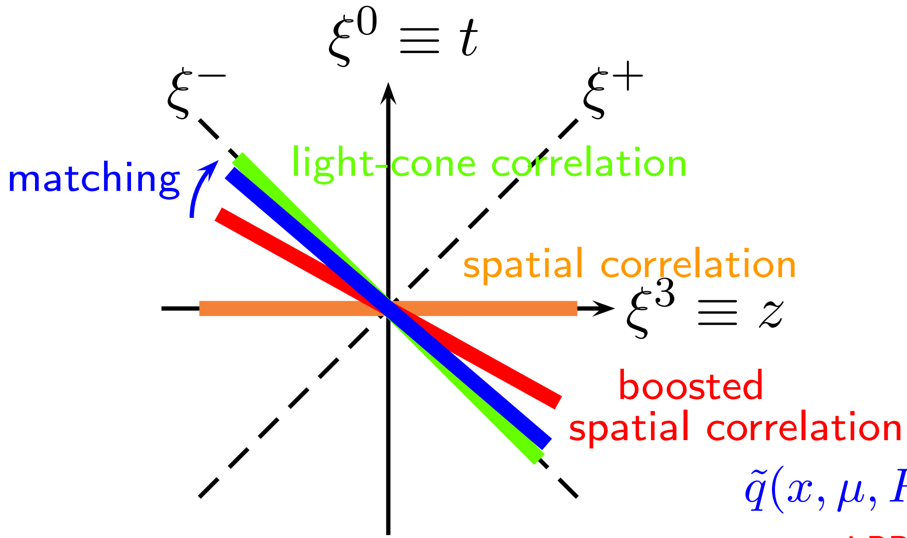

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Need different projectors
to disentangle 2/4 GPDs

UNPOL: $\mathcal{P} = \frac{1+\gamma_0}{4}$

POL-k: $\mathcal{P} = \frac{1+\gamma_0}{4} i\gamma_5\gamma_k$

Quasi-GPDs lattice procedure

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
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lattice computation of bare ME

↓

extraction of amplitudes and/or GPDs
frame-dependent formulas

↓

renormalization of bare GPDs
intermediate RI scheme

↓

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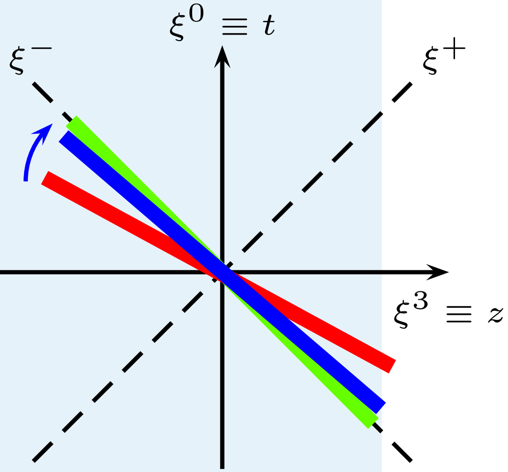

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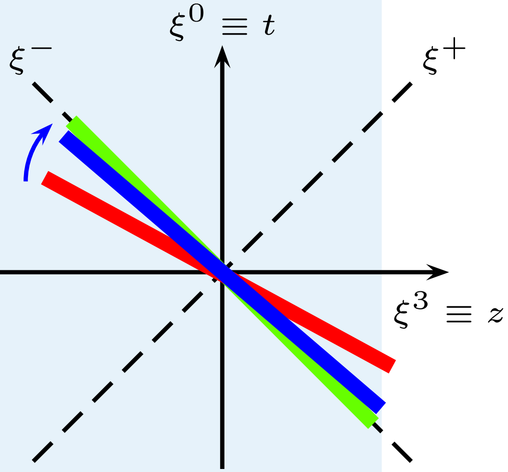
the final desired object!



Setup

Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L \approx 3$ fm,
- $m_\pi \approx 260$ MeV.

Kinematics:

- three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV,
- momentum transfers: $-t \leq 2.76$ GeV^2, most data: $-t = 0.64,\ 0.69$ GeV^2,
- skewness: $\xi = 0, 1/3$.

$\mathcal{O}(20000)$ measurements (≈ 250 confs, 8 source positions, 8 permutations of $\vec{\Delta}$).

Twist-2 unpolarized+helicity GPDs C. Alexandrou et al. (ETMC), PRL 125(2020)262001

Twist-2 transversity GPDs C. Alexandrou et al. (ETMC), PRD 105(2022)034501

Twist-2 unpolarized GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 106(2022)114512

Twist-2 unpolarized GPDs (OPE) S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507

Twist-3 axial GPDs S. Bhattacharya et al. (ETMC/Temple), PRD 108(2023)054501

Twist-2 helicity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 109(2024)034508

Twist-2 transversity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) in preparation

Twist-2 unpolarized GPDs (pseudo-GPDs) S. Bhattacharya et al. (ETMC/Temple) in preparation

First extractions of x-dependent GPDs

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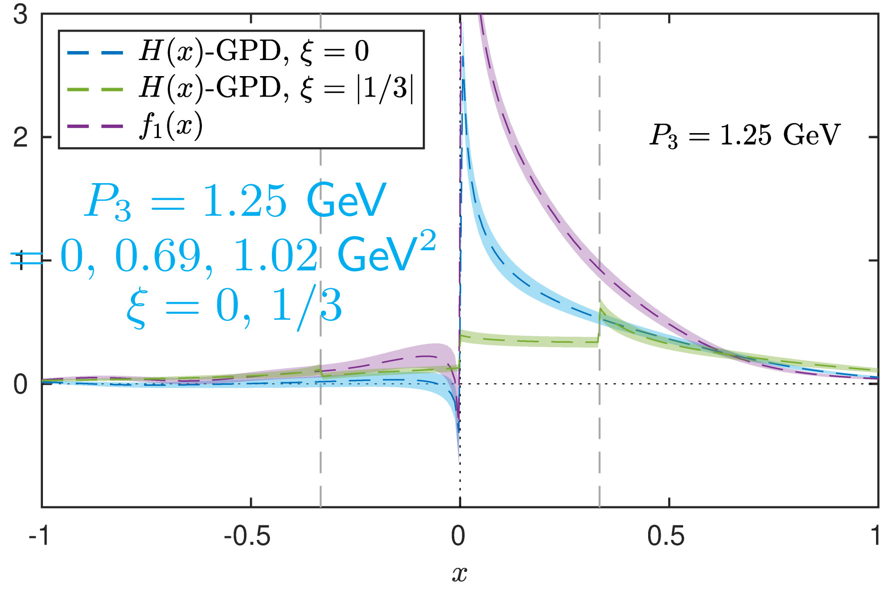

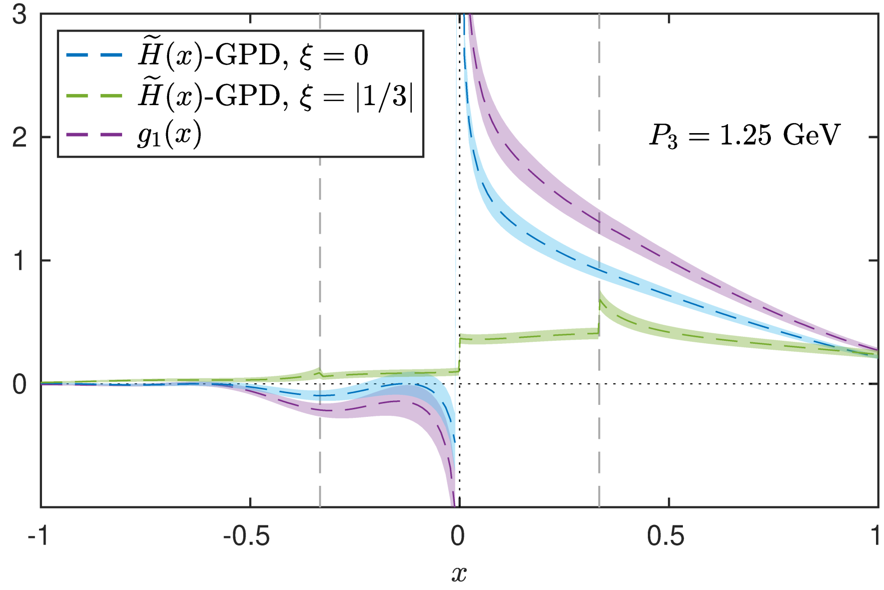

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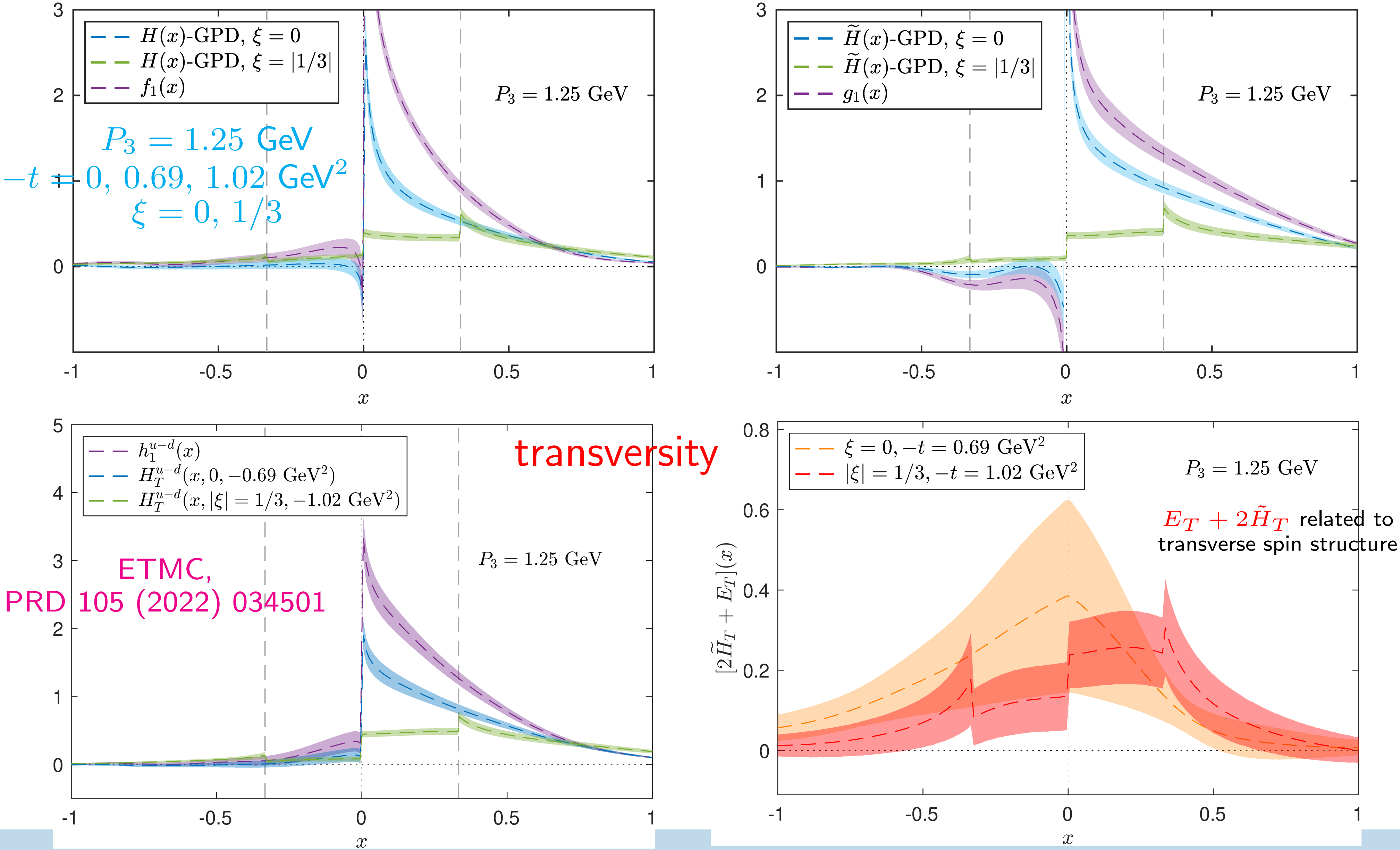

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source momentum: $P_i = (E, \vec{P} - \vec{\Delta}/2)$,
sink momentum: $P_f = (E, \vec{P} + \vec{\Delta}/2)$.

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construction of the 3-point correlation functions required for the MEs needs the calculation of the all-to-all propagator

preferred way: "sequential propagator" – implies separate inversions (most costly part!) for each P_f.

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Asymmetric frame:
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sink momentum: $P_f = (E_f, \vec{P})$.

GPDs in different frames of reference

Standard symmetric (Breit) frame:

source momentum: $P_i = (E, \vec{P} - \vec{\Delta}/2)$,
sink momentum: $P_f = (E, \vec{P} + \vec{\Delta}/2)$.

Lattice perspective:
construction of the 3-point correlation functions required for the MEs needs the calculation of the all-to-all propagator
preferred way: "sequential propagator" – implies separate inversions (most costly part!) for each P_f.
Hence, **separate calculation for each momentum transfer $\vec{\Delta}$!**

Asymmetric frame:

source momentum: $P_i = (E_i, \vec{P} - \vec{\Delta})$,
sink momentum: $P_f = (E_f, \vec{P})$.
Lattice perspective:

Several momentum transfer vectors $\vec{\Delta}$ can be obtained within a single calculation!

Lorentz-covariant parametrization

Main theoretical tool: S. Bhattacharya et al., PRD106(2022)114512

Lorentz-covariant parametrization of matrix elements (e.g. vector case):

$$F^{\mu}(z,P,\Delta)=\bar{u}(p',\lambda')\left[\frac{P^{\mu}}{m}A_1+mz^{\mu}A_2+\frac{\Delta^{\mu}}{m}A_3+im\sigma^{\mu z}A_4+\frac{i\sigma^{\mu\Delta}}{m}A_5+\frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6+\frac{z^{\mu}i\sigma^{z\Delta}}{m}A_7+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p,\lambda),$$

(inspired by: S. Meissner, A. Metz, M. Schlegel, JHEP08(2009)056).

- most general parametrization in terms of 8 linearly-independent Lorentz structures,
- 8 Lorentz-invariant amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

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- 8 Lorentz-invariant amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

Example: (γ_0 insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0)=C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1+\frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5+\frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

asymmetric frame:

$$\Pi_0^a(\Gamma_0) = C\left(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1-\frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3+\frac{(E_i-E_f)P_3z}{4m}A_4\right.$$
$$\left.+\frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5+\frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6+\frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\right).$$

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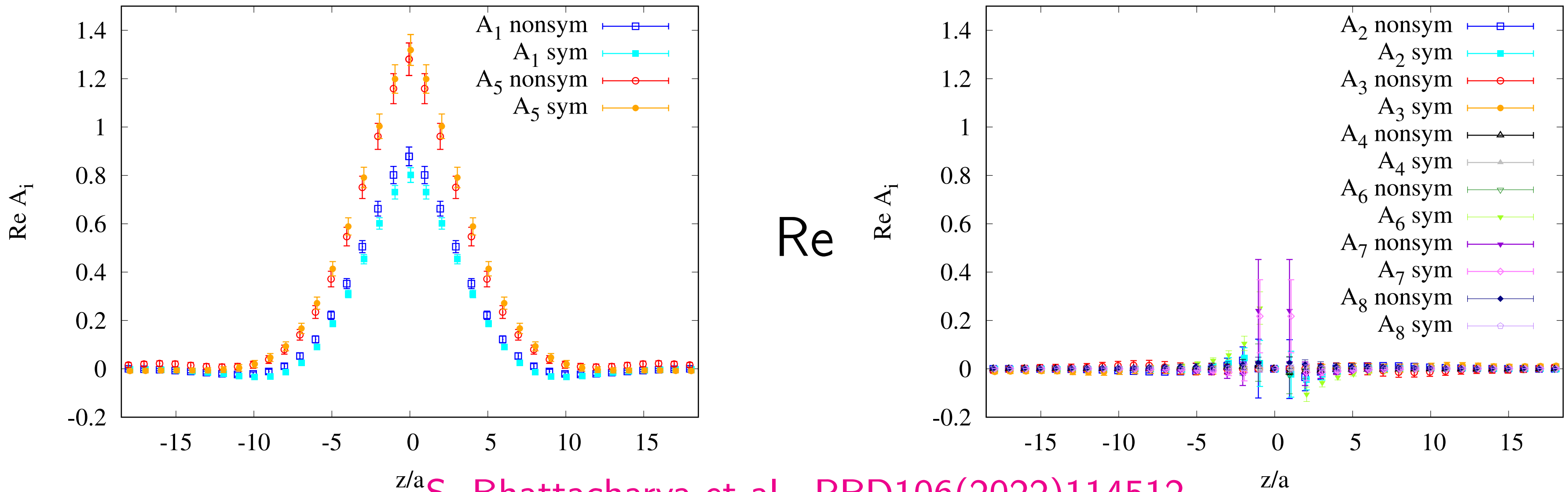
$$+\frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5+\frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6+\frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\Bigg).$$

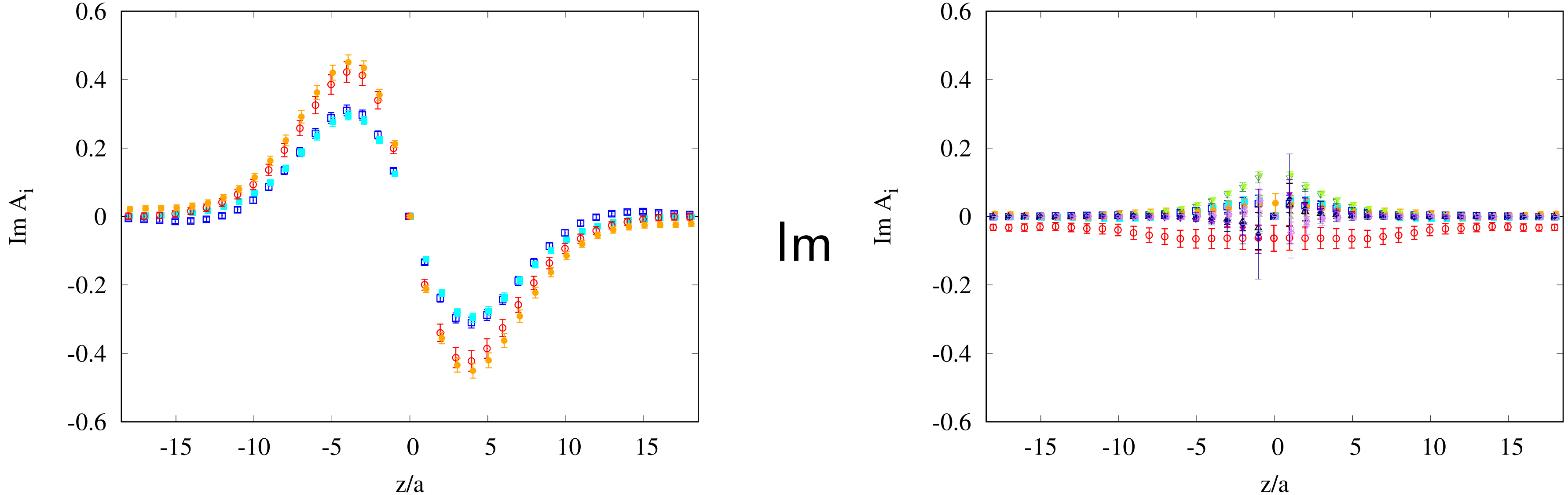
- matrix elements $\Pi_\mu(\Gamma_\nu)$ are **frame-dependent**,
- but the amplitudes A_i are **frame-invariant**.

Proof of concept (comparison between frames)

A_1, A_5 (leading ones) A_2, A_3, A_4, A_6, A_7, A_8 (suppressed ones)



S. Bhattacharya et al., PRD106(2022)114512



Defining H and E GPDs in the standard way, expressions are frame-dependent:

SYMMETRIC frame:

$$F_{H^{(0)}} = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3} A_6 \,,$$

$$F_{E^{(0)}} = -A_1 + 2A_5 + \frac{z\left(4E^2 - \Delta_1^2 - \Delta_2^2\right)}{2P_3} A_6 \,.$$

H and E GPDs – possible definitions

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ASYMMETRIC frame:

$$F_{H^{(0)}} = A_1 + \frac{\Delta_0}{P_0} A_3 + \frac{m^2 z \Delta_0}{2P_0 P_3} A_4 + \frac{z(\Delta_0^2 + \Delta_\perp^2)}{2P_3} A_6 + \frac{z(\Delta_0^3 + \Delta_0 \Delta_\perp^2)}{2P_0 P_3} A_8 \,,$$

$$F_{E^{(0)}} = -A_1 - \frac{\Delta_0}{P_0} A_3 - \frac{m^2 z(\Delta_0 + 2P_0)}{2P_0 P_3} A_4 + 2A_5 - \frac{z\left(\Delta_0^2 + 2P_0\Delta_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_3} A_6 - \frac{z\Delta_0\left(\Delta_0^2 + 2\Delta_0 P_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_0 P_3} A_8 \,.$$

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One can also modify the definition to make it Lorentz-invariant and arrive at:

ANY frame:

$$F_H = A_1 \,,$$

$$F_E = -A_1 + 2A_5 + 2zP_3 A_6 \,.$$

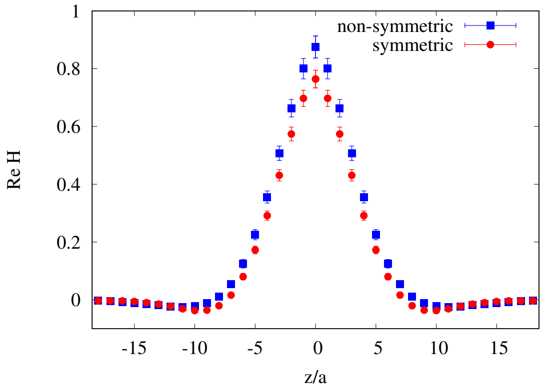
With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

In terms of matrix elements: standard definition – only $\Pi_0(\Gamma_0)$, $\Pi_0(\Gamma_{1/2})$,

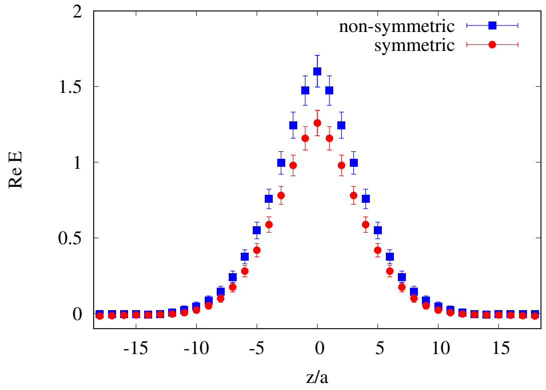
LI definition – additionally: $\Pi_{1/2}(\Gamma_3)$ (both frames), $\Pi_{1/2}(\Gamma_3)$, $\Pi_{1/2}(\Gamma_0)$, $\Pi_1(\Gamma_2)$, $\Pi_2(\Gamma_1)$ (asym.).

STANDARD DEFINITION

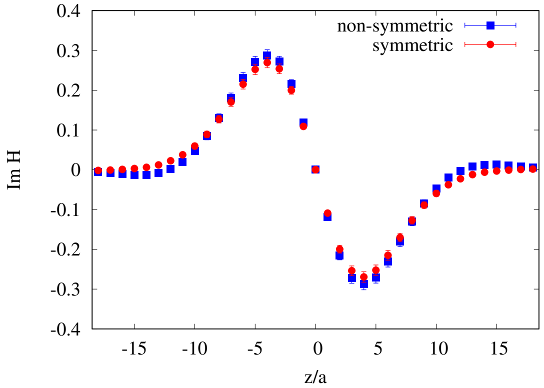
H-GPD

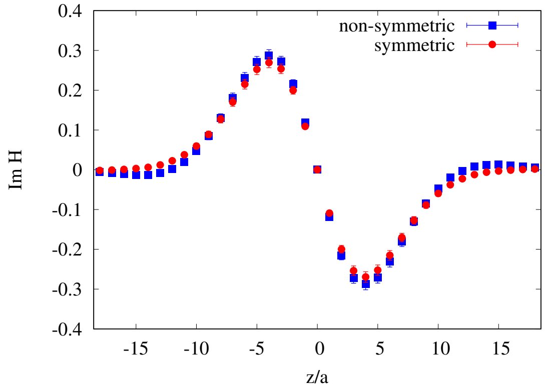


E-GPD



S. Bhattacharya et al., PRD106(2022)114512

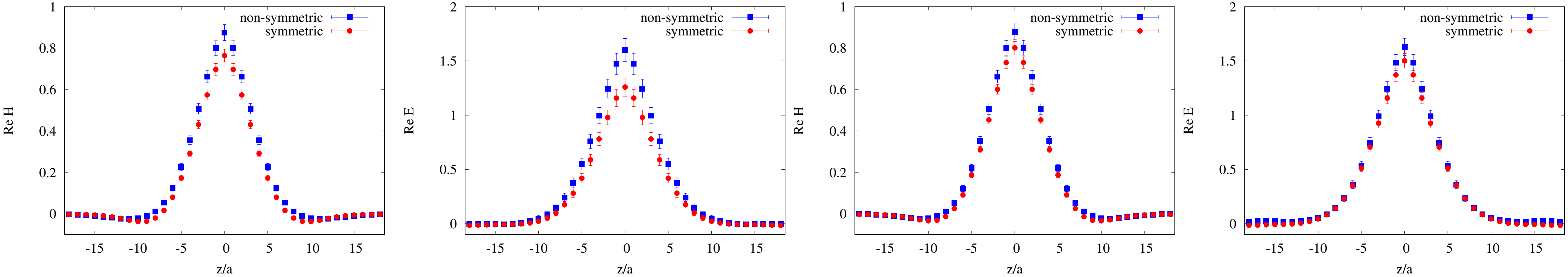




H and E GPDs – comparison of definitions

STANDARD DEFINITION | LORENTZ-INVARIANT DEFINITION

H-GPD | E-GPD | H-GPD | E-GPD



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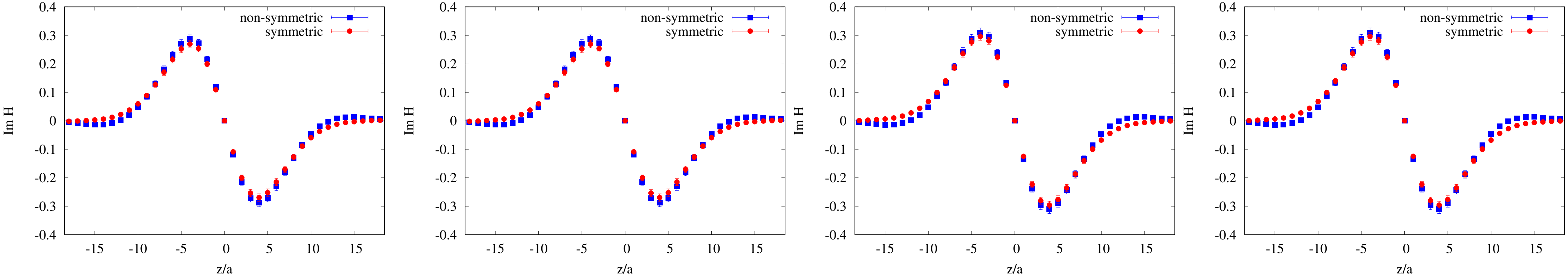

t-dependence of H/E GPDs

All kinematic cases (asymmetric frame):

- $\Delta = (1,0,0) \Rightarrow -t = 0.17$ GeV^2,
- $\Delta = (1,1,0) \Rightarrow -t = 0.33$ GeV^2,
- $\Delta = (2,0,0) \Rightarrow -t = 0.64$ GeV^2,
- $\Delta = (2,1,0) \Rightarrow -t = 0.79$ GeV^2,
- $\Delta = (2,2,0) \Rightarrow -t = 1.22$ GeV^2,
- $\Delta = (3,0,0) \Rightarrow -t = 1.36$ GeV^2,
- $\Delta = (3,1,0) \Rightarrow -t = 1.49$ GeV^2,
- $\Delta = (4,0,0) \Rightarrow -t = 2.24$ GeV^2,

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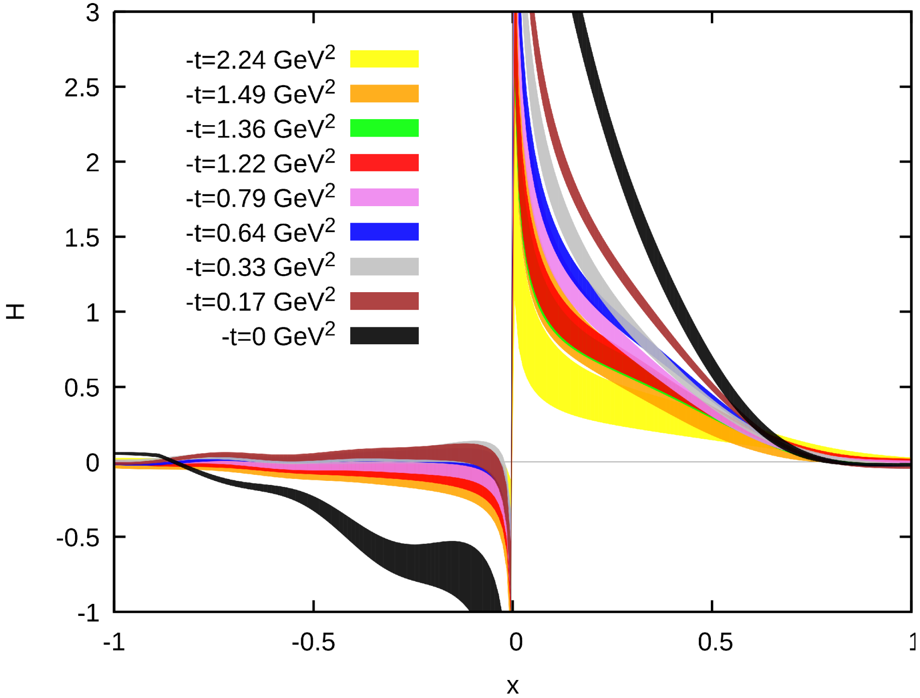

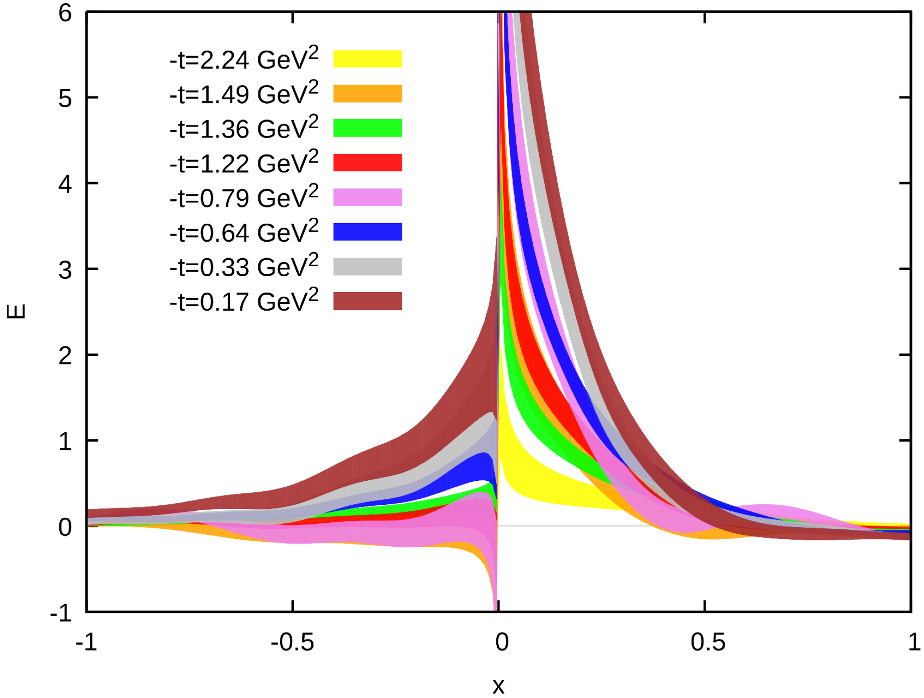

Helicity GPDs

Lorentz-covariant parametrization of matrix elements (axial vector case):

$$F^{[\gamma^\mu \gamma_5]} = \bar{u}(p', \lambda')\left[\frac{i\epsilon^{\mu P z \Delta}}{m} A_1 + \gamma^\mu \gamma_5 A_2 + \gamma_5\left(\frac{P^\mu}{m} A_3 + m z^\mu A_4 + \frac{\Delta^\mu}{m} A_5\right) + m \not{z} \gamma_5 \left(\frac{P^\mu}{m} A_6 + m z^\mu A_7 + \frac{\Delta^\mu}{m} A_8\right)\right] u(p, \lambda)$$

S. Bhattacharya et al., PRD109(2024)034508

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S. Bhattacharya et al., PRD109(2024)034508

Two definitions of $\widetilde{H}$:

standard ($\gamma_5\gamma_3$ operator): $F_{\tilde{H}} = A_2 + zP_3A_6 - m^2z^2A_7$,

another ($\gamma_5\gamma_i$ operators, $i = 0, 1, 2$): $F_{\tilde{H}} = A_2 + zP_3A_6$.

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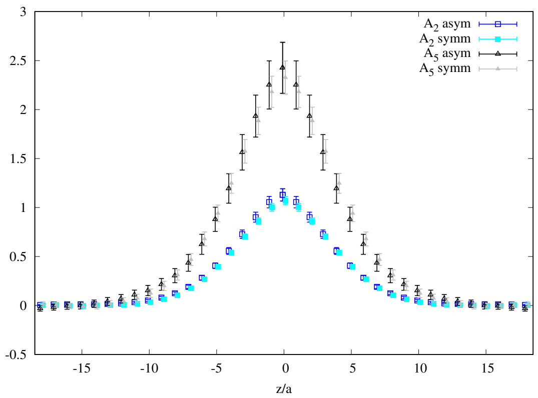
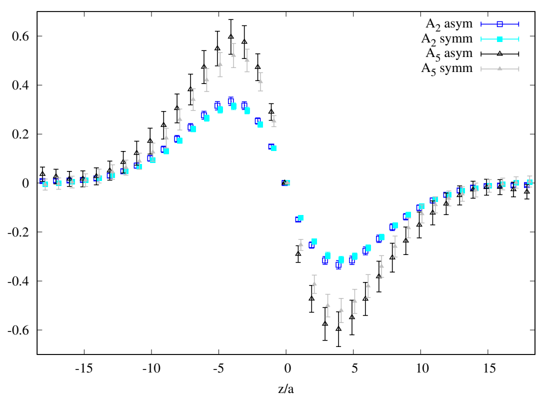
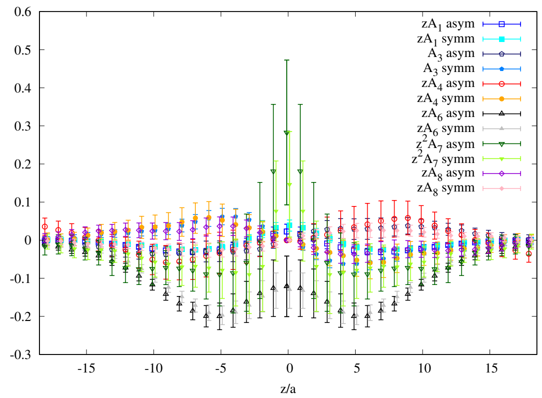
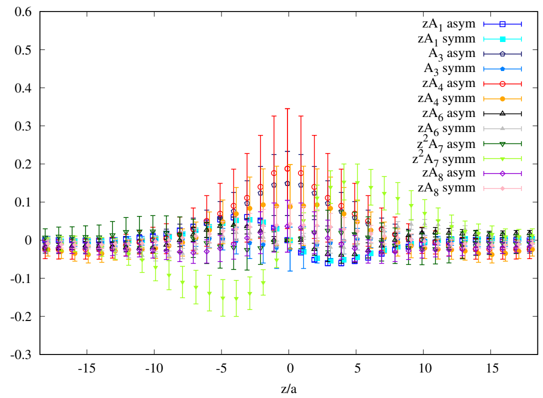
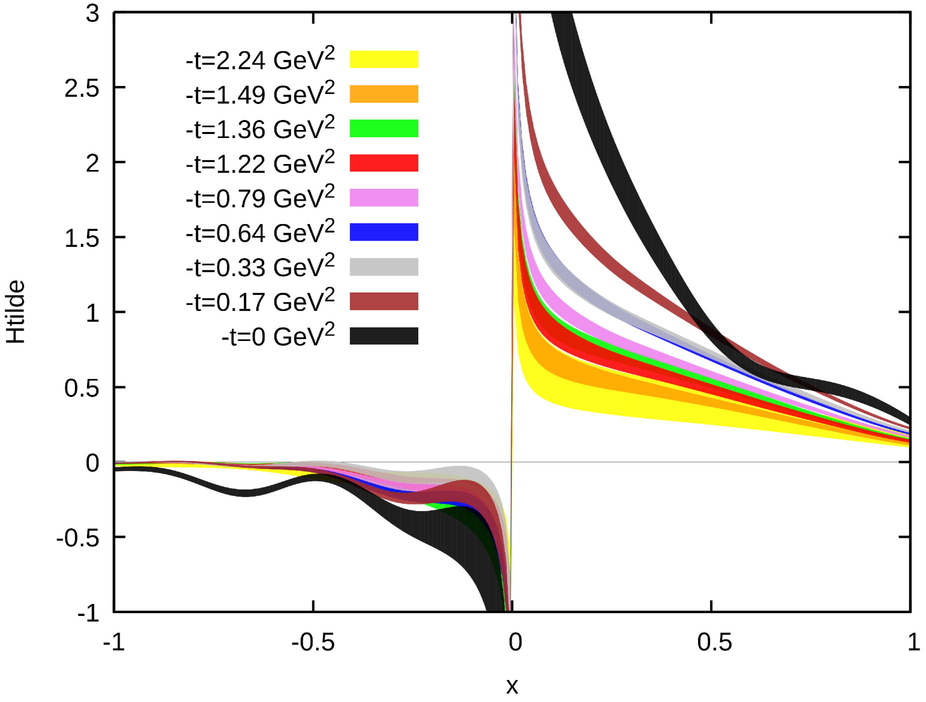








t-dependence of $\tilde{H}/H/E$ GPDs



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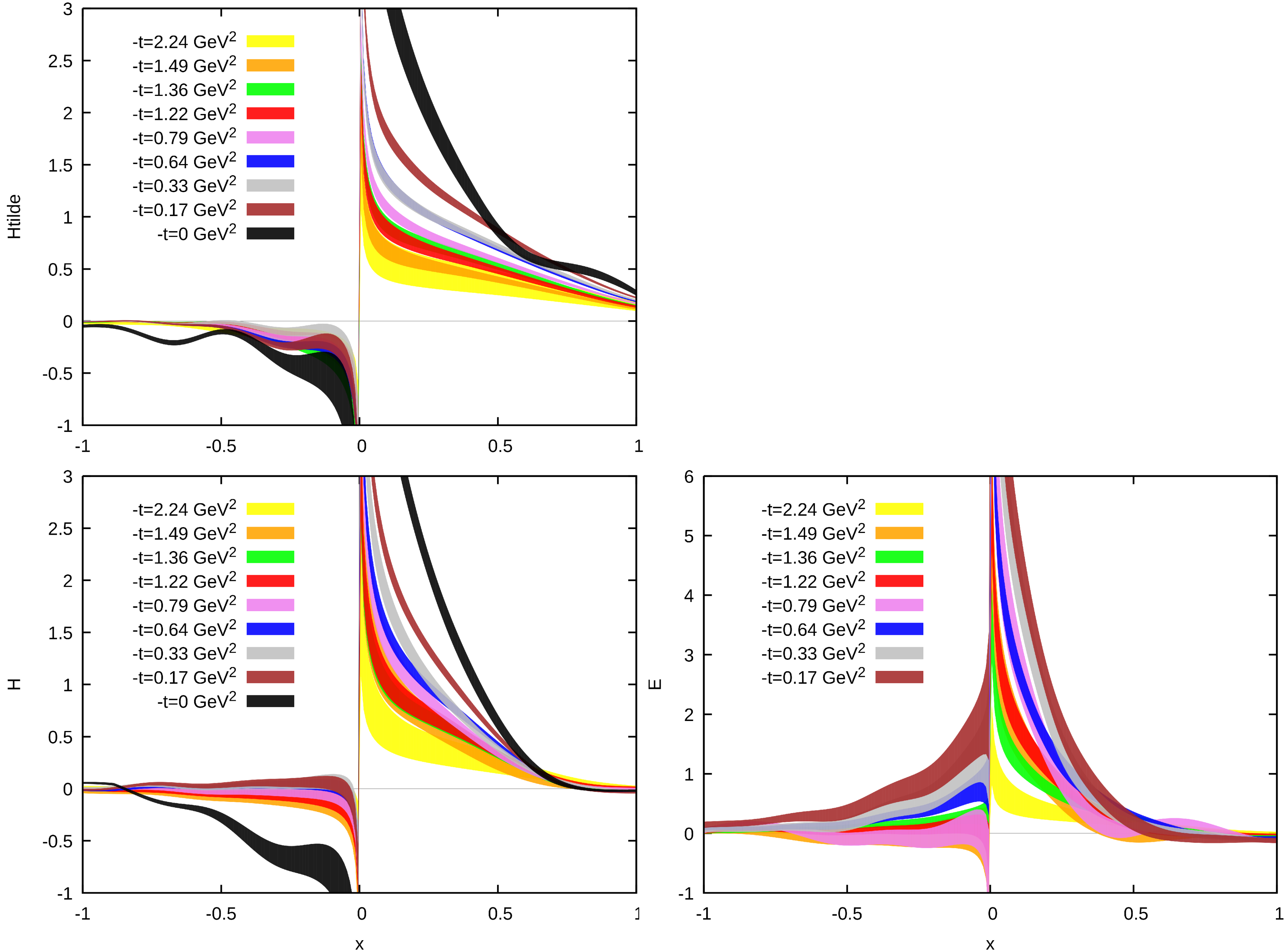

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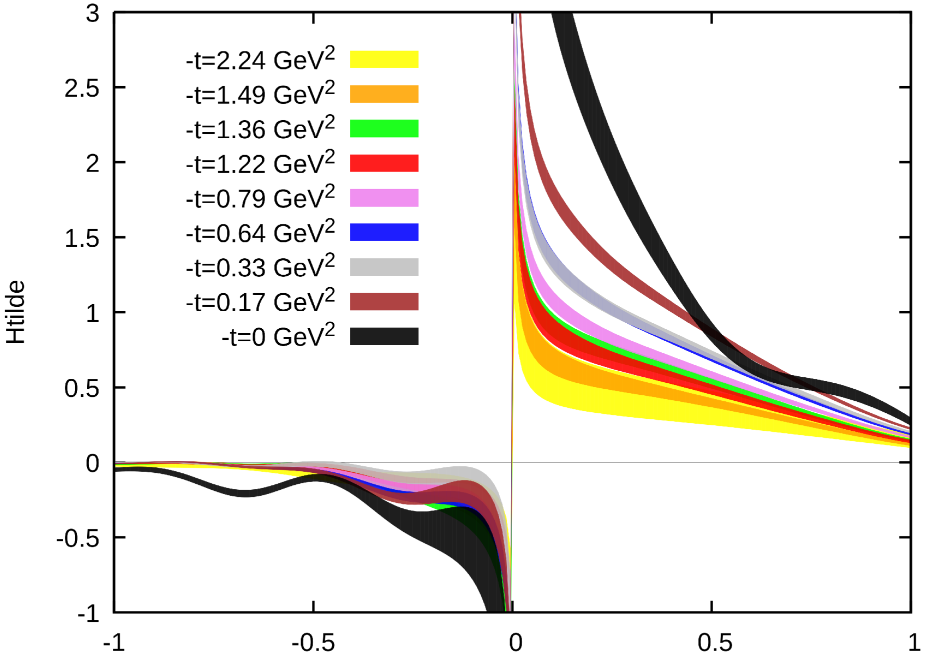
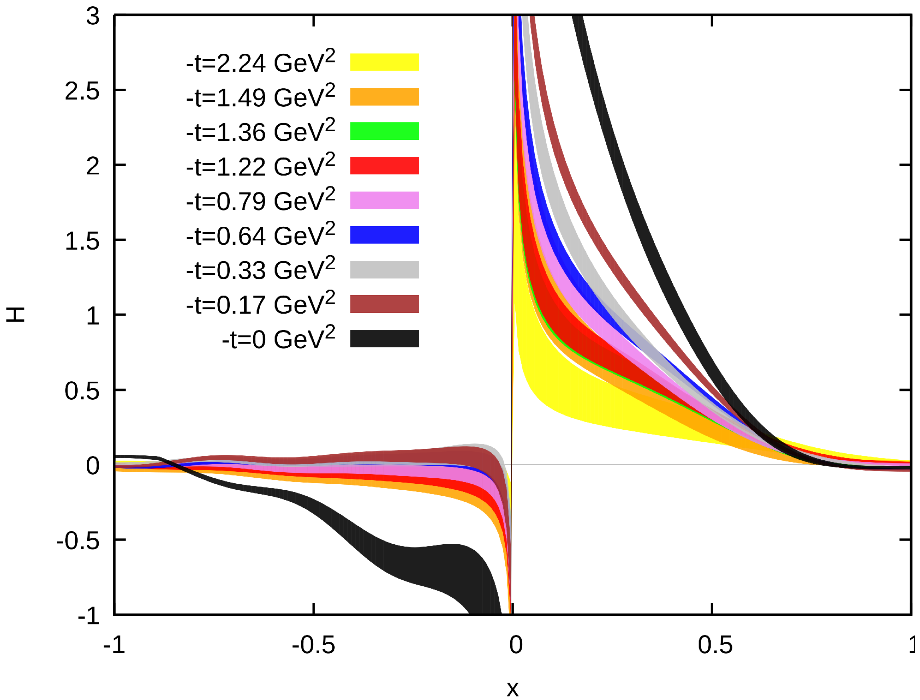
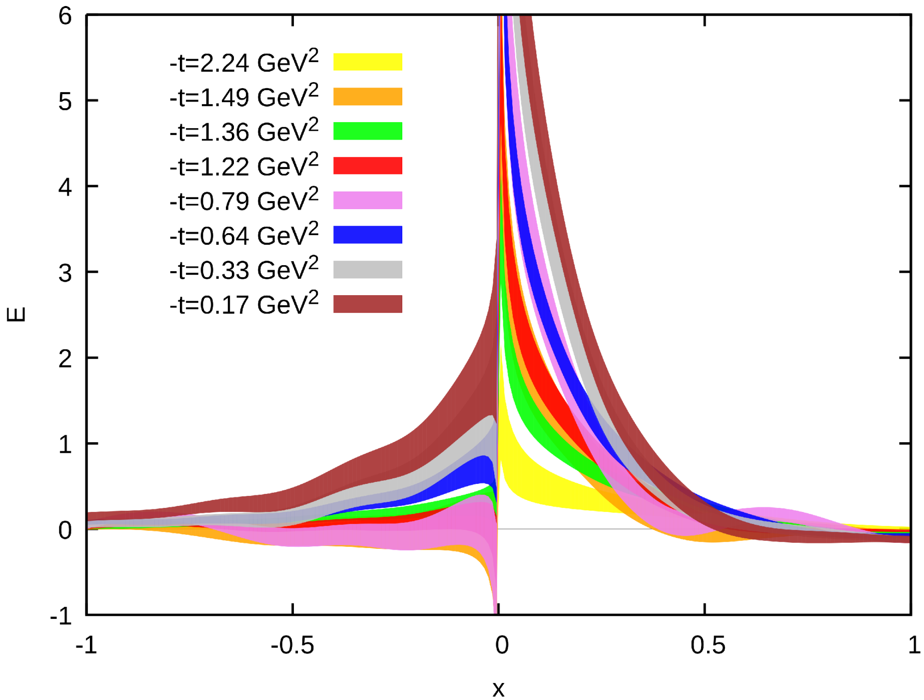


Impact parameter distribution:

$$GPD(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} GPD(x,t)$$

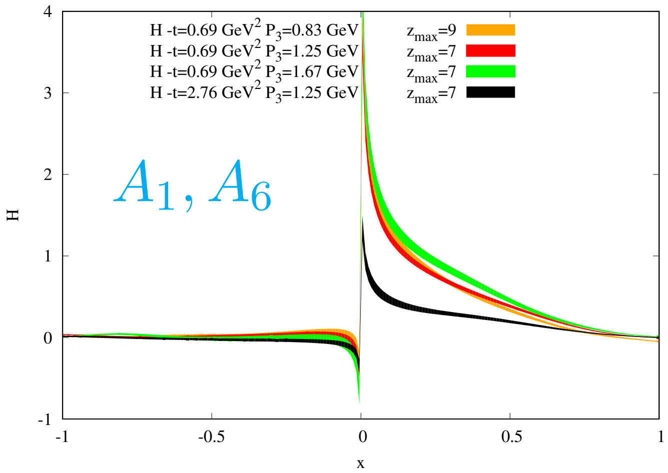


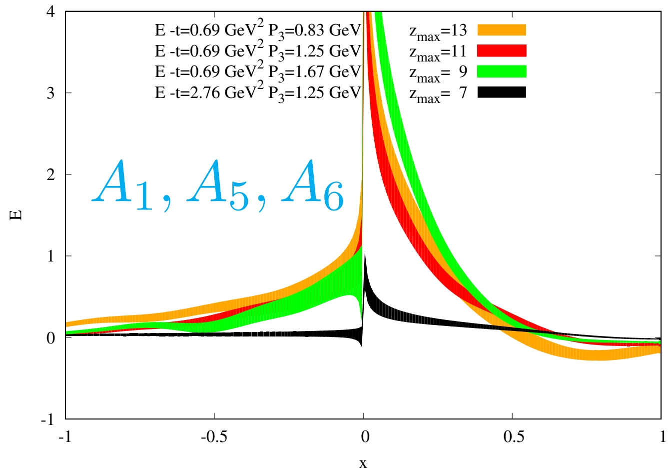


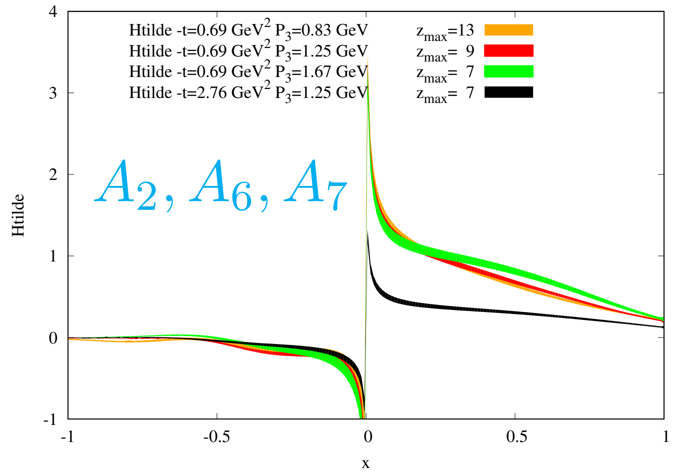
UNPOLARIZED

HELICITY

STANDARD







γ_0 operator (non-LI) $\gamma_5\gamma_3$ operator (LI)

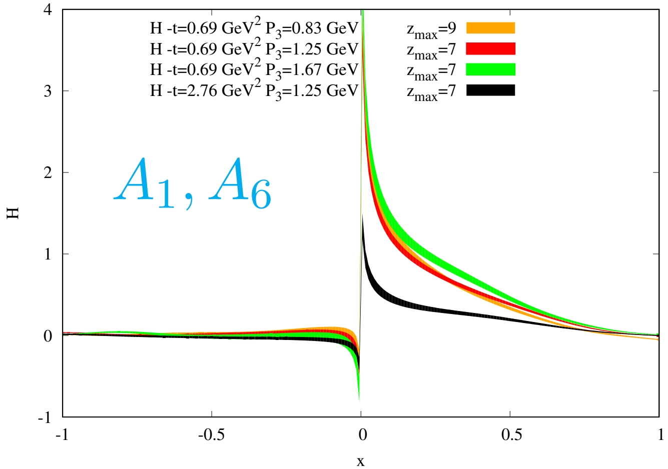
H-GPD E-GPD $\widetilde{H}$-GPD

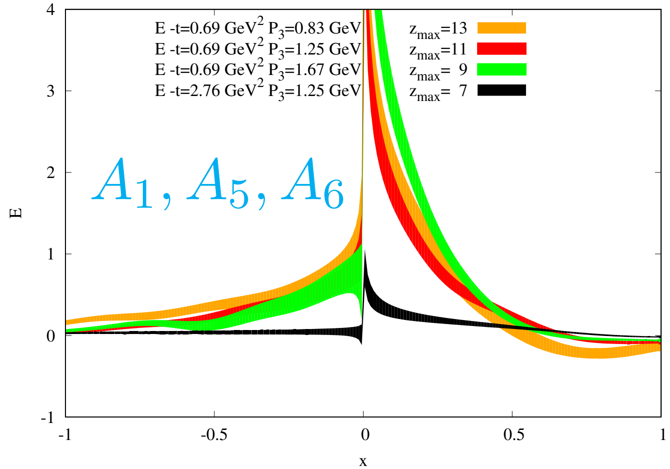
Convergence of alternative definitions of $\tilde{H}/H/E$

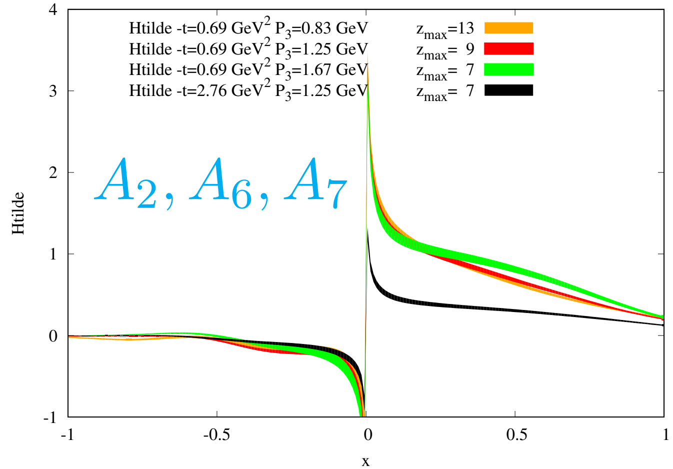
STANDARD

UNPOLARIZED

HELICITY







γ_0 operator (non-LI)

$\gamma_5\gamma_3$ operator (LI)

ALTERNATIVE

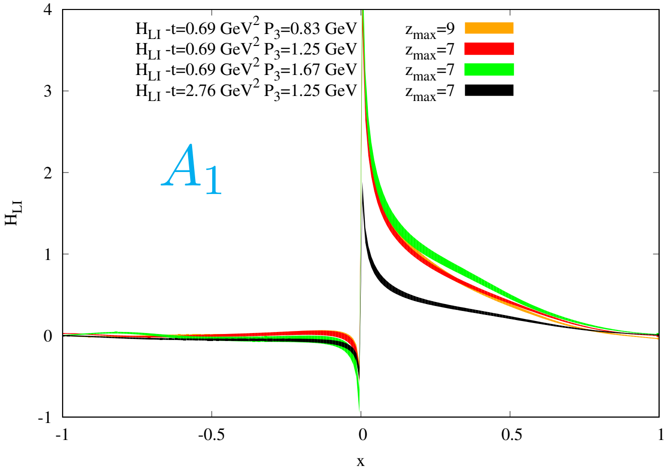
H-GPD

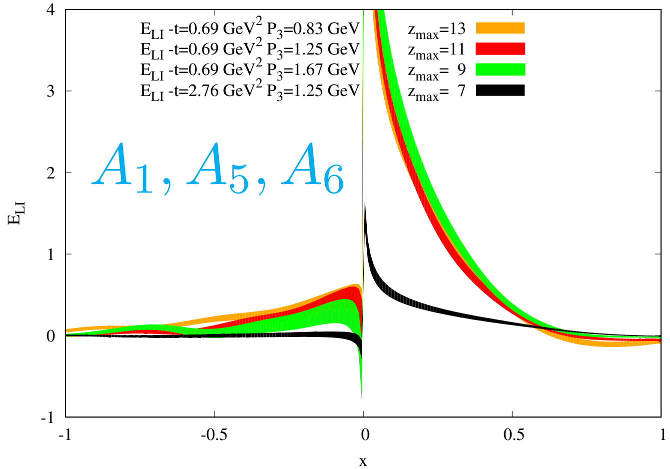
E-GPD

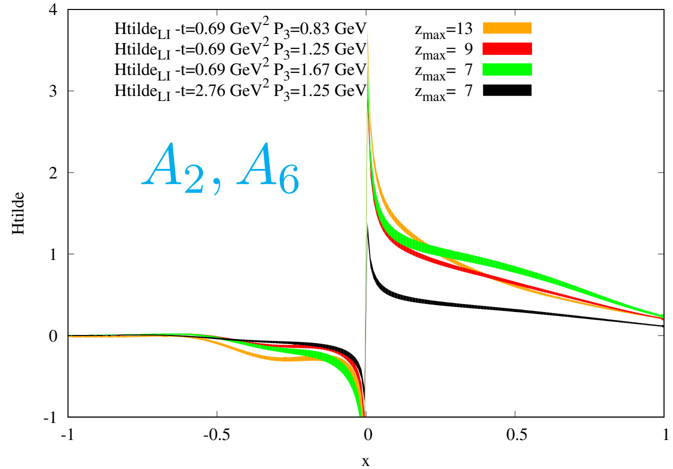
$\widetilde{H}$-GPD

γ_0,γ_T operators (LI)

$\gamma_5\gamma_0,\ \gamma_5\gamma_T$ operators (LI)



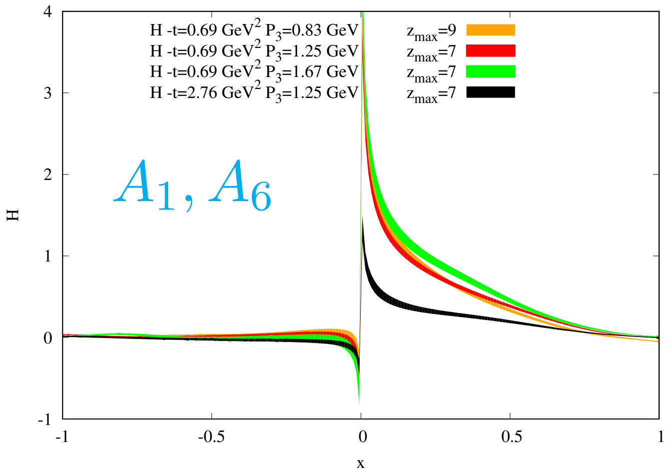


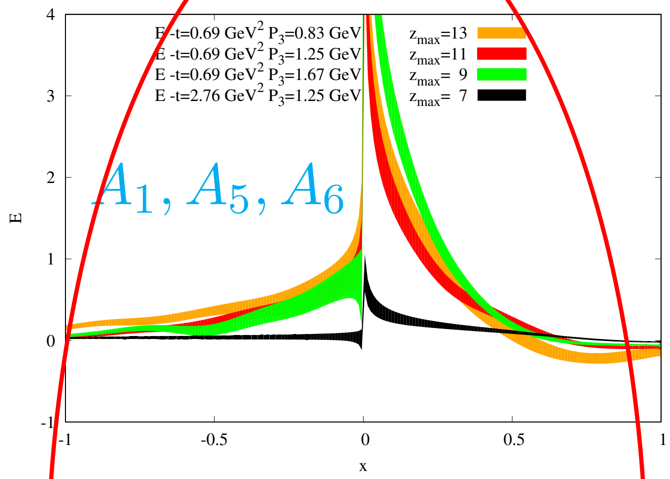


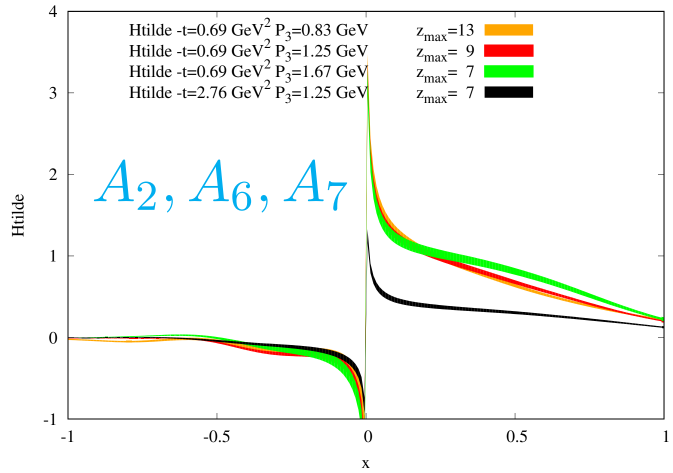
Convergence of alternative definitions of $\tilde{H}/H/E$

UNPOLARIZED | HELICITY

STANDARD







γ_0 operator (non-LI) | $\gamma_5\gamma_3$ operator (LI)

H-GPD | E-GPD | $\widetilde{H}$-GPD

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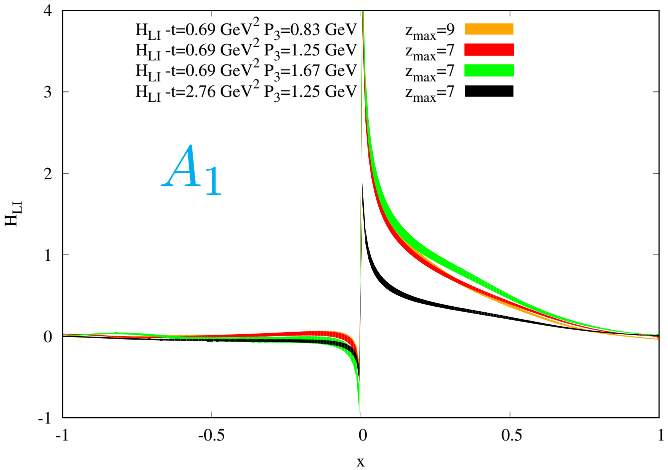
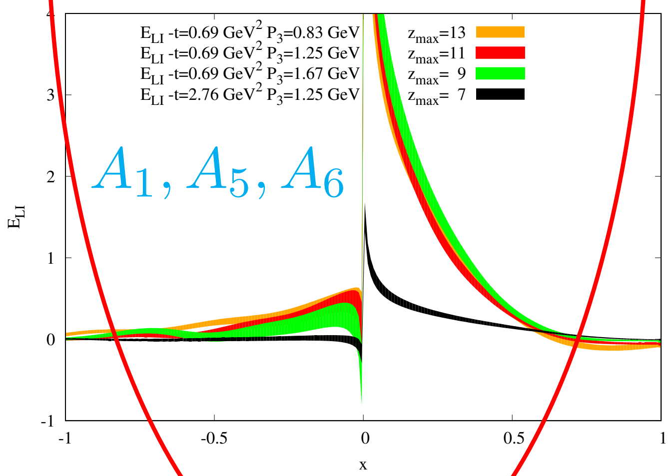
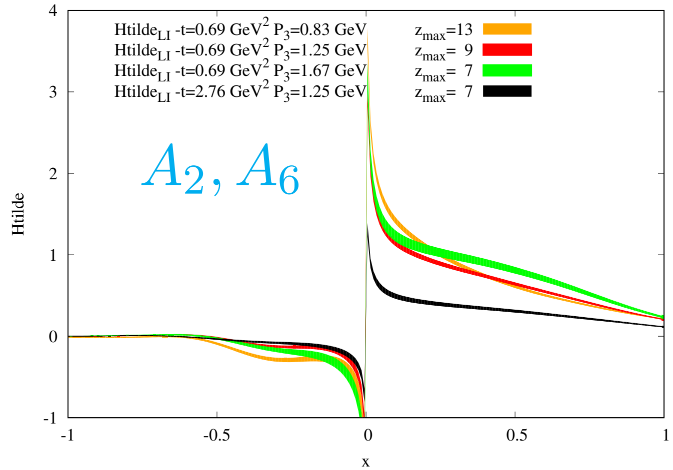
ALTERNATIVE







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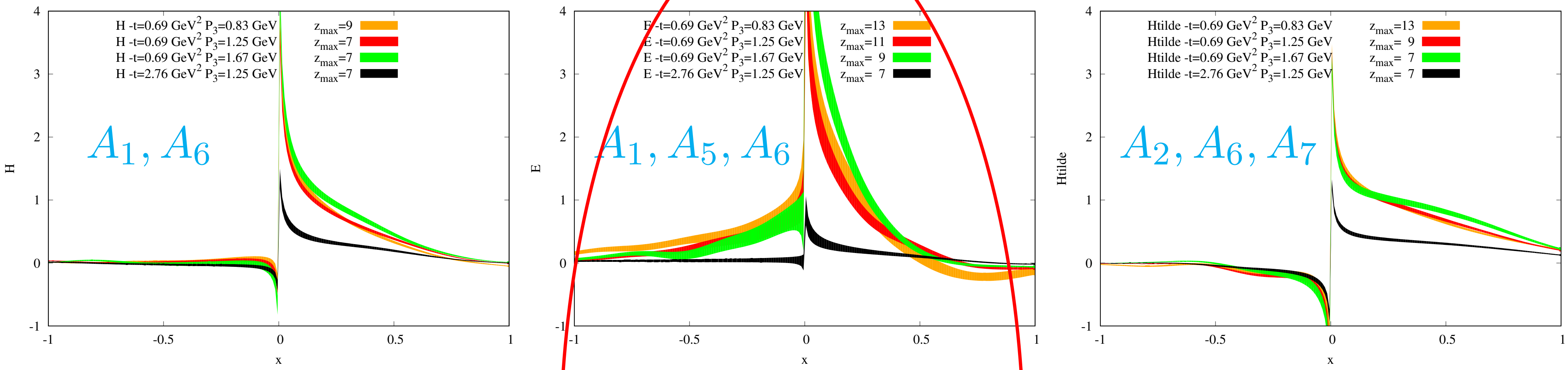

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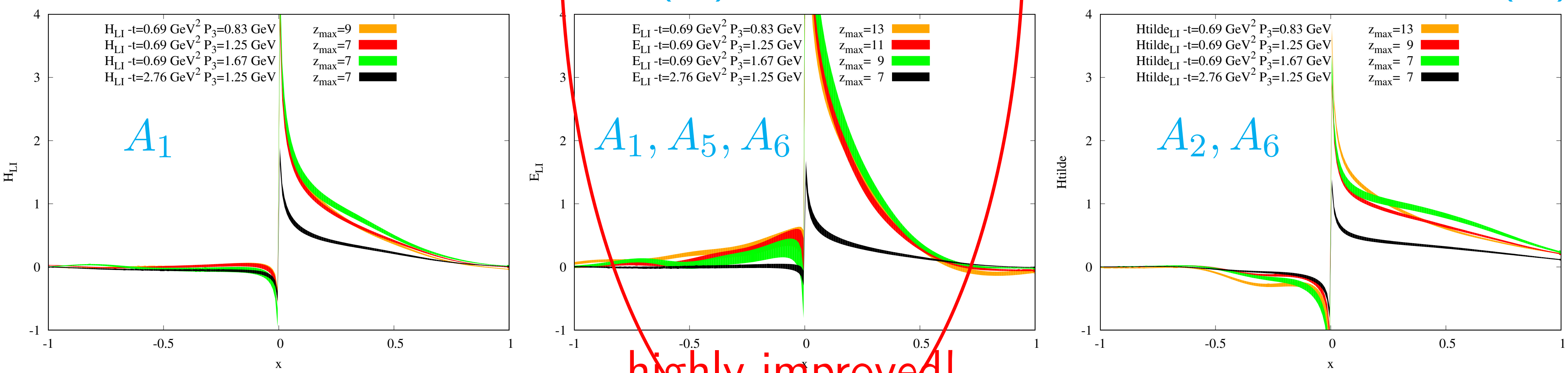

highly-improved!

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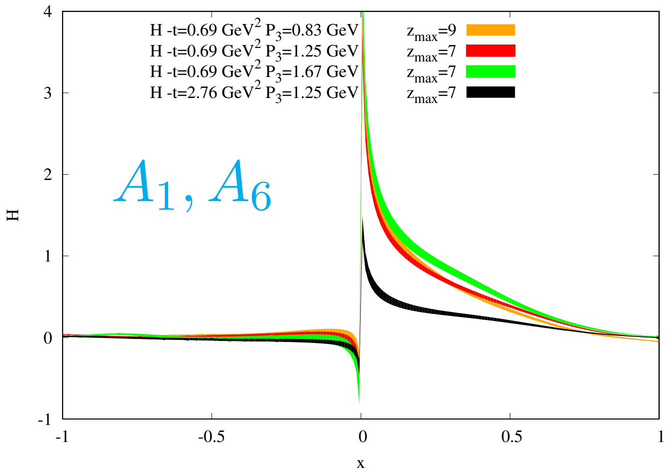

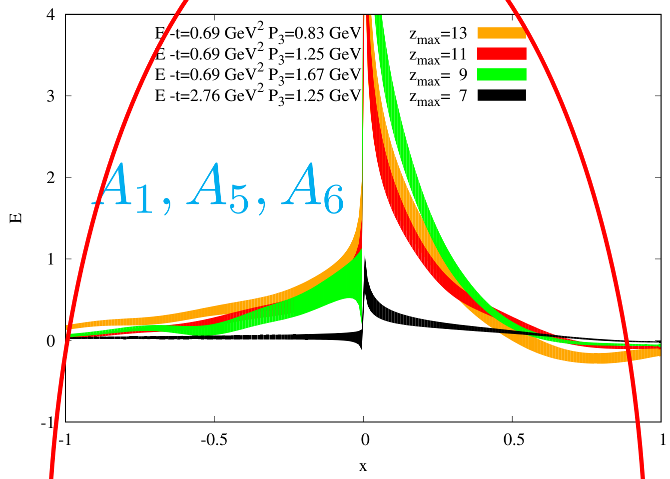

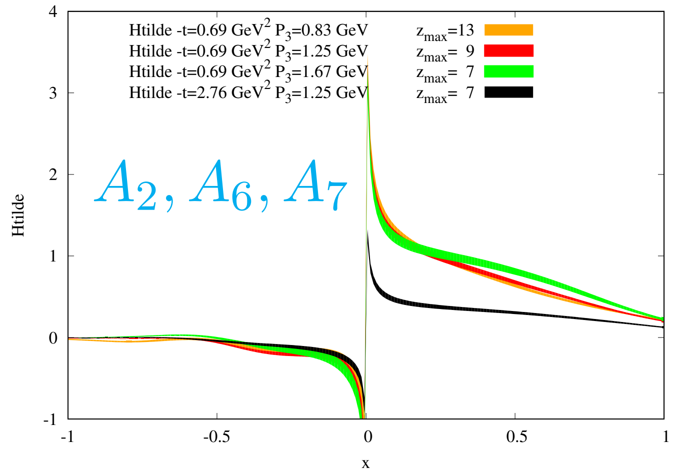

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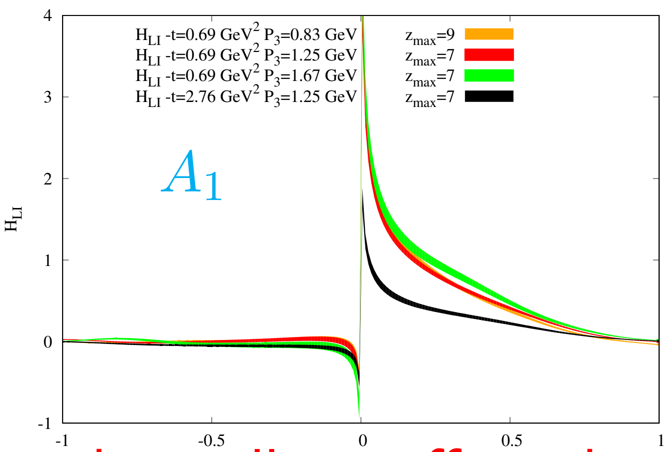
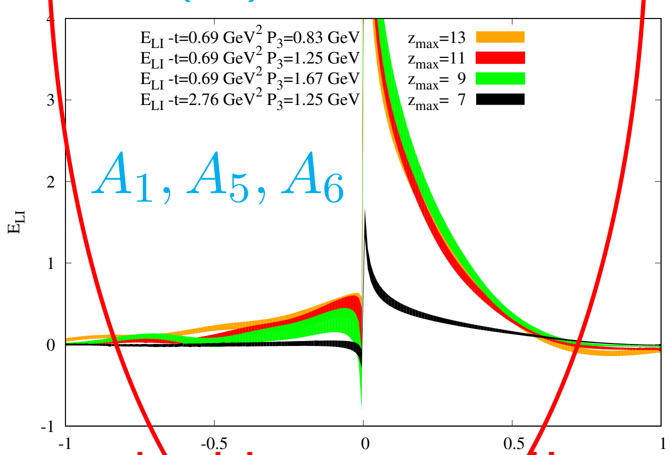
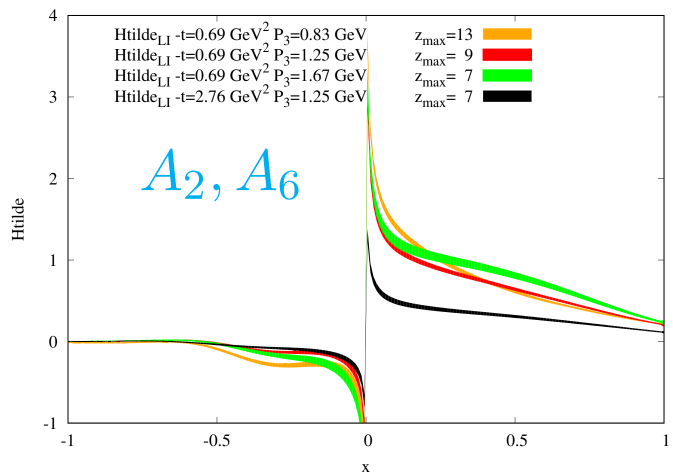




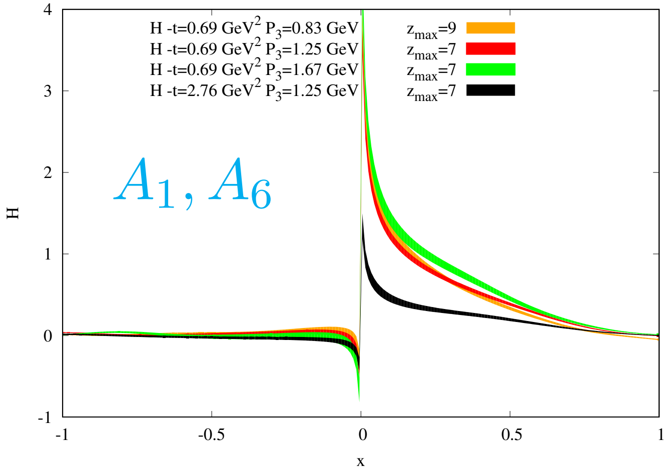


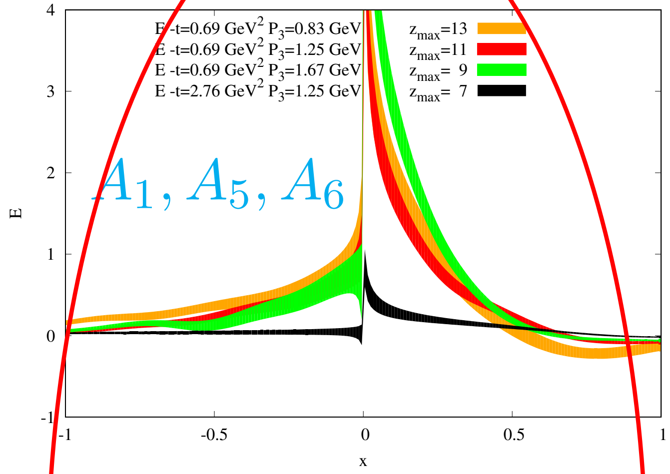
basically unaffected | highly-improved!

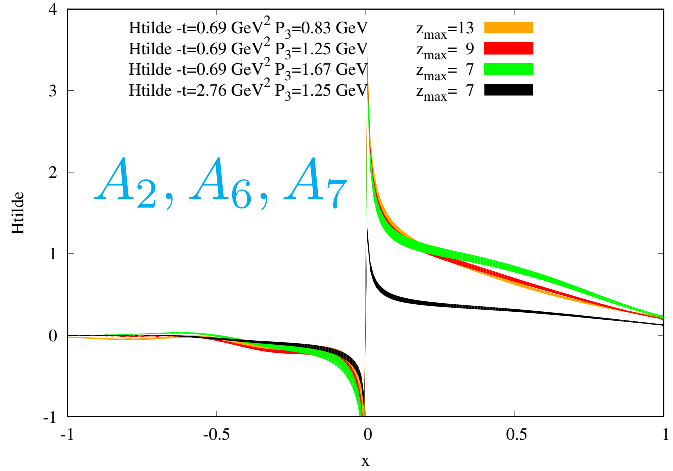
Convergence of alternative definitions of $\tilde{H}/H/E$

UNPOLARIZED | HELICITY

STANDARD







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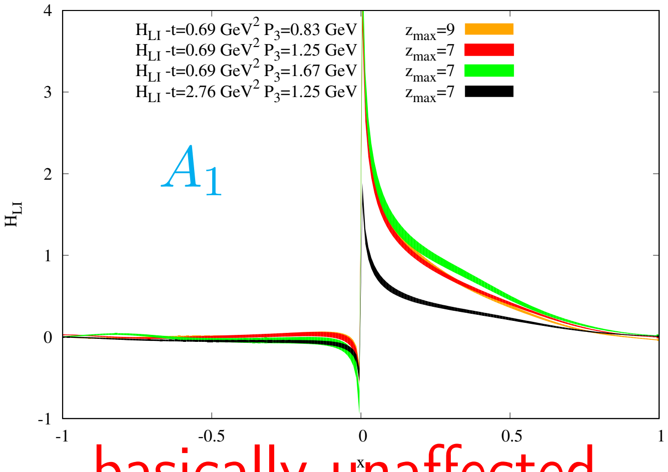

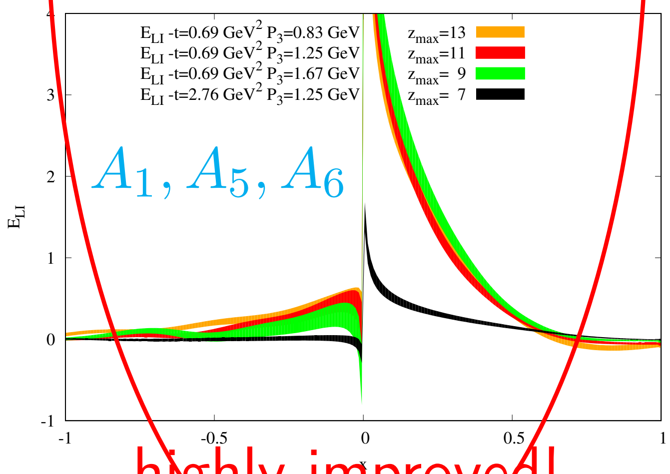

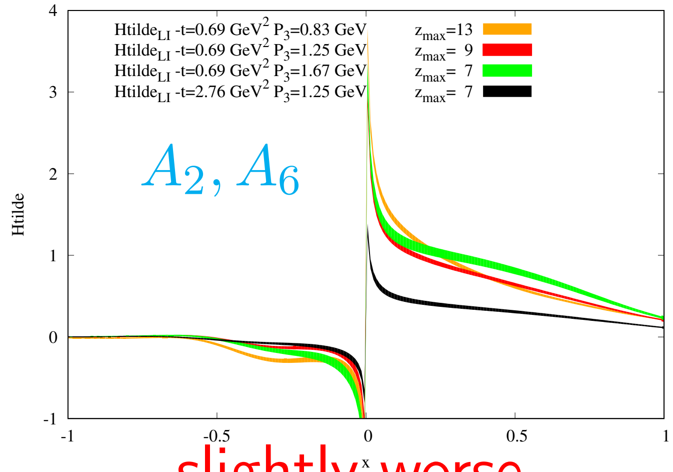

basically unaffected | highly-improved! | slightly worse

Twist-3

PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

LT: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

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Exploratory studies:

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  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
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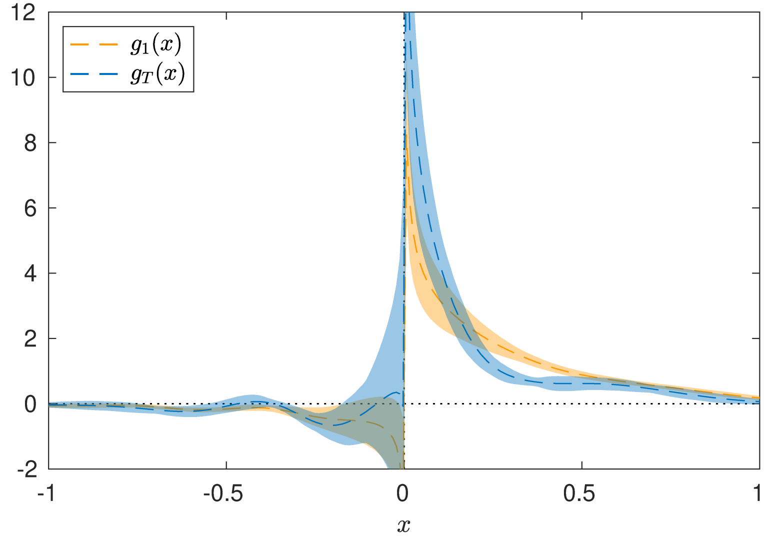

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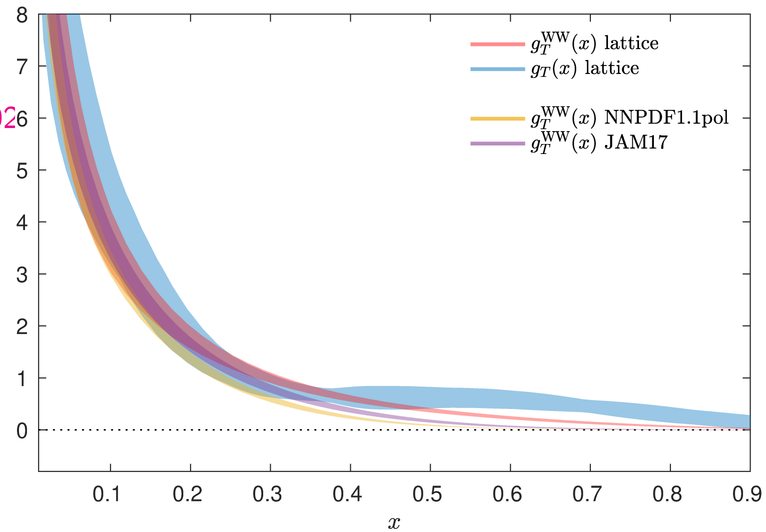

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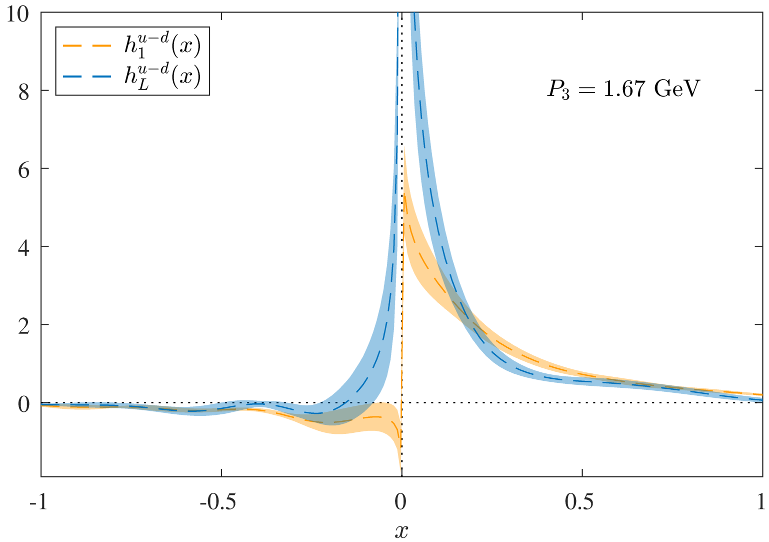

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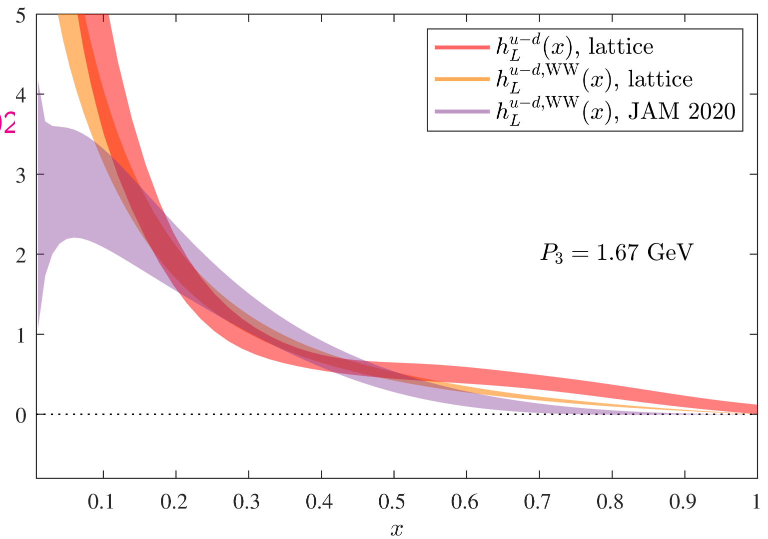

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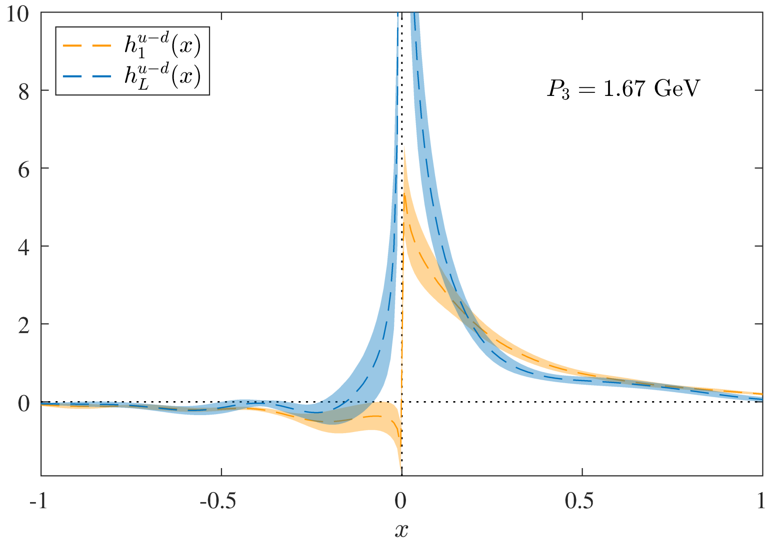

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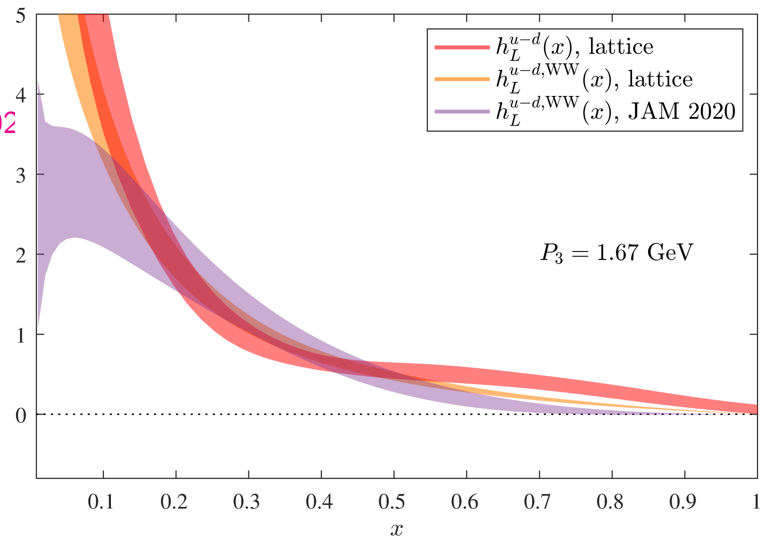
- first exploration of twist-3 GPDs
  S. Bhattacharya et al., 2306.05533



Very recently, we combined our explorations of GPDs and of twist-3 distributions
S. Bhattacharya et al., PRD108(2023)054501

Twist-3 axial GPDs: $\widetilde{G}_1, \widetilde{G}_2, \widetilde{G}_3, \widetilde{G}_4$

$$\mathcal{F}^{[\gamma_j \gamma_5]} = -i \frac{\Delta_j \gamma_5}{2m} F_{\widetilde{E}+\widetilde{G}_1} + \gamma_j \gamma_5 \; F_{\widetilde{H}+\widetilde{G}_2} + \frac{\Delta_j \gamma_3 \gamma_5}{P_3} F_{\widetilde{G}_3} - \frac{\mathrm{sign}[P_3] \, \varepsilon_{\perp}^{j\,\rho} \Delta_\rho \gamma_3}{P_3} F_{\widetilde{G}_4}$$

Contributions from different insertions and projectors ($\vec{\Delta} = (\Delta_1, 0, 0)$):

$\Pi(\gamma^2\gamma^5, \Gamma_0)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^2\gamma^5, \Gamma_2)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^1\gamma^5, \Gamma_1)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{E} + \widetilde{G}_1$,
$\Pi(\gamma^1\gamma^5, \Gamma_3)$: $\widetilde{G}_3$.

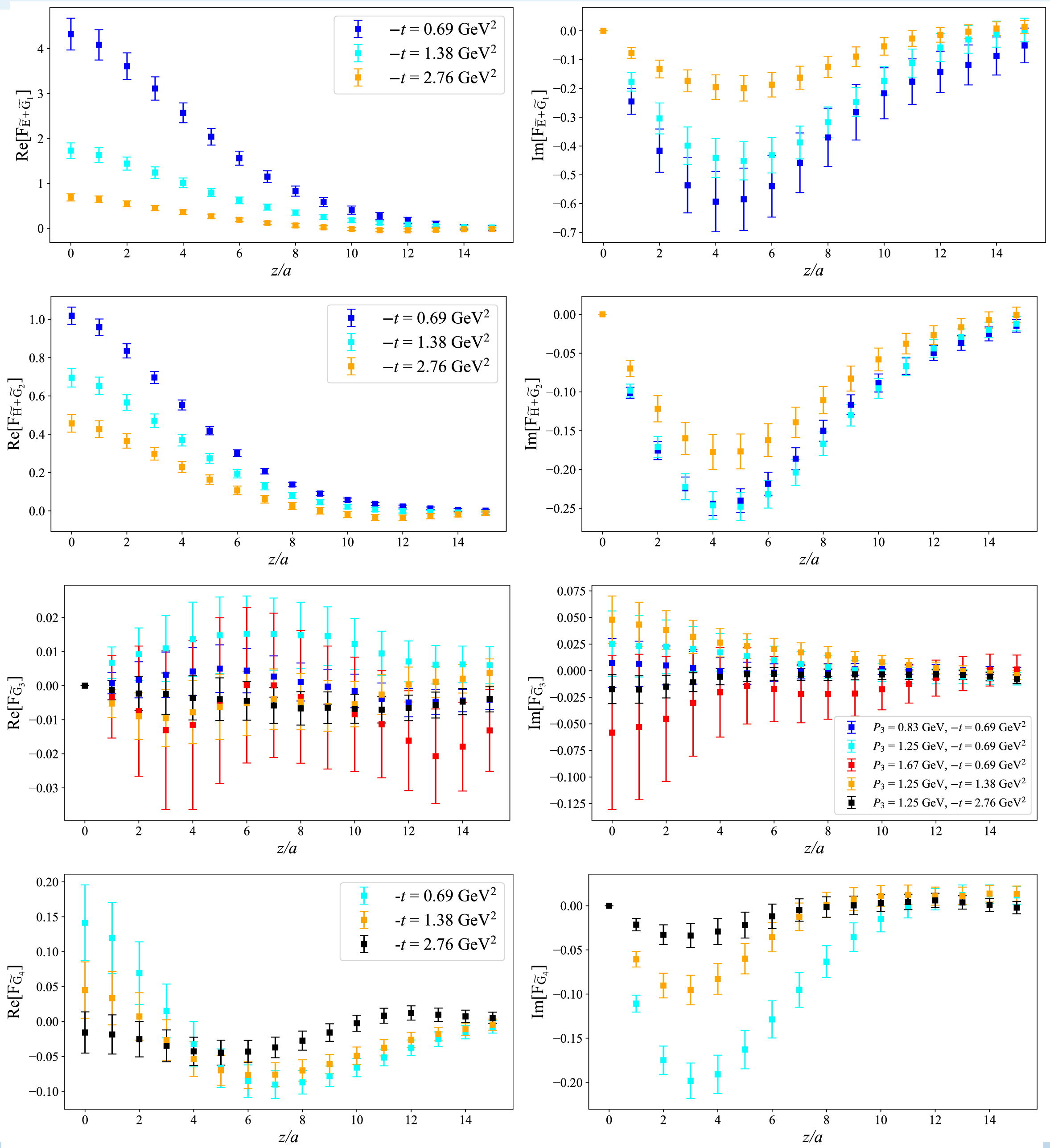
Twist-3 GPDs in coordinate space

$\widetilde{E}+\widetilde{G}_1$

$\widetilde{H}+\widetilde{G}_2$

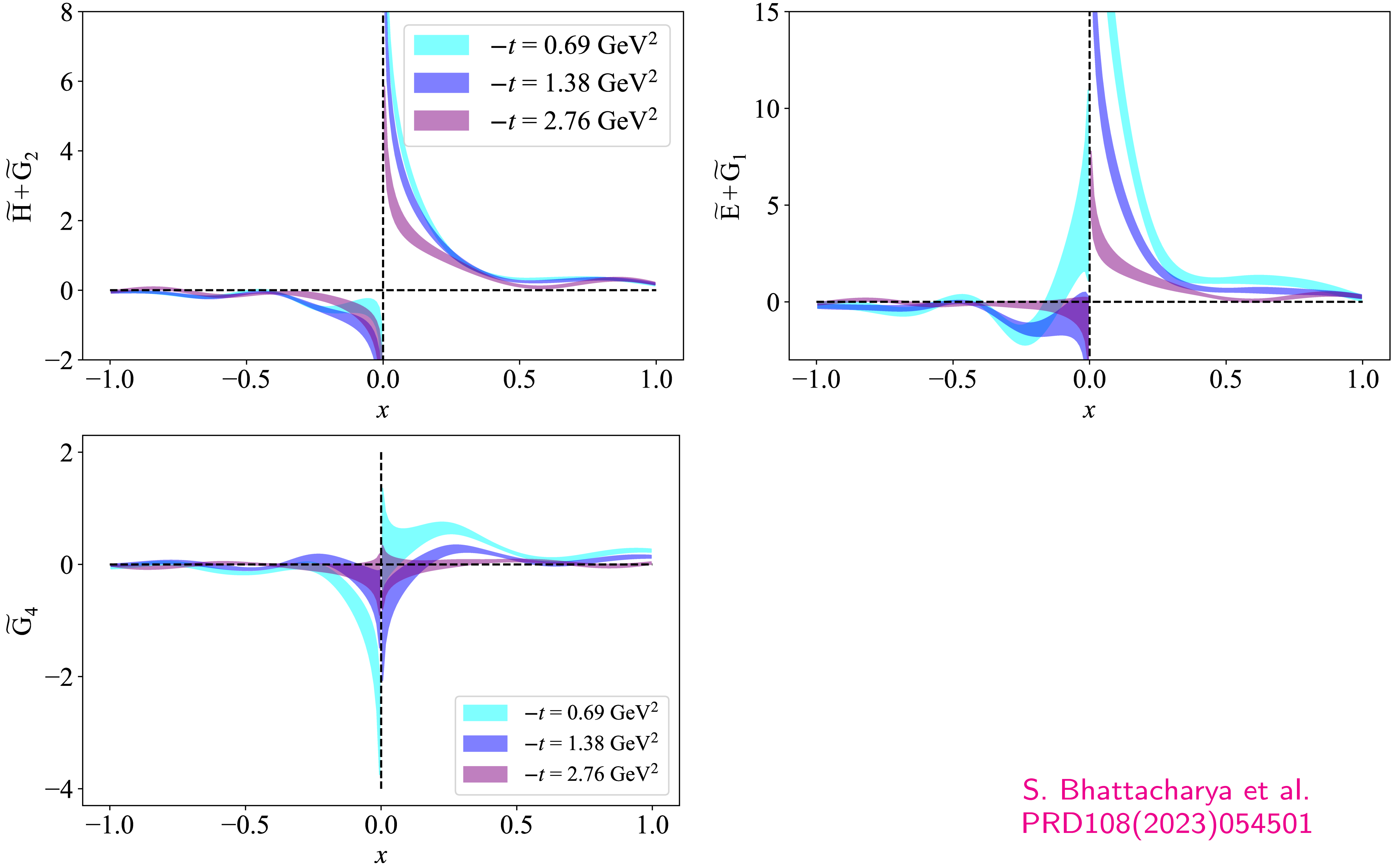
$\widetilde{G}_3$

$\widetilde{G}_4$



S. Bhattacharya et al.
PRD108(2023)054501

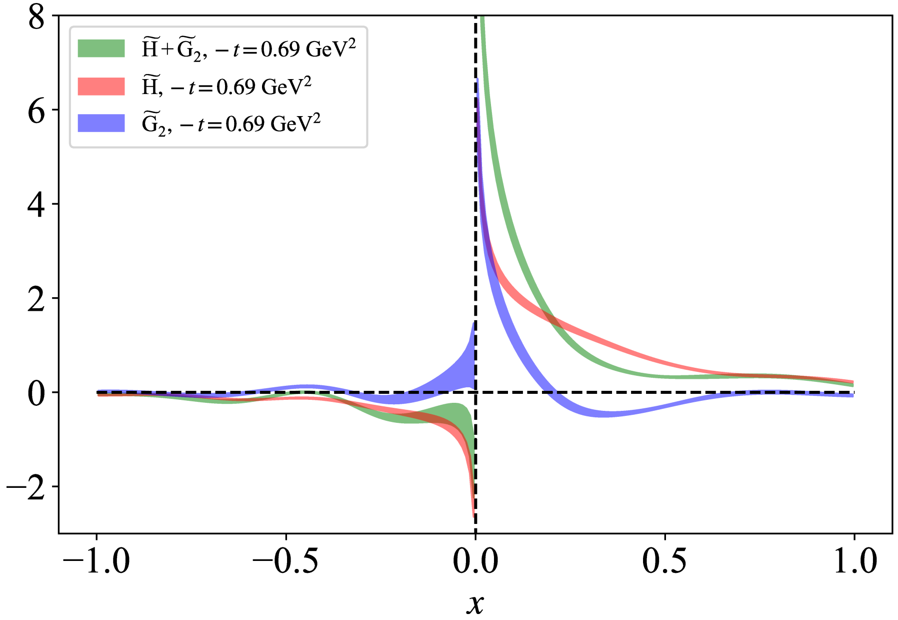
Twist-3 GPDs in x-space

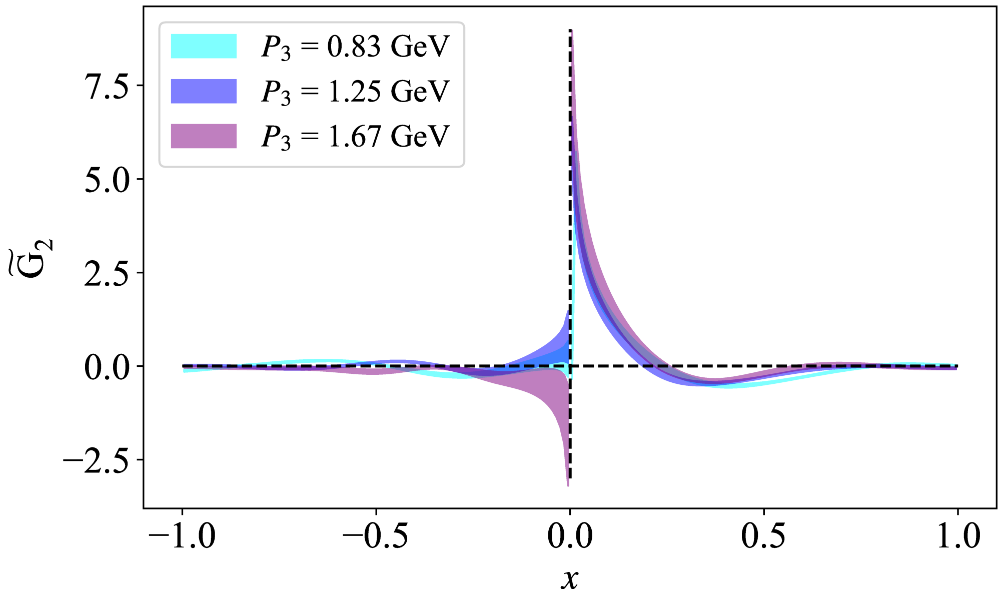


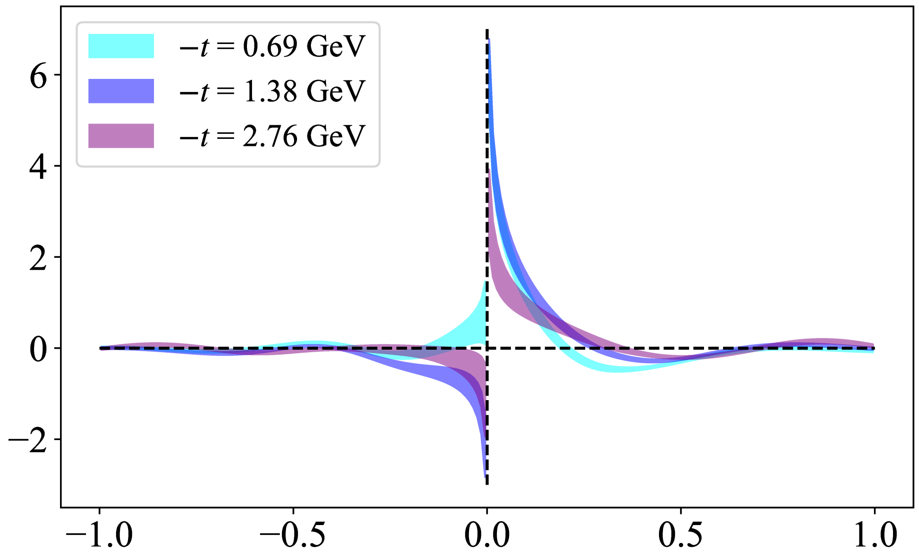
S. Bhattacharya et al.
PRD108(2023)054501

Isolating $\widetilde{G}_2$

S. Bhattacharya et al.
PRD108(2023)054501







Consistency checks

Burkhardt-Cottingham-type sum rules:

$$G_P(t) = \int_{-1}^{1} dx \left(\widetilde{E}(x,\xi,t) + \widetilde{G}_1(x,\xi,t)\right) = \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t)$$

$$G_A(t) = \int_{-1}^{1} dx \left(\widetilde{H}(x,\xi,t) + \widetilde{G}_2(x,\xi,t)\right) = \int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t)$$

$$\Rightarrow \int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0$$

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GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

- satisfied for $\widetilde{H} + \widetilde{G}_2$ – same local limit and norm as $\widetilde{H}$,
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$$\int dx\, x\, \widetilde{G}_3(x,\xi,t) = \frac{\xi}{4} G_E(t)\,, \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E(t)\,.$$

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$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

- satisfied for $\widetilde{H} + \widetilde{G}_2$ – same local limit and norm as $\widetilde{H}$,
- cannot be tested for $\widetilde{E} + \widetilde{G}_1$ – $\widetilde{E}$ inaccessible at $\xi = 0$.
- norms of $\widetilde{G}_2$ and $\widetilde{G}_4$ close to vanishing.

Efremov-Leader-Teryaev-type sum rules:

$$\int dx\, x\, \widetilde{G}_3(x,\xi,t) = \frac{\xi}{4} G_E(t)\,, \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E(t)\,.$$

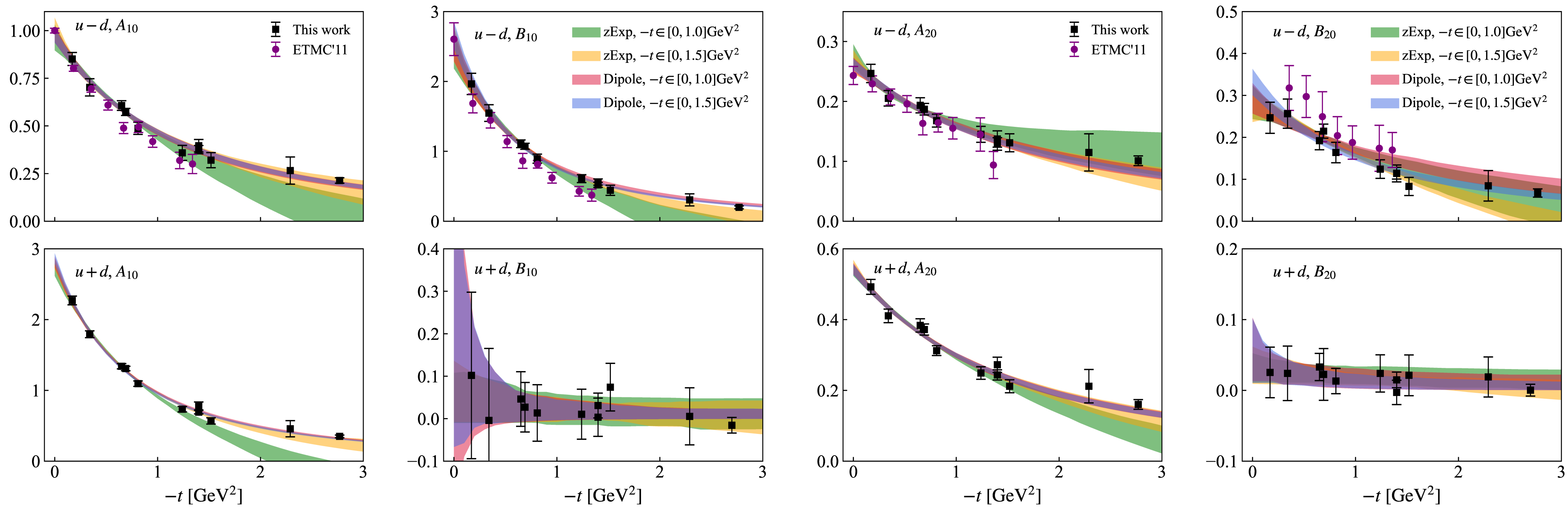
- $\widetilde{G}_3$ indeed vanishes at $\xi = 0$,
- $\widetilde{G}_4$ non-vanishing and small.

GPDs moments from OPE of non-local operators

Short-distance factorization of ratio-renormalized H/E:

$$\mathcal{F}^{\overline{\rm MS}}(z,P,\Delta)=\sum_{n=0}\frac{(-izP)^n}{n!}C_n^{\overline{\rm MS}}(\mu^2z^2)\langle x^n\rangle+\mathcal{O}(\Lambda^2_{\rm QCD}z^2),$$

$C_n^{\overline{\rm MS}}(\mu^2z^2)$ – Wilson coefficients (NNLO for $u-d$, NLO for $u+d$)



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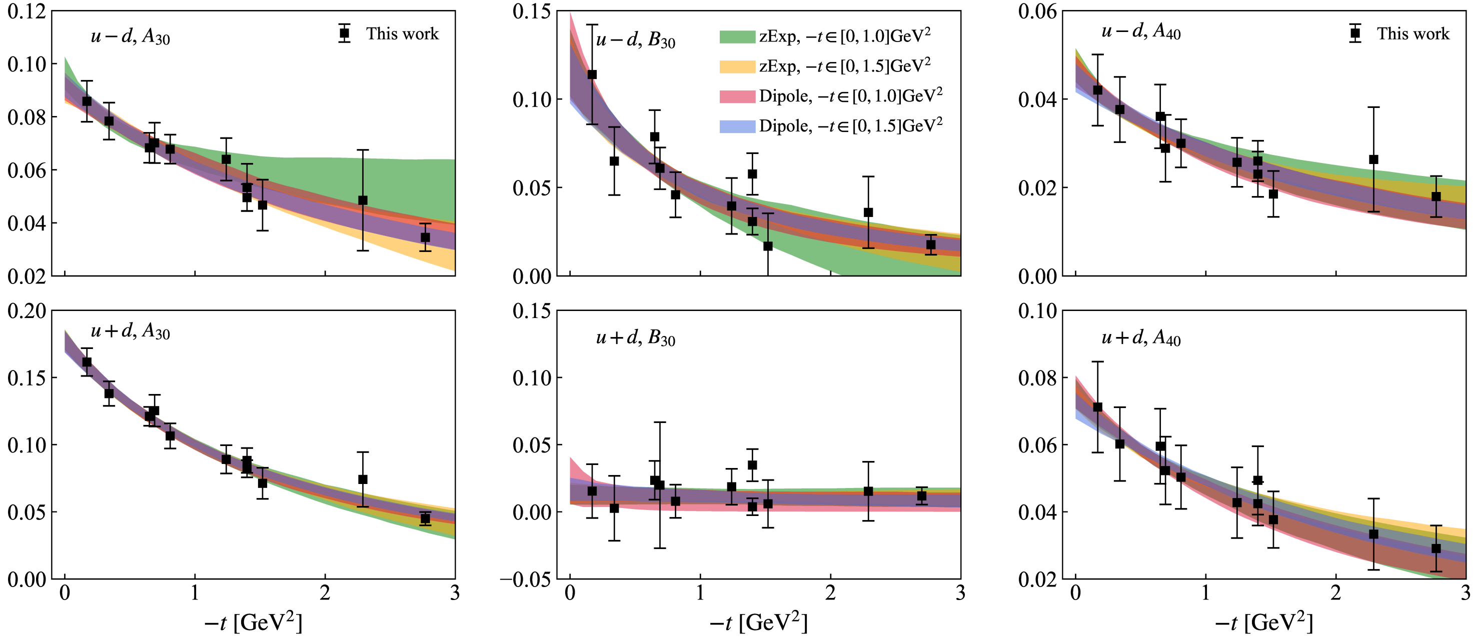
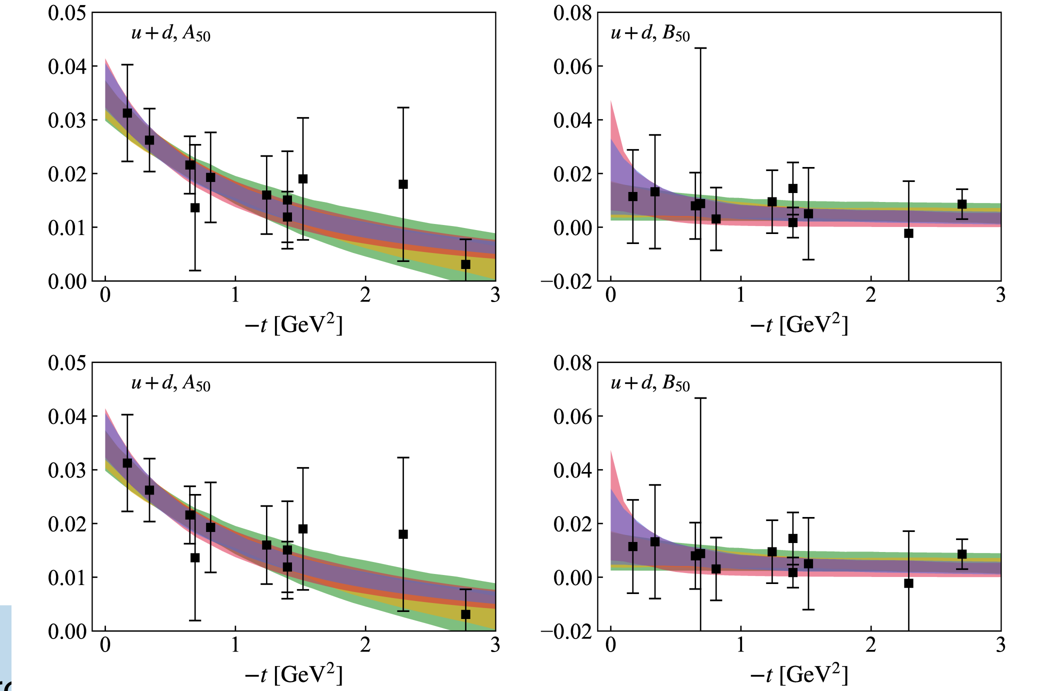
GPDs moments from OPE of non-local operators



Also
higher moments!



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PRD 108(2023)014507

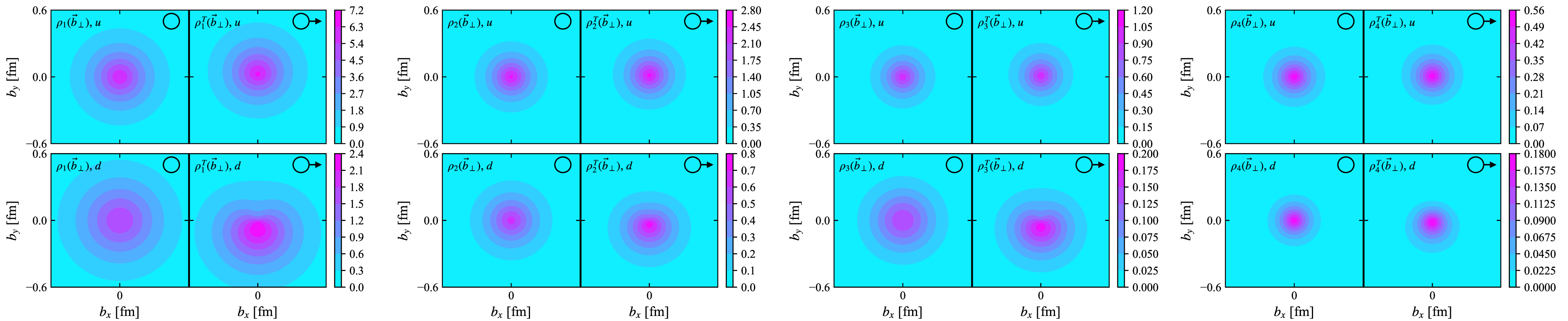
GPDs moments from OPE of non-local operators

Moments of impact parameter parton distributions in the transverse plane:

$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\vec{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp},$$

$$\rho^T_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} [A_{n+1,0}(-\vec{\Delta}_\perp^2) + i\frac{\Delta_y}{2M} B_{n+1,0}(-\vec{\Delta}_\perp^2)] e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}.$$

1st moment | 2nd moment | 3rd moment | 4th moment



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Conclusions and prospects

- Main message: **probing nucleon's 3D structure with LQCD becomes feasible!**
- Recent breakthrough for GPDs: **computationally more efficient calculations in non-symmetric frames**.
- Also, new definitions of GPDs with different convergence properties – e.g. faster convergence for the unpolarized GPD E.
- A lot of follow-up work in progress: transversity GPDs, pion and kaon GPDs, other twist-3 GPDs, extension of kinematics.
- Obviously, GPDs much more challenging than PDFs.
- Several challenges have to be overcome – control of lattice and other systematics.
- Consistent progress will ensure complementary role to pheno!

Conclusions and prospects

- Main message: **probing nucleon's 3D structure with LQCD becomes feasible!**
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- Several challenges have to be overcome – control of lattice and other systematics.
- Consistent progress will ensure complementary role to pheno!

Thank you for your attention!

Backup slides

Bare matrix elements

Lattice matrix elements need to be computed with 2 different projections (unpolarized/polarized).
Below for the unpolarized Dirac insertion (for unpolarized GPDs)

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
of bare ME
intermediate RI scheme

↓

reconstruction of x-dependence
z-space→x-space
Backus-Gilbert

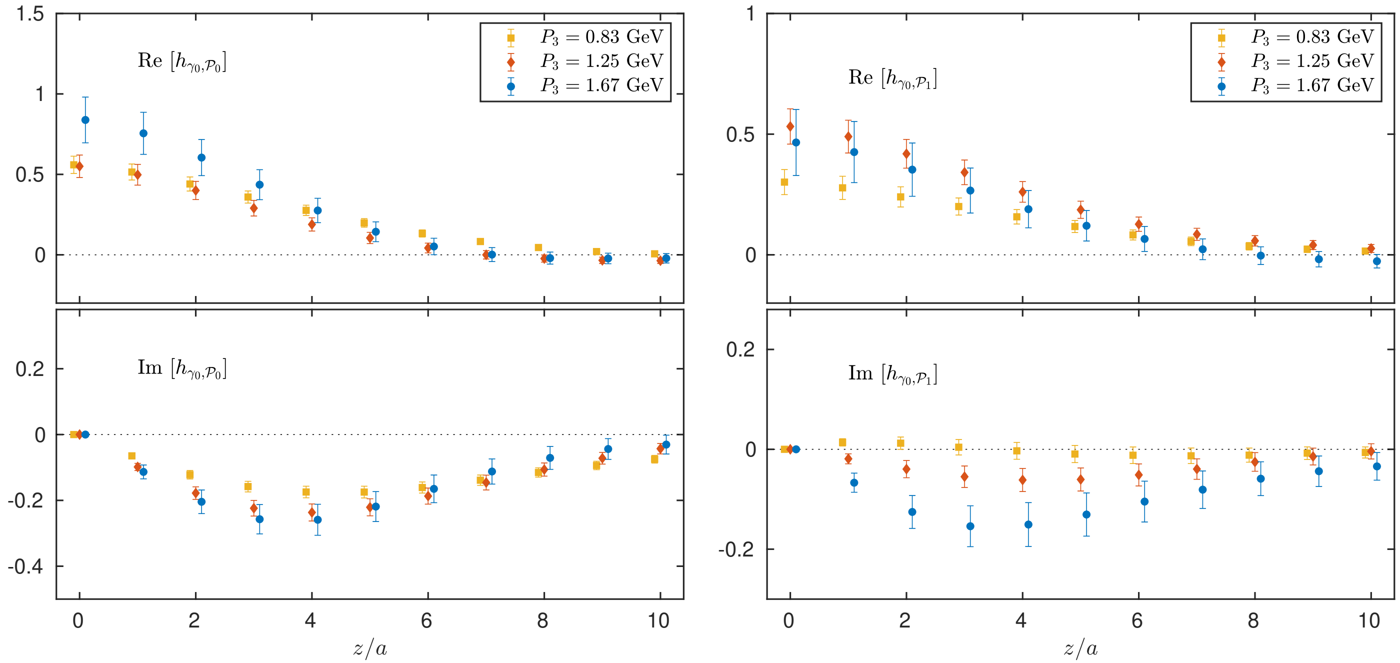
↓

matching to light cone
RI→ $\overline{\text{MS}}$
(incl. evolution to $\mu = 2$ GeV)

↓

light-cone GPD

unpolarized projector — polarized projector



Three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV
Momentum transfer: $-t = 0.69$ GeV^2
Zero skewness: $\xi = 0$

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Disentangled renormalized matrix elements

Removal of divergences and disentangling of H- and E-GPDs.
Unpolarized Dirac insertion (for unpolarized GPDs)

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
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↓

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RI→ $\overline{\text{MS}}$
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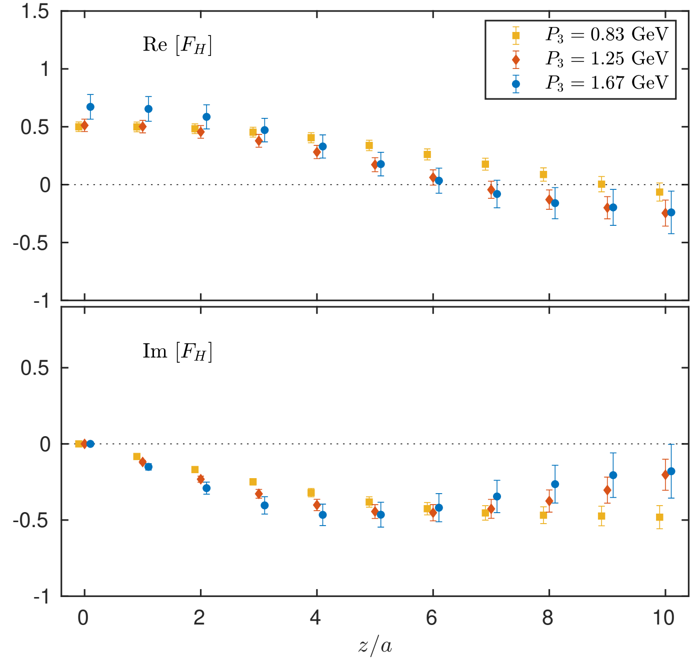
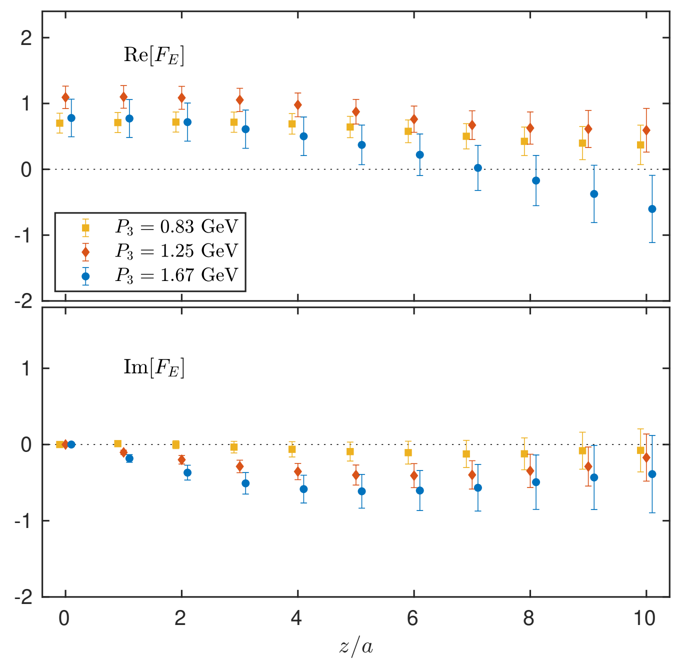
↓

light-cone GPD

ME of H-function



ME of E-function



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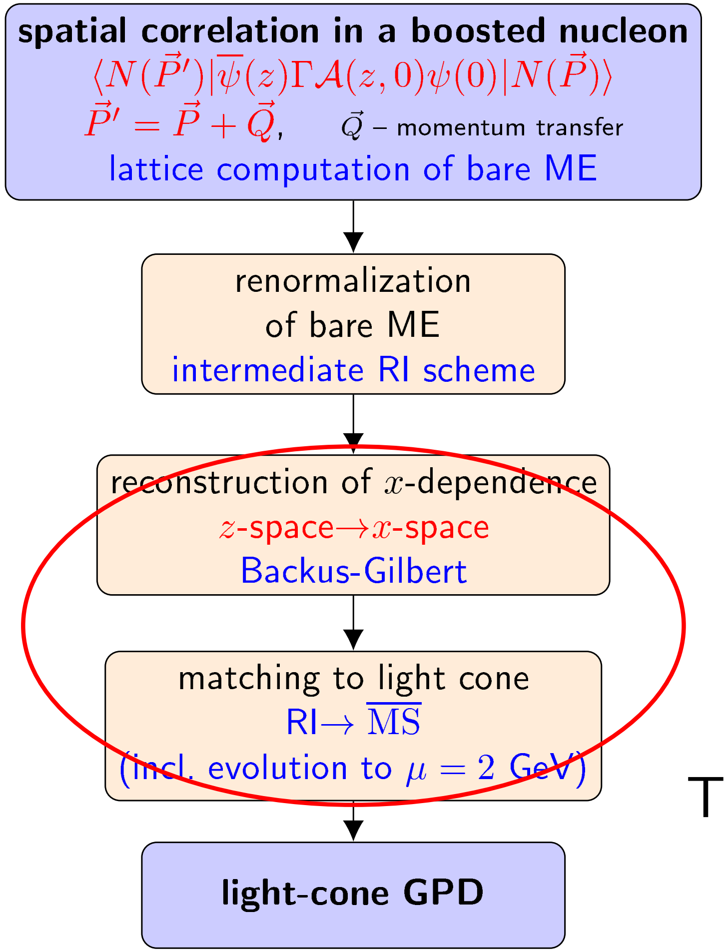
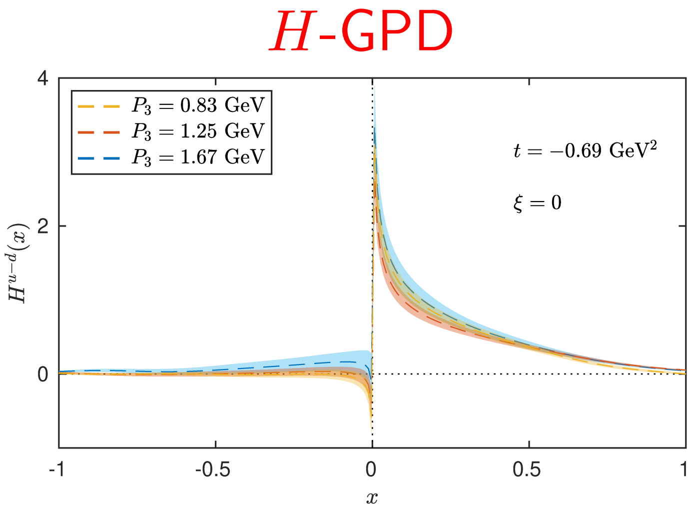
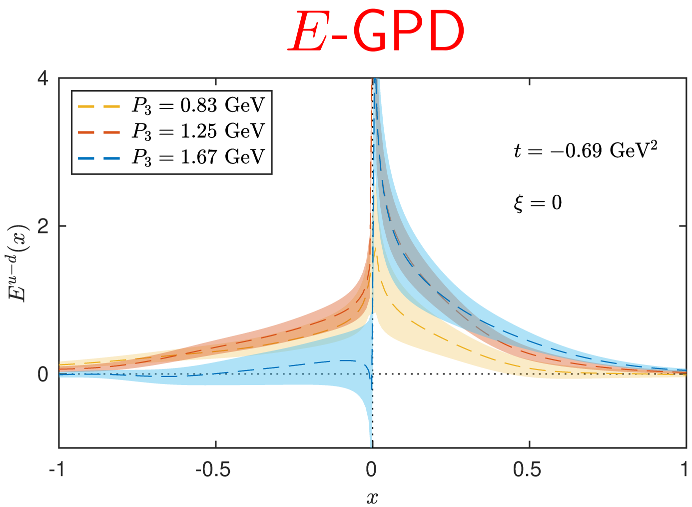
Light-cone distributions

Reconstruction of x-dependence and matching to light cone.
Unpolarized Dirac insertion (for unpolarized GPDs)







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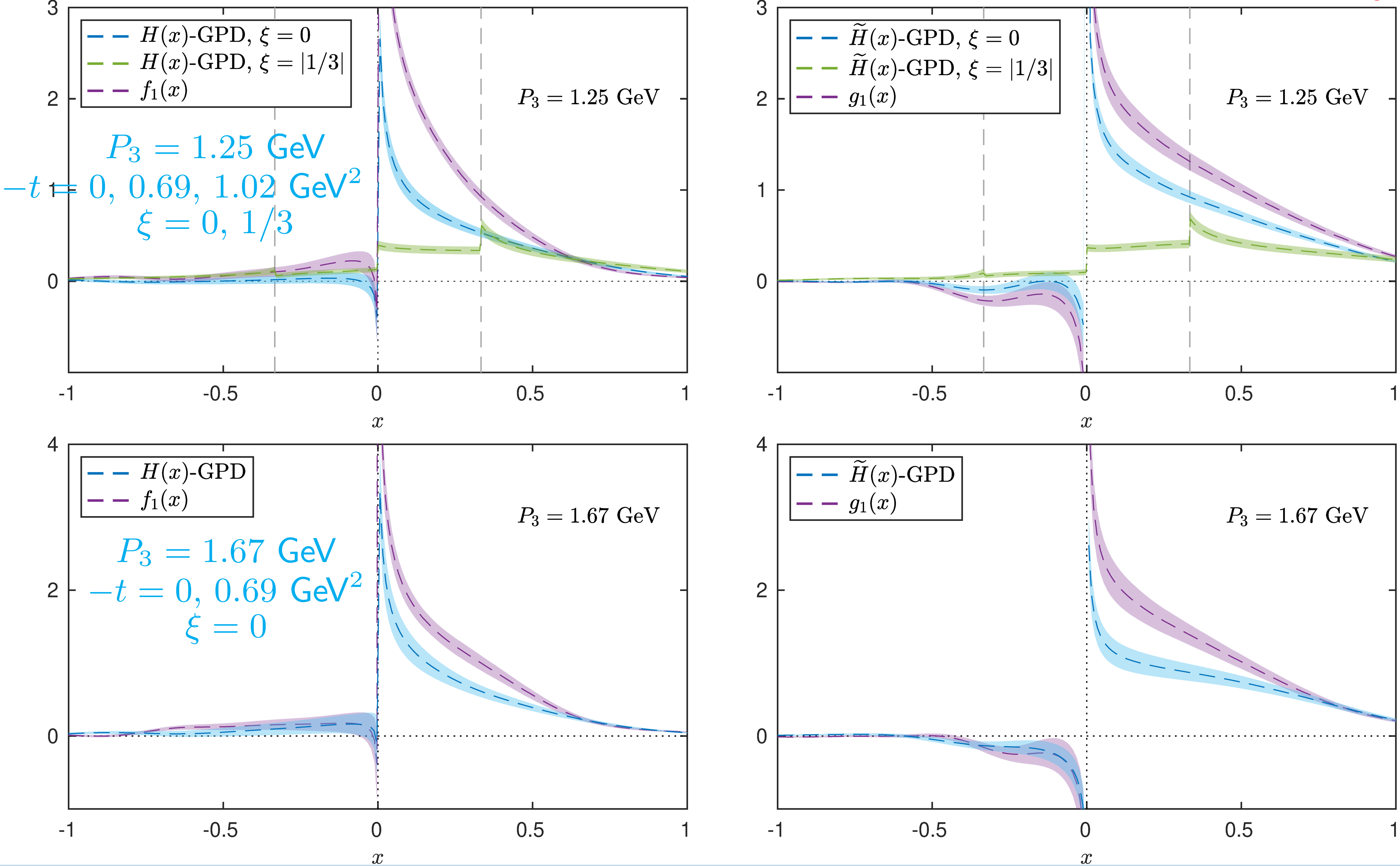
Comparison of PDFs and H-GPDs

unpolarized | ETMC, Phys. Rev. Lett. 125 (2020) 262001 | helicity



Transversity GPDs

Transversity GPDs: ETMC, Phys. Rev. D105 (2022) 034501
4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
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lattice computation of bare ME

↓

renormalization
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intermediate RI scheme

↓

reconstruction of x-dependence
z-space→x-space
Backus-Gilbert

↓

matching to light cone
RI→ $\overline{\text{MS}}$
(incl. evolution to $\mu = 2$ GeV)

↓

light-cone GPD

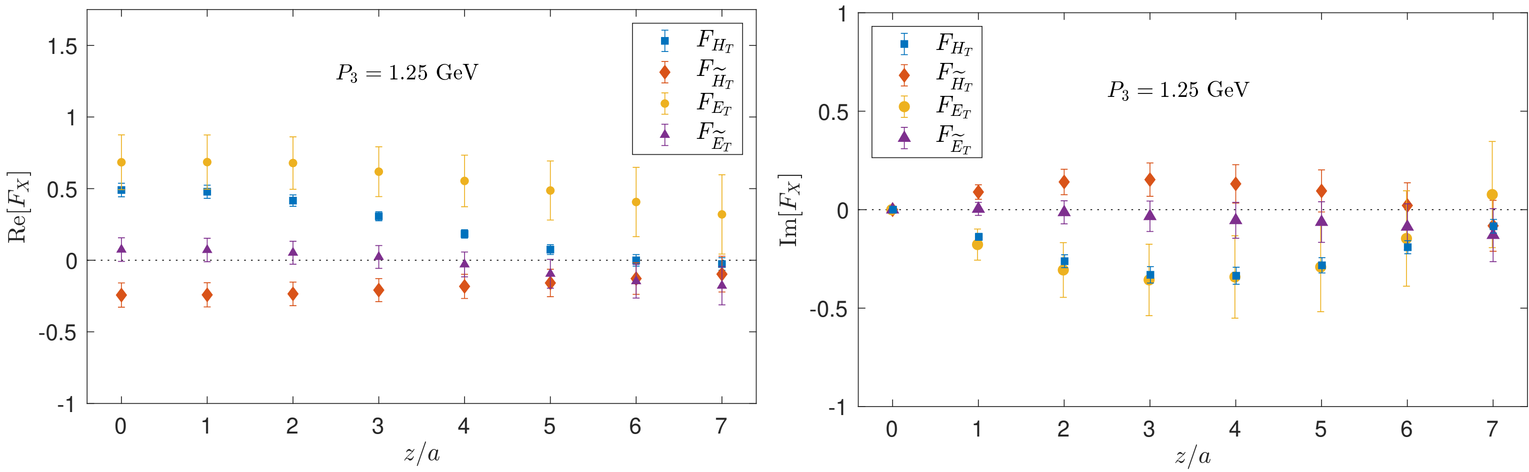
Three nucleon boosts ($\xi = 0$): $P_3 = 0.83, 1.25, 1.67$ GeV
Nucleon boost ($\xi \neq 0$): $P_3 = 1.25$ GeV

Momentum transfer ($\xi = 0$): $-t = 0.69$ GeV^2
Momentum transfer ($\xi \neq 0$): $-t = 1.02$ GeV^2

Renormalized ME

Real part | Imaginary part

$\xi = 1/3$



Transversity GPDs

ETMC, Phys. Rev. D105 (2022) 034501

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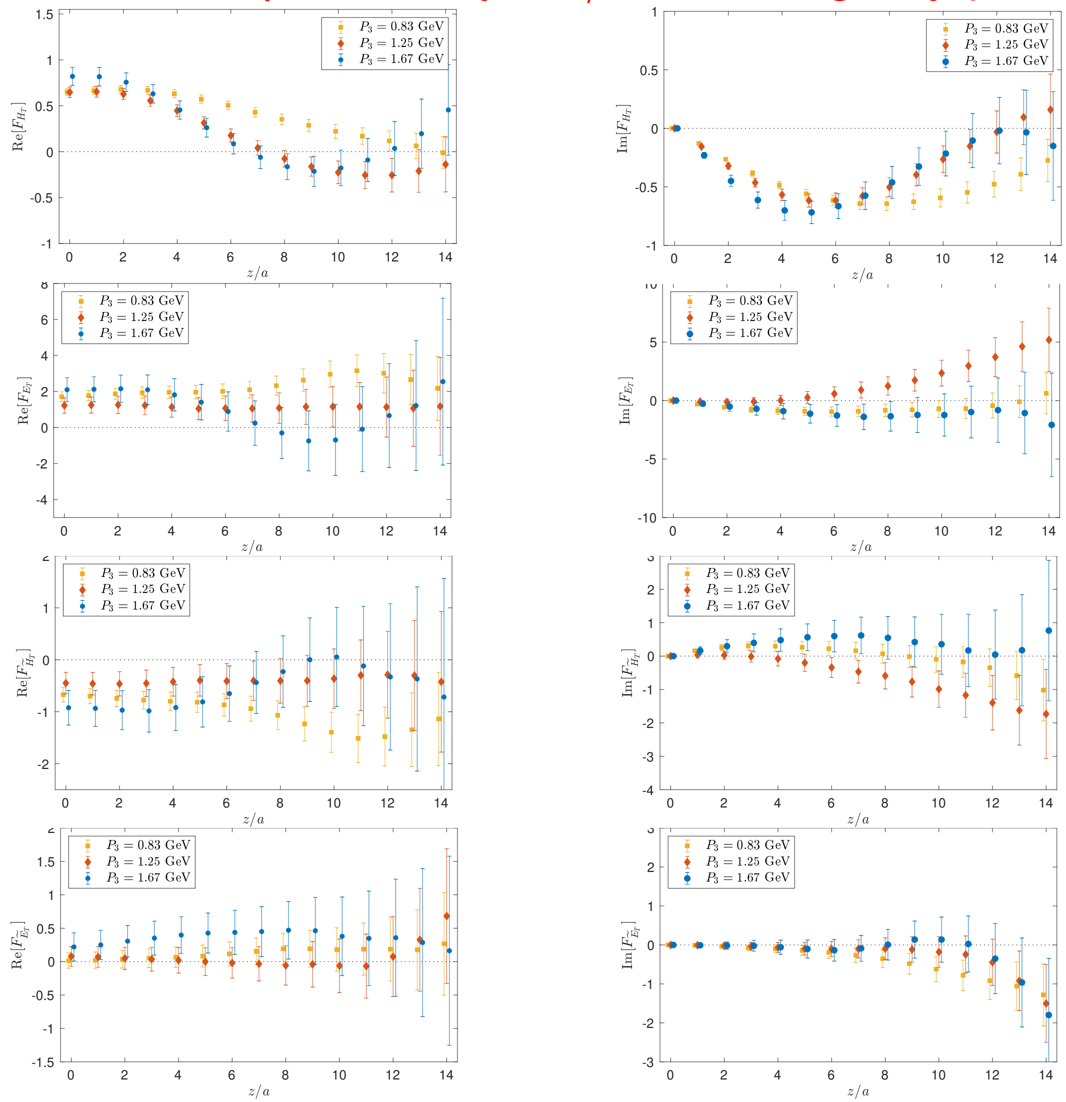
light-cone GPD

Real part $\xi = 1/3$ Imaginary part



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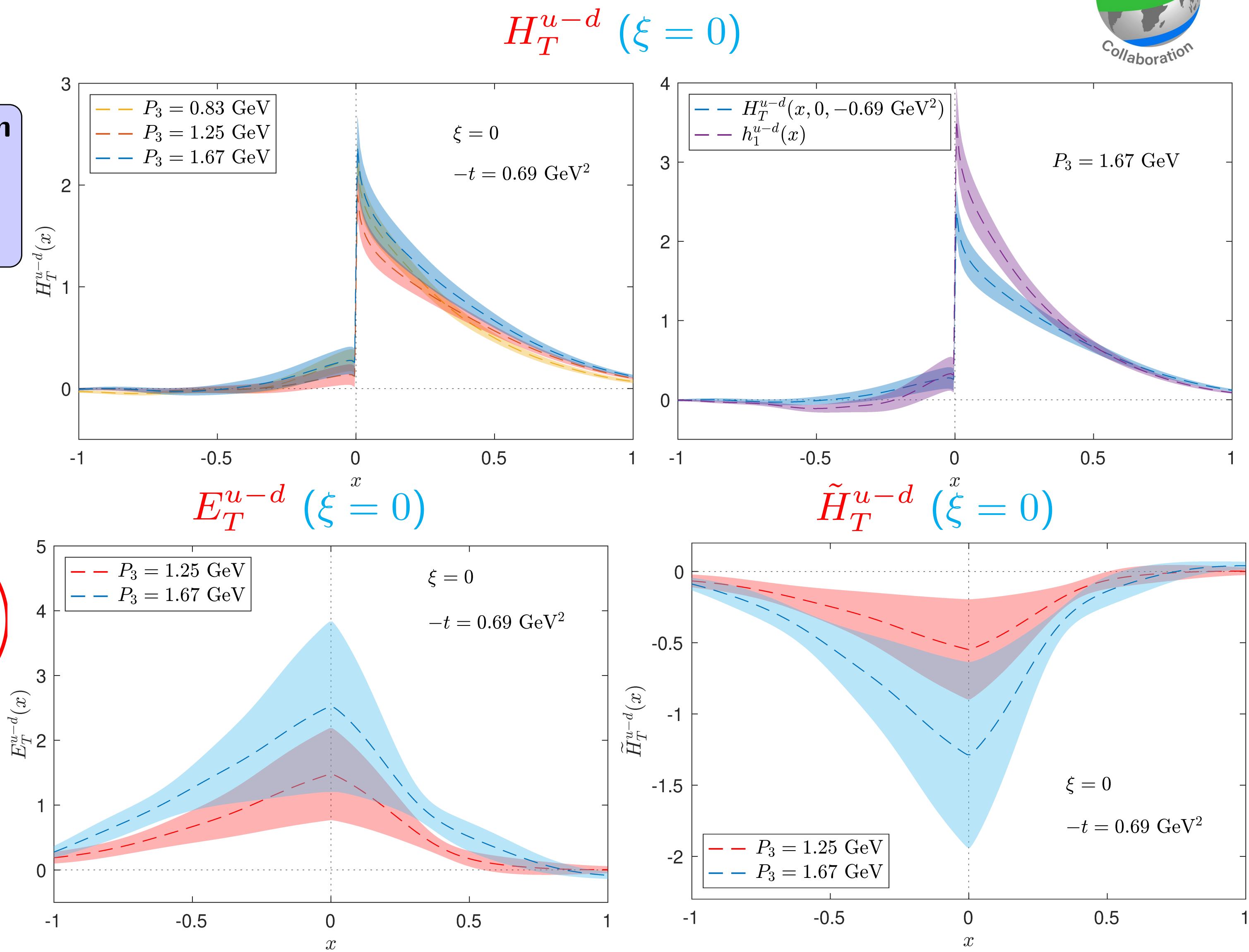

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light-cone GPD

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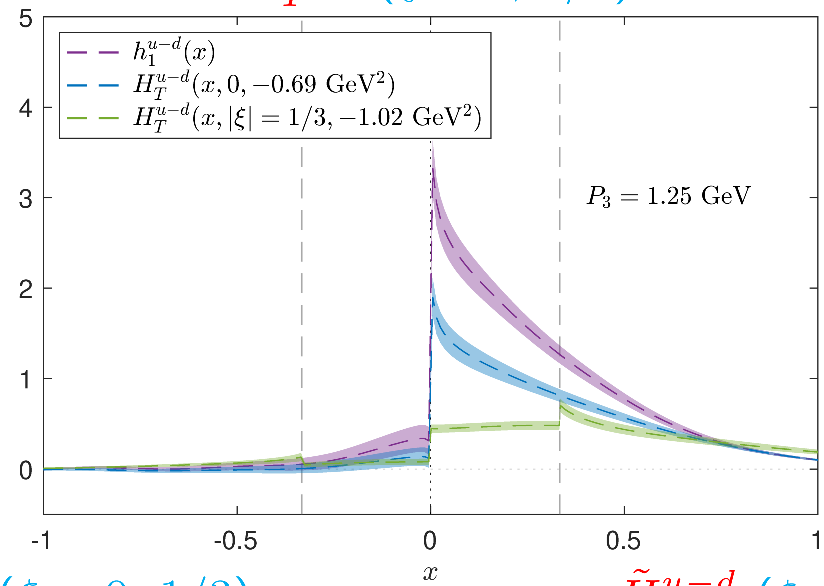
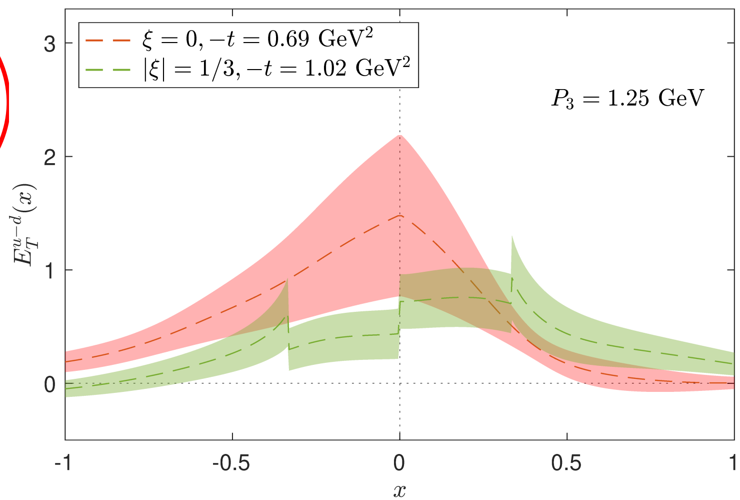
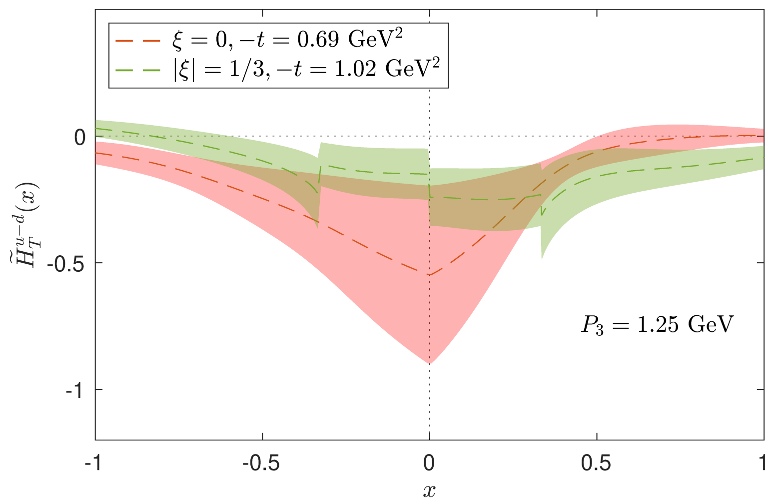
H_T^{u-d} $(\xi = 0,\ 1/3)$



E_T^{u-d} $(\xi = 0,\ 1/3)$



$\tilde{H}_T^{u-d}$ $(\xi = 0,\ 1/3)$



Transversity GPDs

ETMC, Phys. Rev. D105 (2022) 034501

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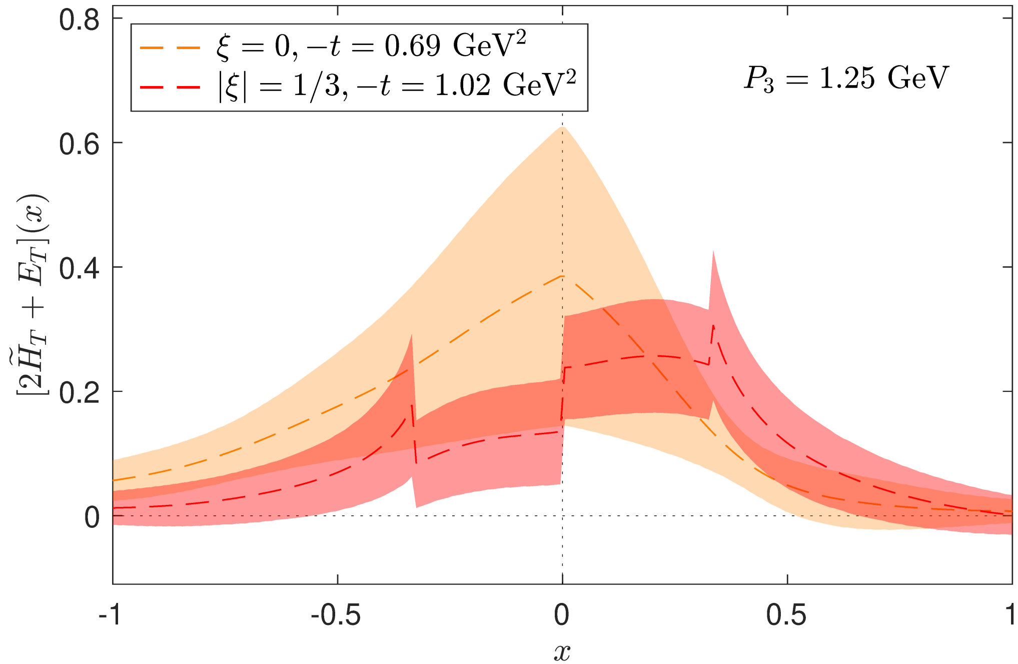
↓

light-cone GPD

More fundamental quantity: $E_T + 2\tilde{H}_T$

– related to the transverse spin structure of the proton
– physically interpreted as lateral deformation in the distribution of transversely polarized quarks in an unpolarized proton
– lowest Mellin moment in the forward limit: transverse spin-flavor dipole moment in an unpolarized target (k_T)
– second moment related to the transverse-spin quark angular momentum in an unpolarized proton



Comparison of different types of PDFs/GPDs

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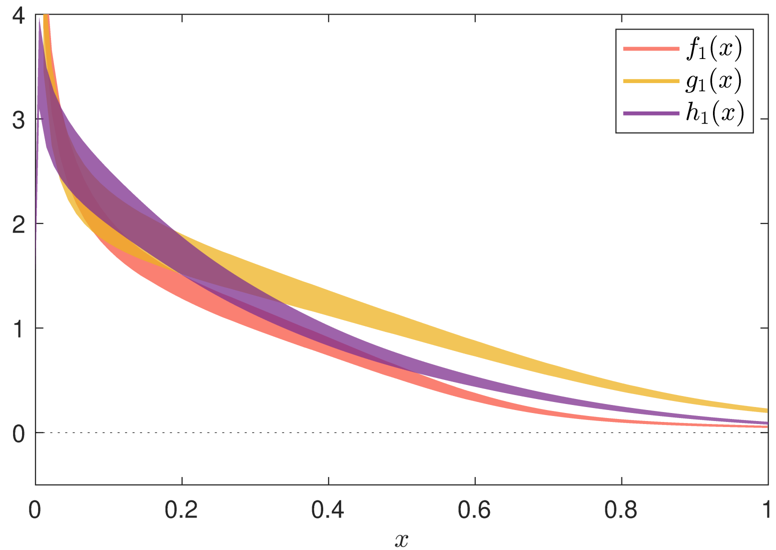
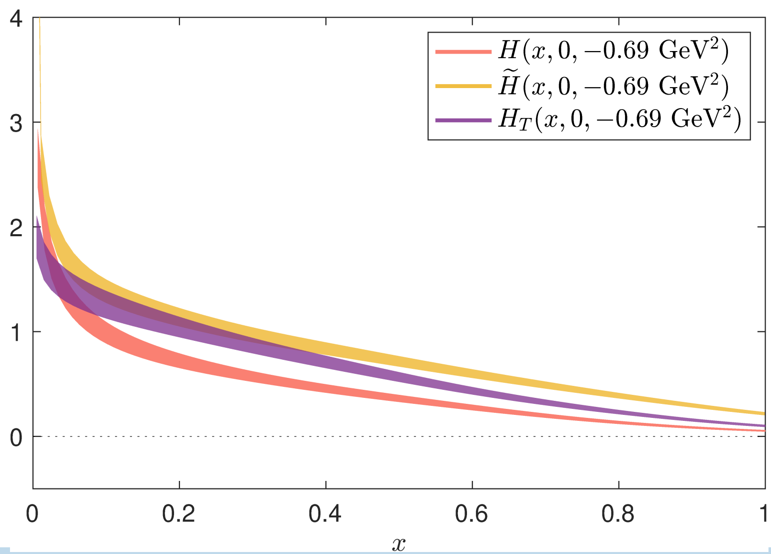
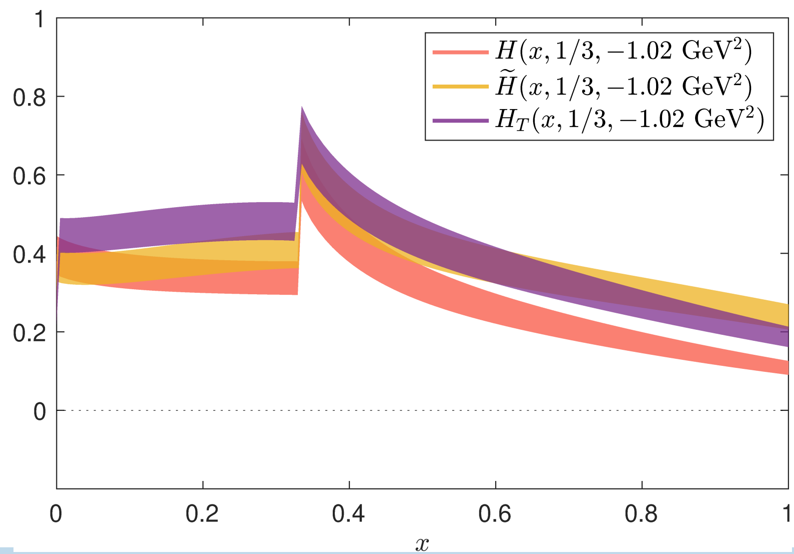






Moments of transversity GPDs

$n = 0$ Mellin moments:

$$\begin{aligned}
\int_{-1}^{1} dx\, H_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, H_{Tq}(x,\xi,t,P_3) = A_{T10}(t)\,, \\
\int_{-1}^{1} dx\, E_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, E_{Tq}(x,\xi,t,P_3) = B_{T10}(t)\,, \\
\int_{-1}^{1} dx\, \widetilde{H}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{H}_{Tq}(x,\xi,t,P_3) = \widetilde{A}_{T10}(t)\,, \\
\int_{-1}^{1} dx\, \widetilde{E}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{E}_{Tq}(x,\xi,t,P_3) = 0\,,
\end{aligned} \tag{1}$$

- lowest moments of GPDs skewness-independent,
- lowest moments of quasi-GPDs boost-independent.

$n = 1$ Mellin moments (related to GFF of one-derivative tensor operator):

$$\begin{aligned}
\int_{-1}^{1} dx\, x\, H_T(x,\xi,t) &= A_{T20}(t)\,, \\
\int_{-1}^{1} dx\, x\, E_T(x,\xi,t) &= B_{T20}(t)\,, \\
\int_{-1}^{1} dx\, x\, \widetilde{H}_T(x,\xi,t) &= \widetilde{A}_{T20}(t)\,, \qquad (3) \\
\int_{-1}^{1} dx\, x\, \widetilde{E}_T(x,\xi,t) &= 2\xi \widetilde{B}_{T21}(t)\,, \qquad (2)
\end{aligned}$$

- skewness-dependence only in for $\widetilde{E}_T$ (only ξ-odd GPD).

Moments of transversity GPDs

Moments of	$H_T(x, \xi = 0, t = -0.69\,\text{GeV}^2)$			$H_T(x, \xi = 1/3, t = -1.02\,\text{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
H_{Tq}	0.65(4)	0.64(6)	0.81(10)	0.49(5)
H_T	0.69(4)	0.67(6)	0.84(10)	0.45(4)
xH_T	0.20(2)	0.21(2)	0.24(3)	0.15(2)
$A_{T10}\ (z = 0)$	0.65(4)	0.65(6)	0.82(10)	0.49(5)

Mellin moments P_3-independent, preserved by matching, suppressed with increasing $-t$.

Moments of	$E_T(x, \xi = 0, t = -0.69\,\text{GeV}^2)$			$H_T(x, \xi = 1/3, t = -1.02\,\text{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
E_{Tq}		1.20(42)	2.05(65)	0.67(19)
E_T		1.15(43)	2.10(67)	0.73(19)
xE_T		0.06(4)	0.13(5)	0.11(11)
$B_{T10}\ (z = 0)$	1.71(28)	1.22(43)	2.10(67)	0.68(19)

Moments of	$\widetilde{H}_T(x, \xi = 0, t = -0.69\,\text{GeV}^2)$			$\widetilde{H}_T(x, \xi = 1/3, t = -1.02\,\text{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
$\widetilde{H}_{Tq}$		-0.44(20)	-0.90(32)	-0.26(9)
$\widetilde{H}_T$		-0.42(21)	-0.92(33)	-0.27(9)
$x\widetilde{H}_T$		-0.17(8)	-0.30(10)	-0.05(5)
$\widetilde{A}_{T10}\ (z = 0)$	-0.67(14)	-0.45(21)	-0.92(33)	-0.24(8)

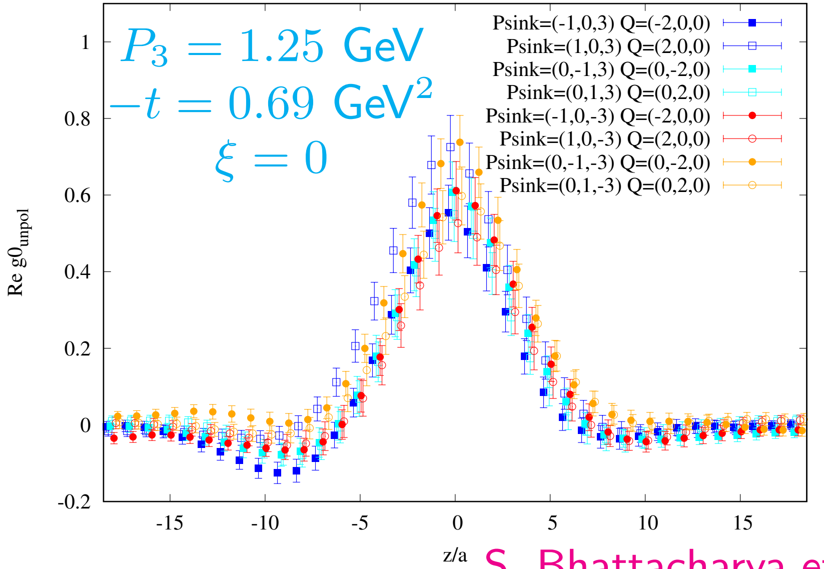
Similar conclusions (but very large errors).

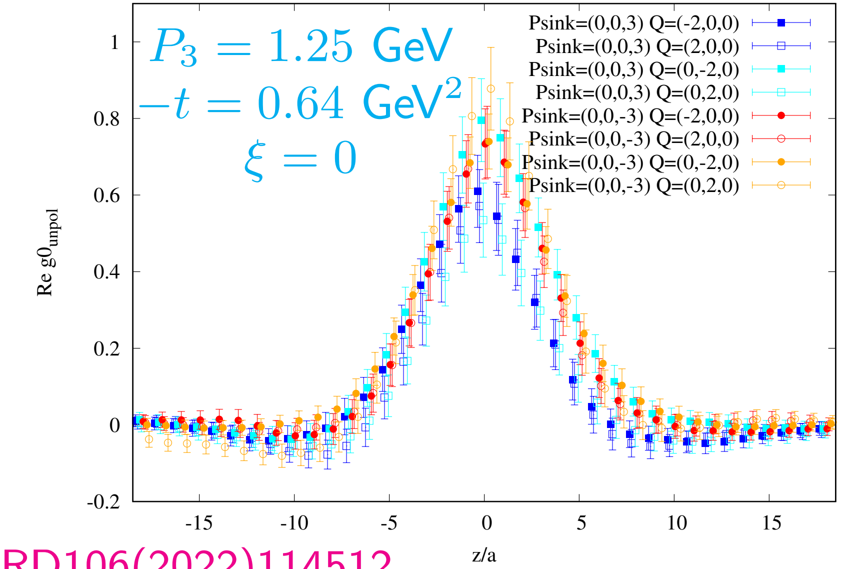
Bare matrix elements of $\Pi_0(\Gamma_0)$

symmetric frame

non-symmetric frame

Re





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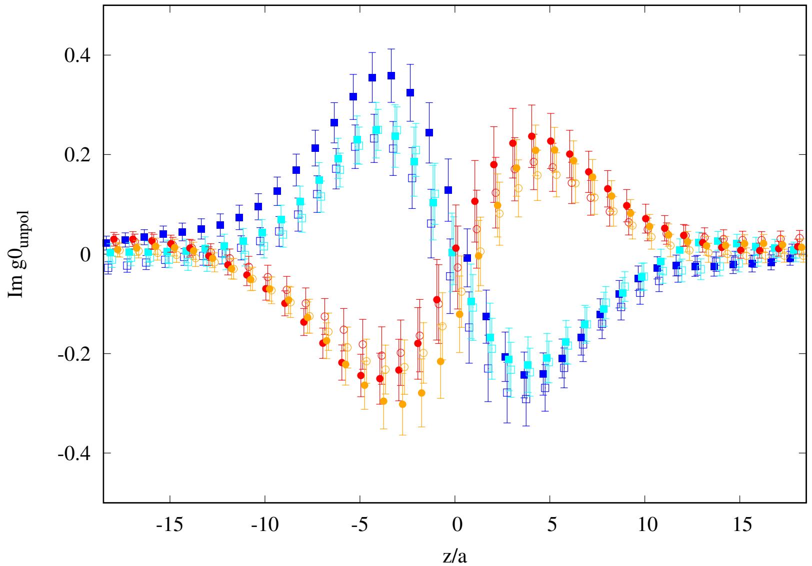
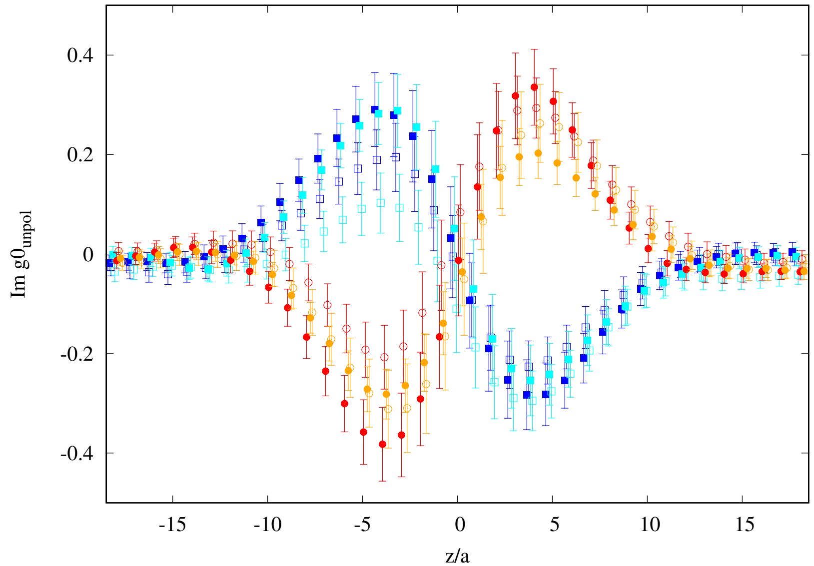
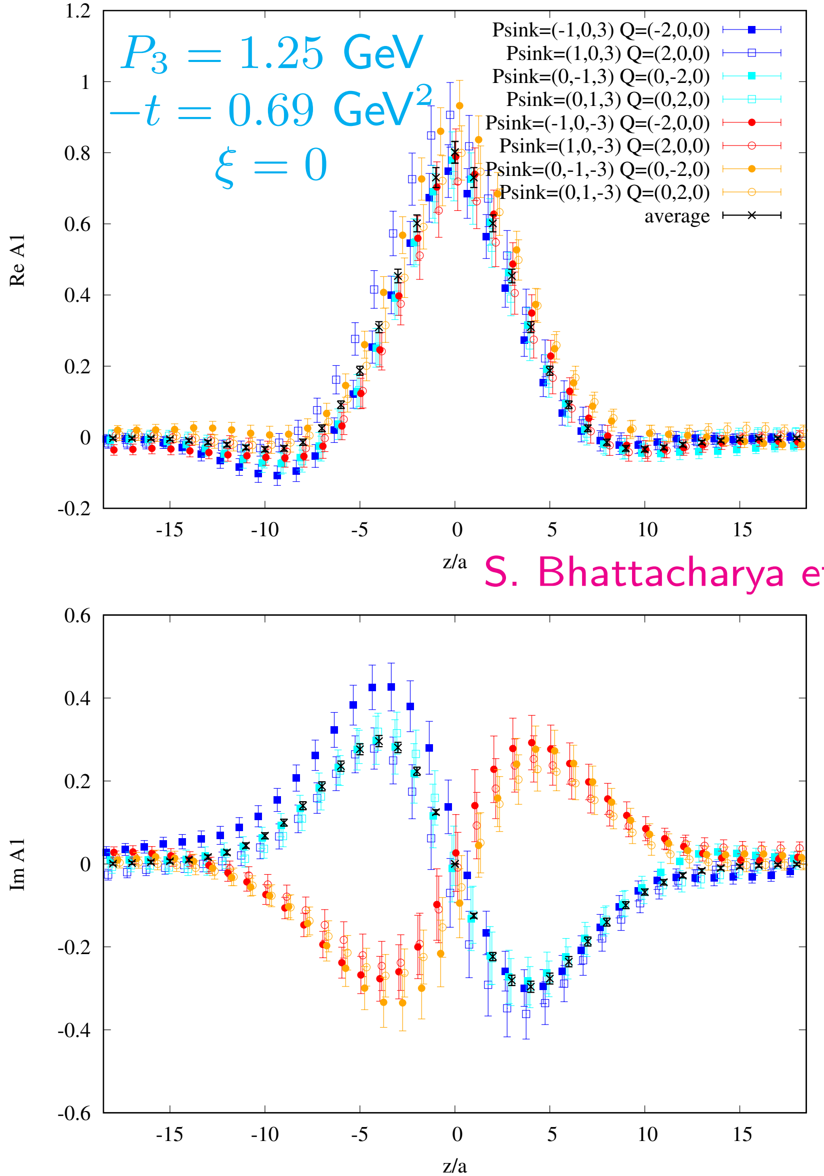
Im



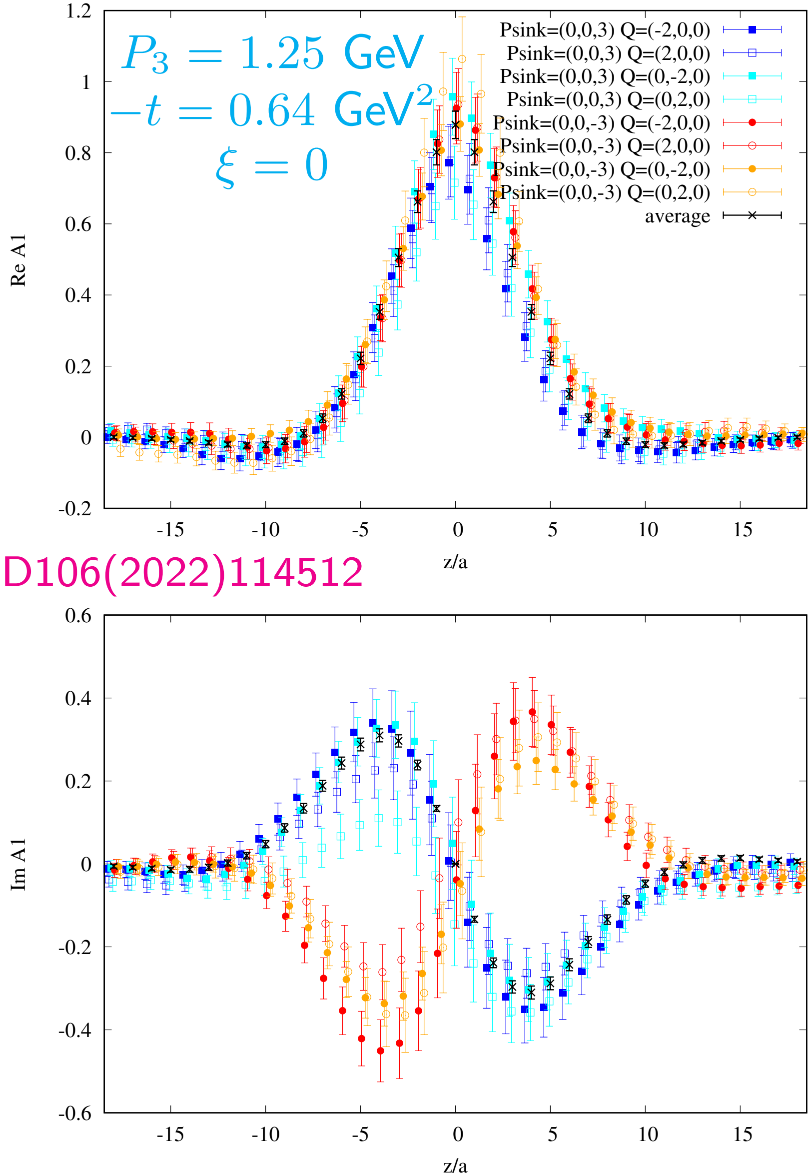


Example amplitude A_1

symmetric frame



non-symmetric frame



Re

Im

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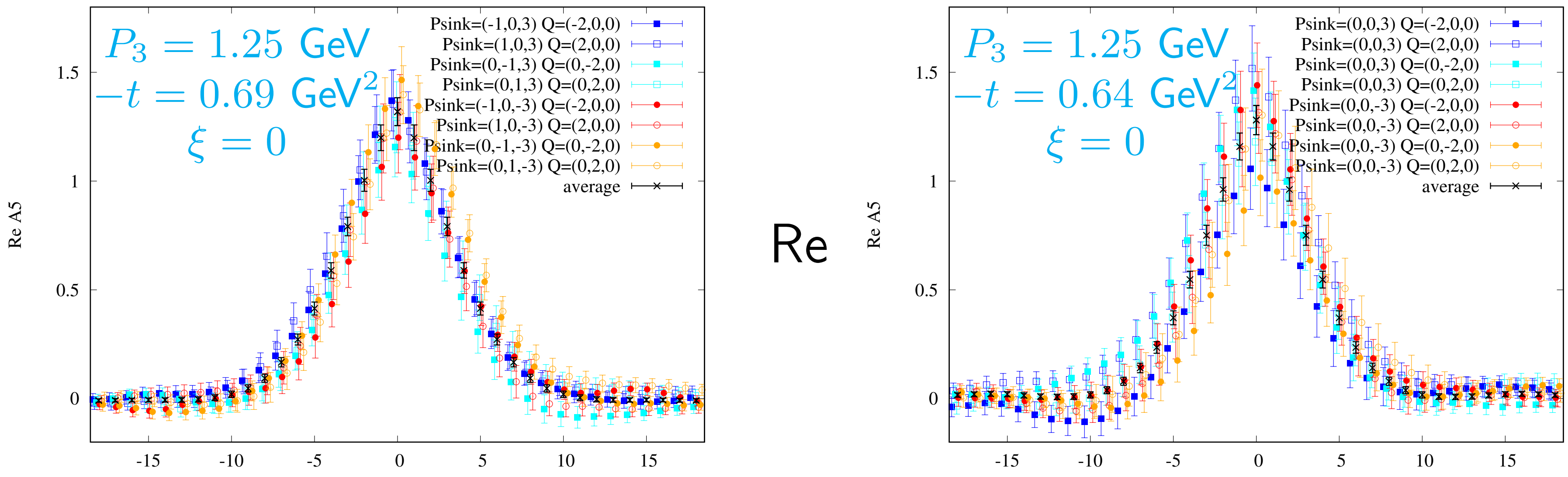
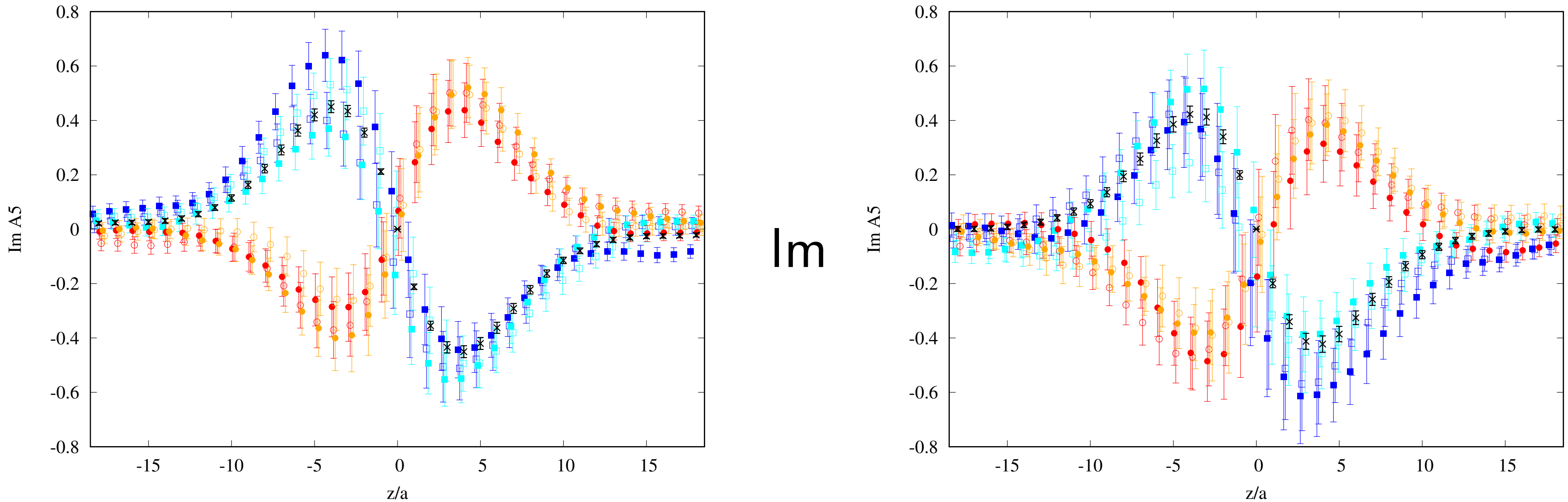
Example amplitude A_5

symmetric frame — non-symmetric frame

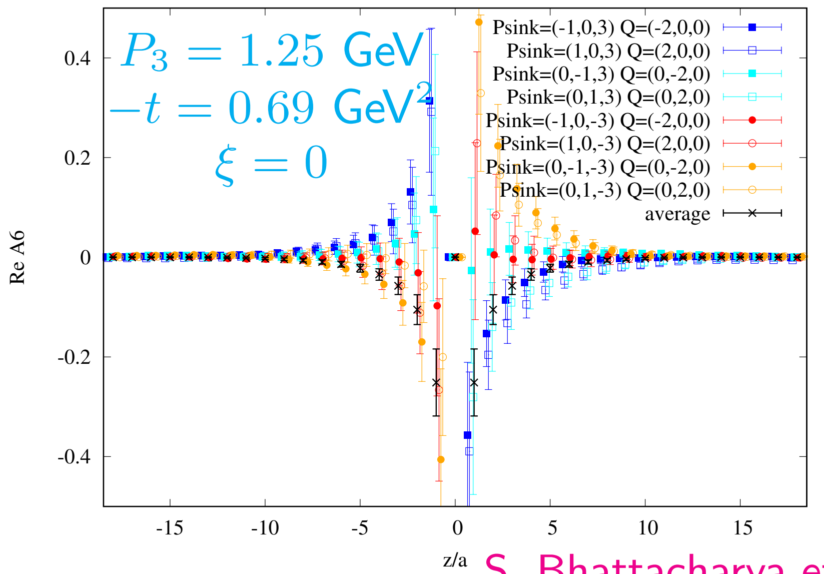


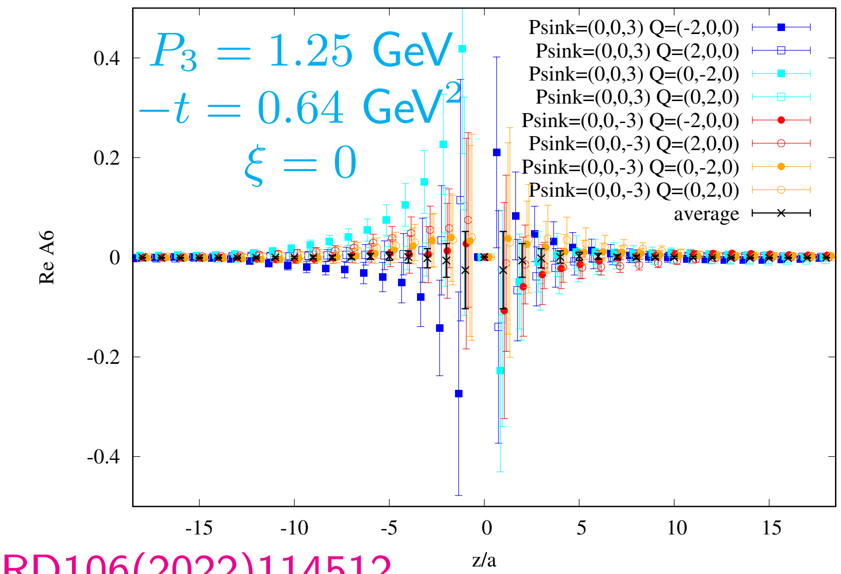
z/a S. Bhattacharya et al., PRD106(2022)114512



symmetric frame

non-symmetric frame





Re

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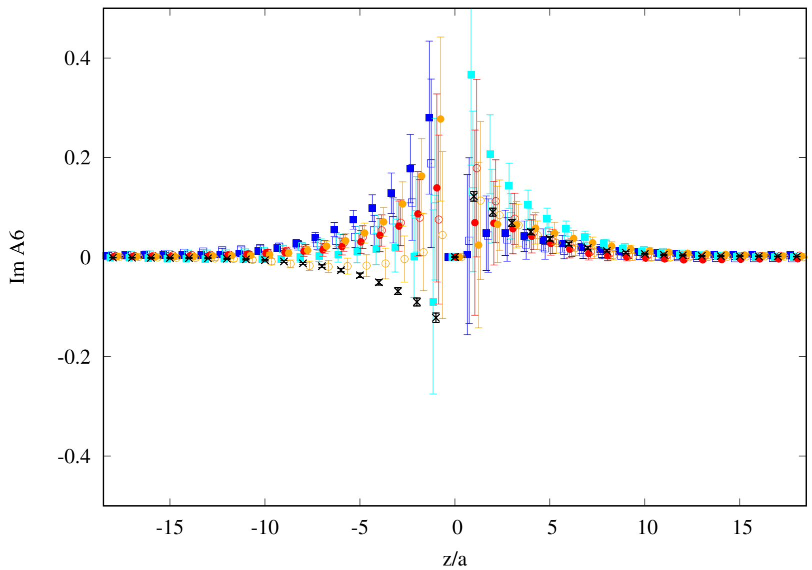
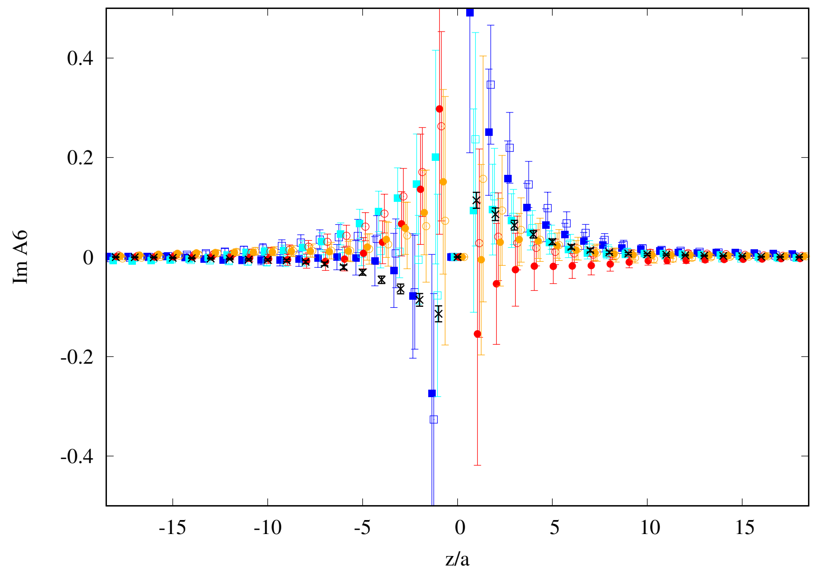


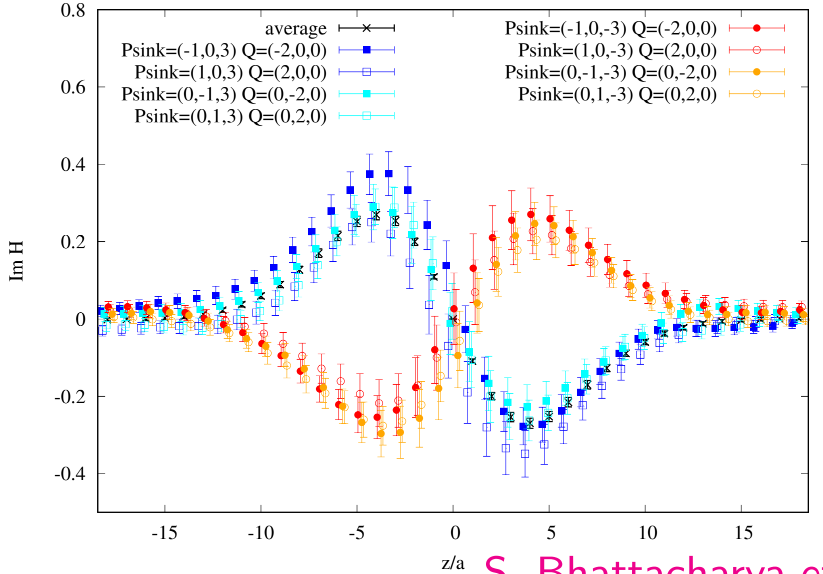


Im

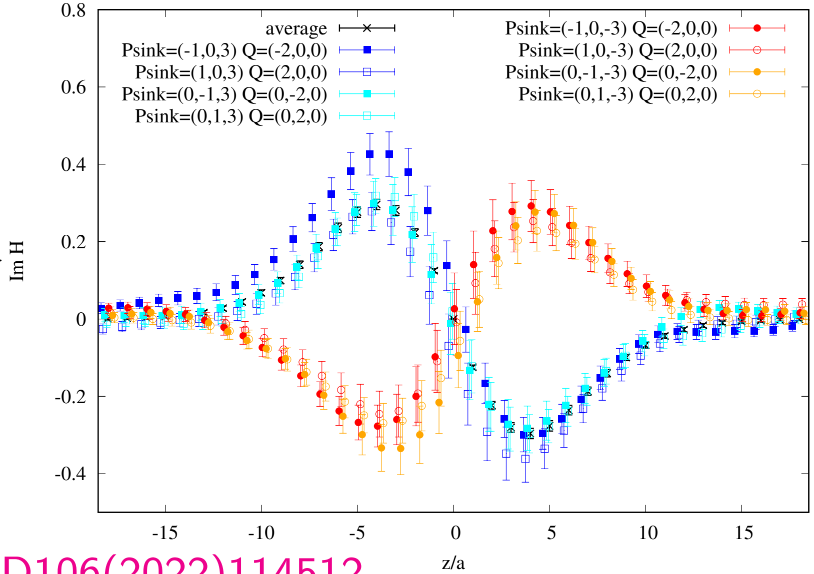
H and E GPDs – signal improvement

standard

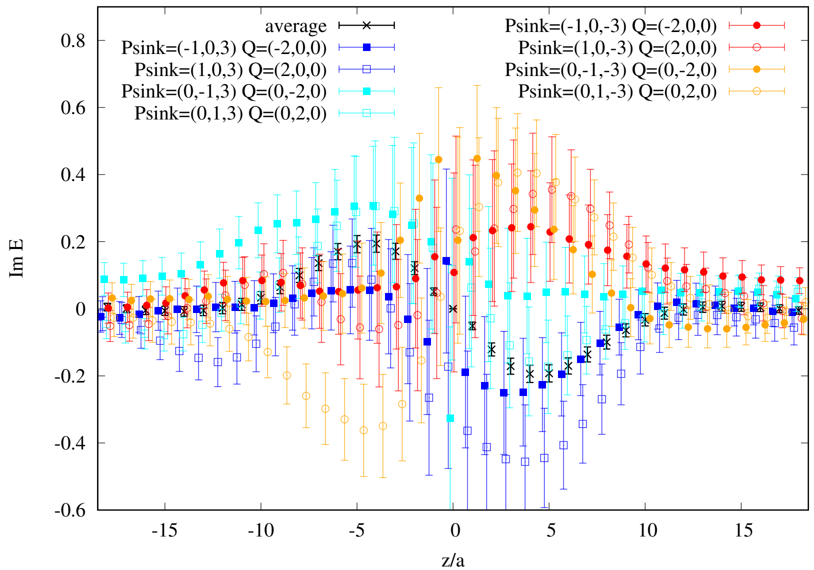
Lorentz-invariant



Im H



S. Bhattacharya et al., PRD106(2022)114512



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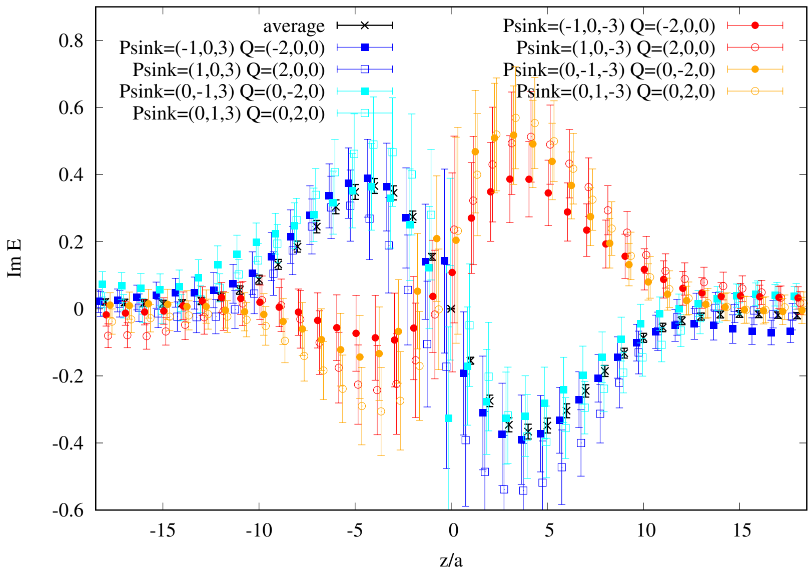
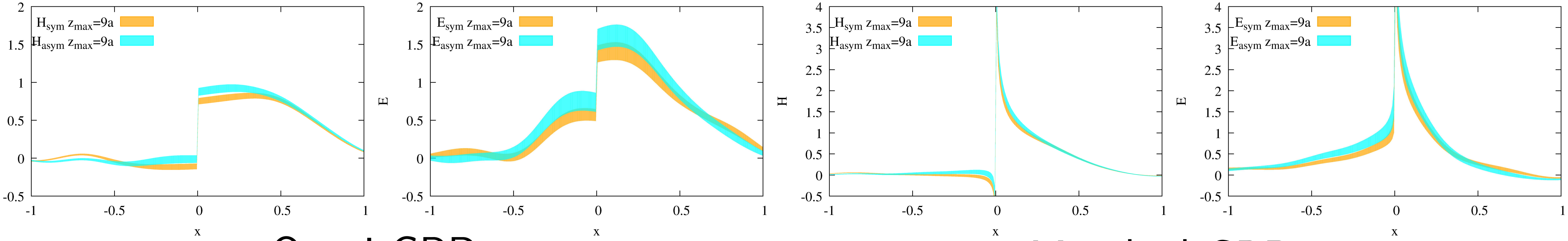


STANDARD DEFINITION



Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

H-GPD E-GPD H-GPD E-GPD

LORENTZ-INVARIANT DEFINITION