Particle-core coupling in deformed nuclei: odd-A and doubly even-A identical bands

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Fundamentals of Nuclear Models: Foundational Models
--David J. Rowe and JLW, World Scientific, 2010 [R&W]
+ second volume nearing completion

Identical bands 23 years ago:

recognized but not understood (and abandoned)

Annu. Rev. Nucl. Part. Sci. 1995. 45:485–541

IDENTICAL BANDS IN DEFORMED AND SUPERDEFORMED NUCLEI

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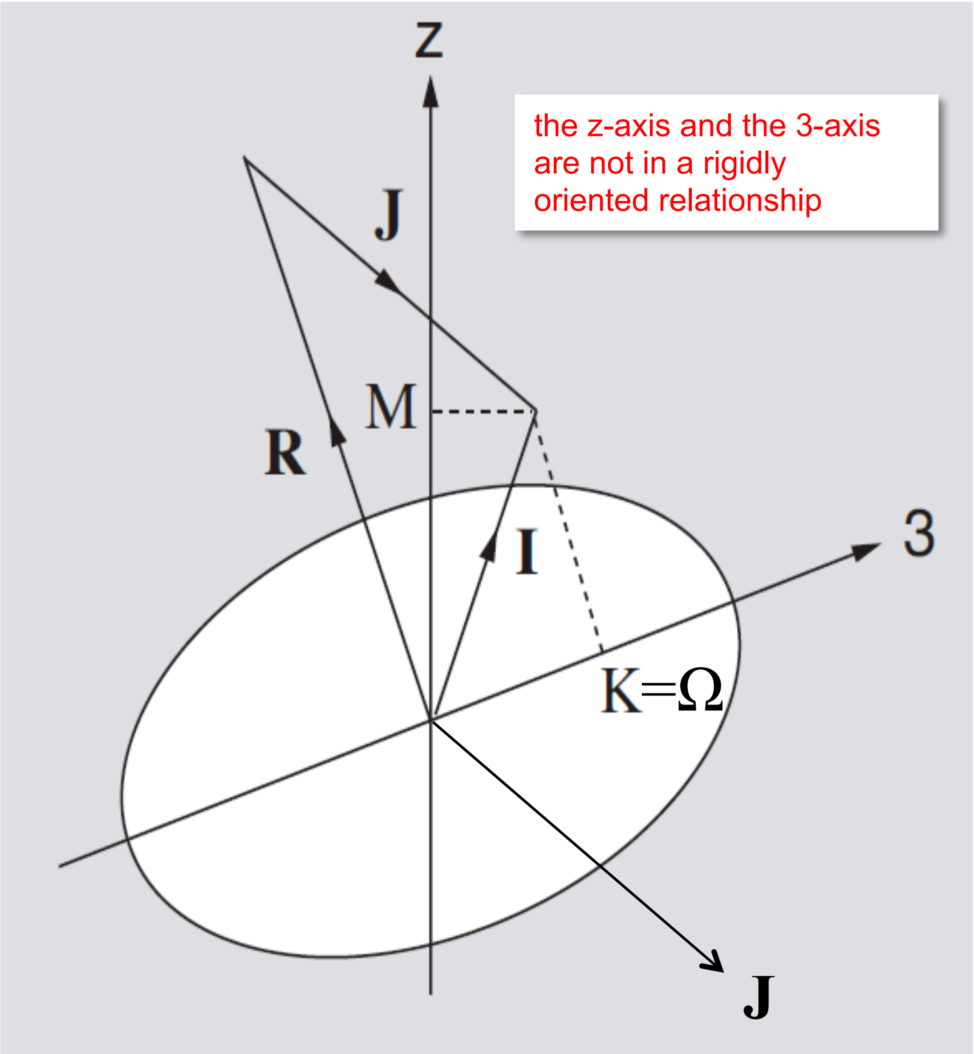
"In particular, the reduction of the BCS pairing correlations due to the blocking of one and two orbitals implies large changes (up to 30%) in the moments of inertia and cannot be reconciled with these [identical band] systematics."

Symmetric-top model:
quantum numbers



R: collective angular momentum

J: intrinsic spin

I: total spin / angular momentum

M: laboratory-frame,
z-component of I

K: body-frame (symmetry axis),
3-component of I; K = Ω

"Coriolis" interaction:

$$\mathbf{R}\cdot\mathbf{R} = (\mathbf{I} - \mathbf{J})\cdot(\mathbf{I} - \mathbf{J}) = \mathbf{I}\cdot\mathbf{I} - 2\,\mathbf{I}\cdot\mathbf{J} + \mathbf{J}\cdot\mathbf{J}$$

$$\hat{H}_{rot} = \frac{\hbar^2 \hat{\mathbf{R}}^2}{2\Im}$$

R&W Fig. 1.46

The key concept for modeling deformed nuclei: the symmetric-top + Nilsson model

$$\hat{H} := \frac{\hbar^2 \hat{\mathbf{R}}^2}{2\Im} + \hat{h},$$

"coupling" = add Hamiltonians
(wave functions will be direct products)

$$\hat{h} := \frac{\hat{p}^2}{2M} + \tfrac{1}{2} M \left[\omega_\perp^2 (\bar{x}^2 + \bar{y}^2) + \omega_z^2 \bar{z}^2\right] + D\hat{\mathbf{l}}^2 + \xi\, \hat{\mathbf{l}} \cdot \hat{\mathbf{s}}.$$

$$\hat{\mathbf{I}} := \hat{\mathbf{R}} + \hat{\mathbf{j}}$$

"coupling" = add spins/angular momenta

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{I}}^2}{2\Im} + \hat{h} + \frac{\hbar^2 \hat{\mathbf{j}}^2}{2\Im} - \frac{\hbar^2}{\Im} \hat{\mathbf{I}} \cdot \hat{\mathbf{j}}$$

Nilsson model plus rotations

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{I}}^2}{2\Im} + h_{\text{Nilsson}} + \frac{\hbar^2 \hat{\mathbf{j}}^2}{2\Im} - \frac{\hbar^2}{\Im}\hat{\mathbf{I}} \cdot \hat{\mathbf{j}}.$$

"Coriolis" or rotational-particle coupling

$$\langle \vec{I} \cdot \vec{j} \rangle = \langle \frac{1}{2}(\hat{I}_+ \hat{j}_- + \hat{I}_- \hat{j}_+) + \hat{I}_z \hat{j}_z \rangle$$

$I_+ := I_x + iI_y$, $I_- = I_x - iI_y$
$j_+ := j_x + ij_y$, $j_- = j_x - ij_y$

$$|KIM\rangle + \varepsilon(-1)^{I+K} |-K, IM\rangle$$

ε = +1 reflection symmetric
ε = -1 reflection asymmetric

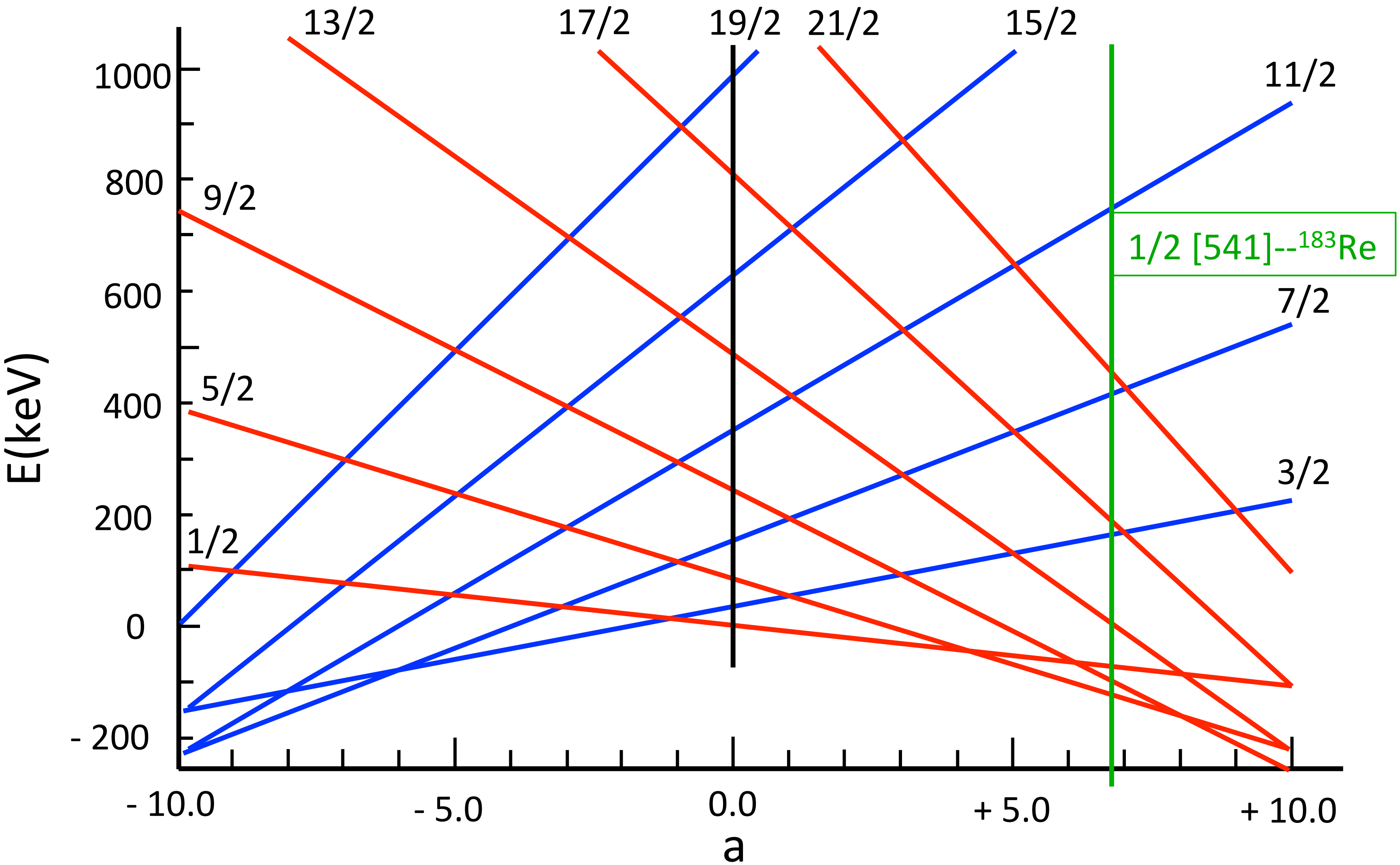
$$E_I = E_0 + A[I(I+1) + (-1)^{I+1/2}(I + 1/2)\, a\, \delta_{K,1/2}]$$

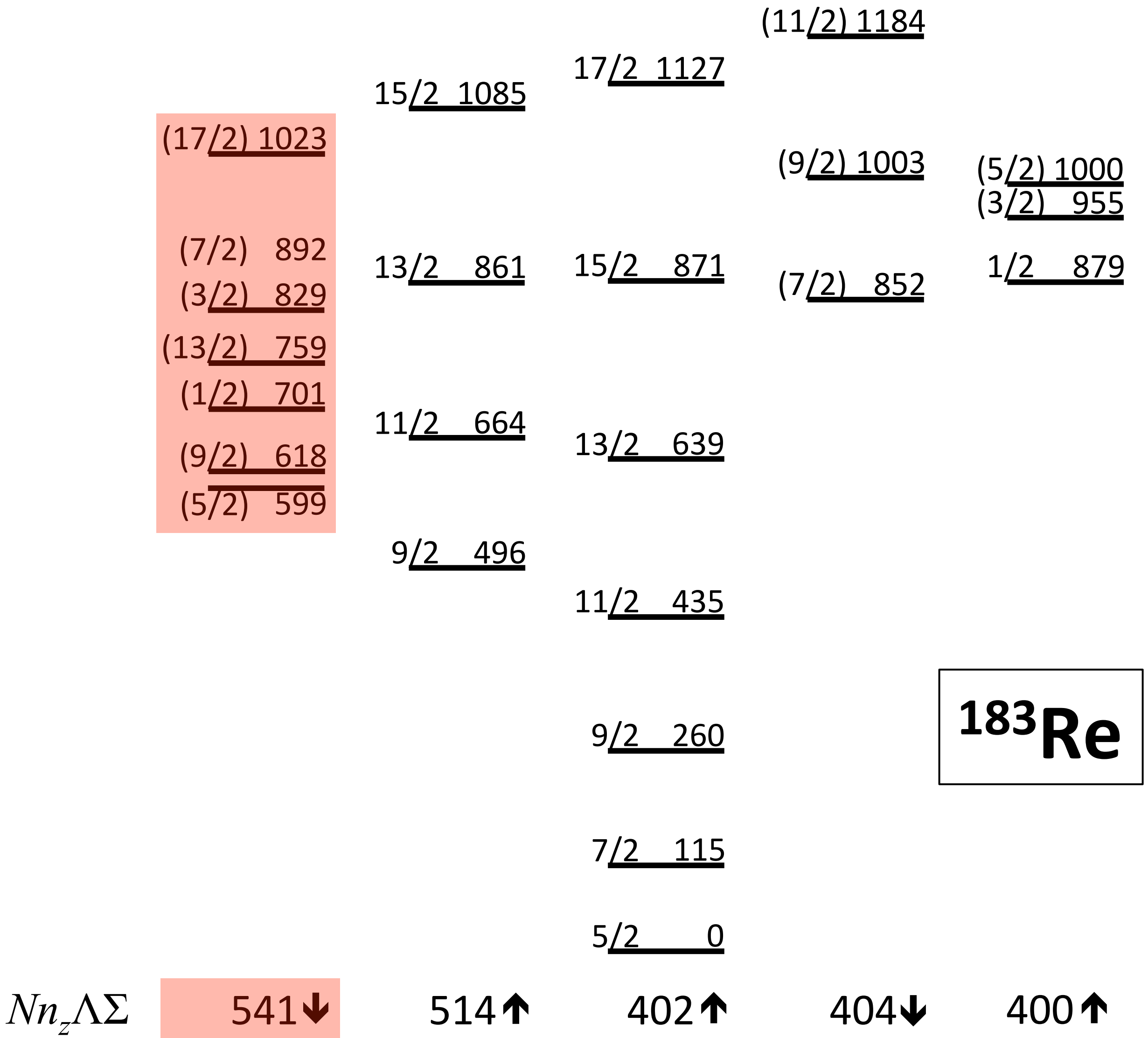
$\delta_{K,1/2} = 1,\ K = 1/2$
$\delta_{K,1/2} = 0$ otherwise

Quantum mechanics of particle-rotor coupling: K = ½ bands and "Coriolis" decoupling

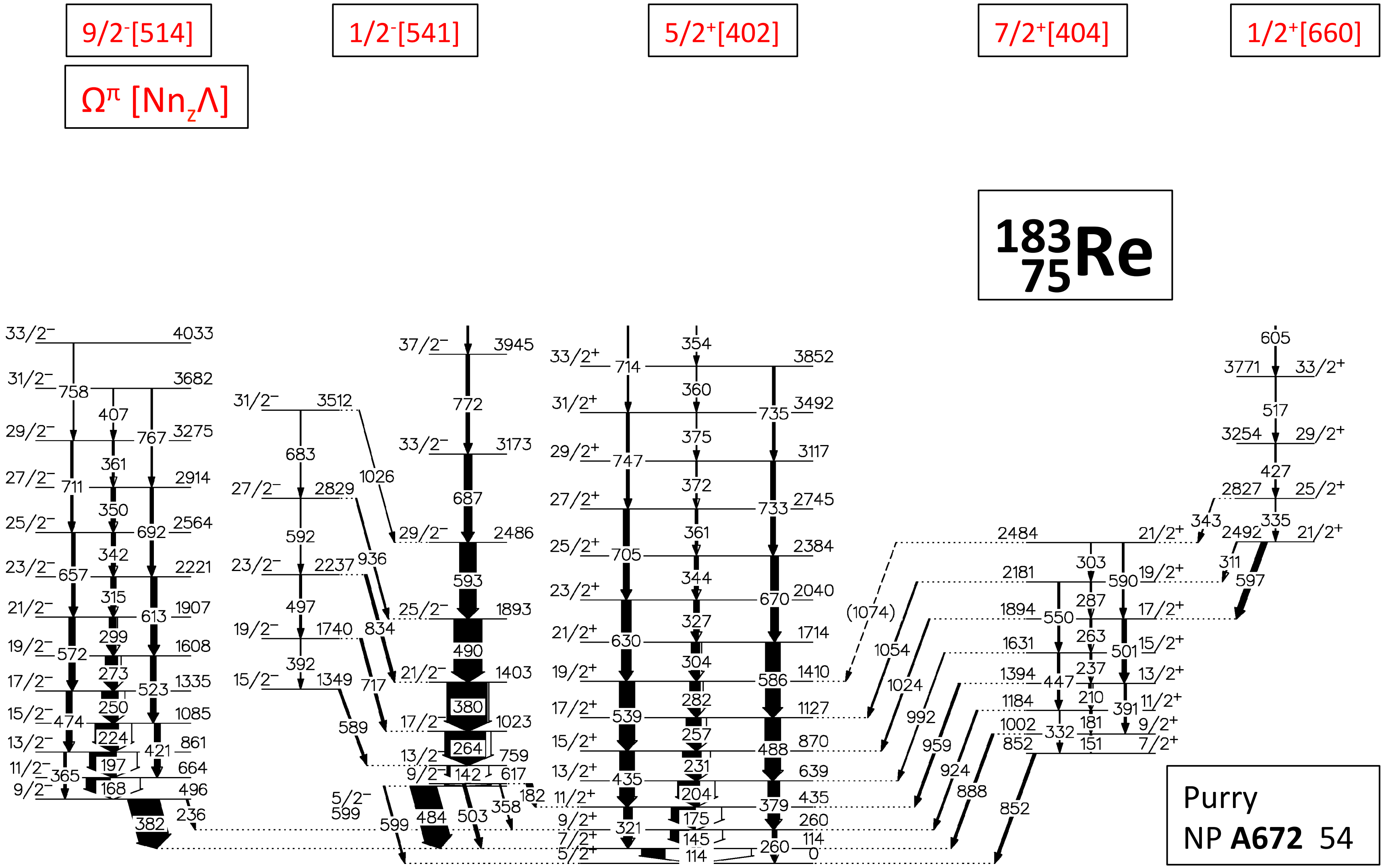
$$E_I = E_0 + A[I(I+1) + (-1)^{I+1/2}(I + 1/2)\, a\, \delta_{K,1/2}]$$

$A = 10.0$ keV

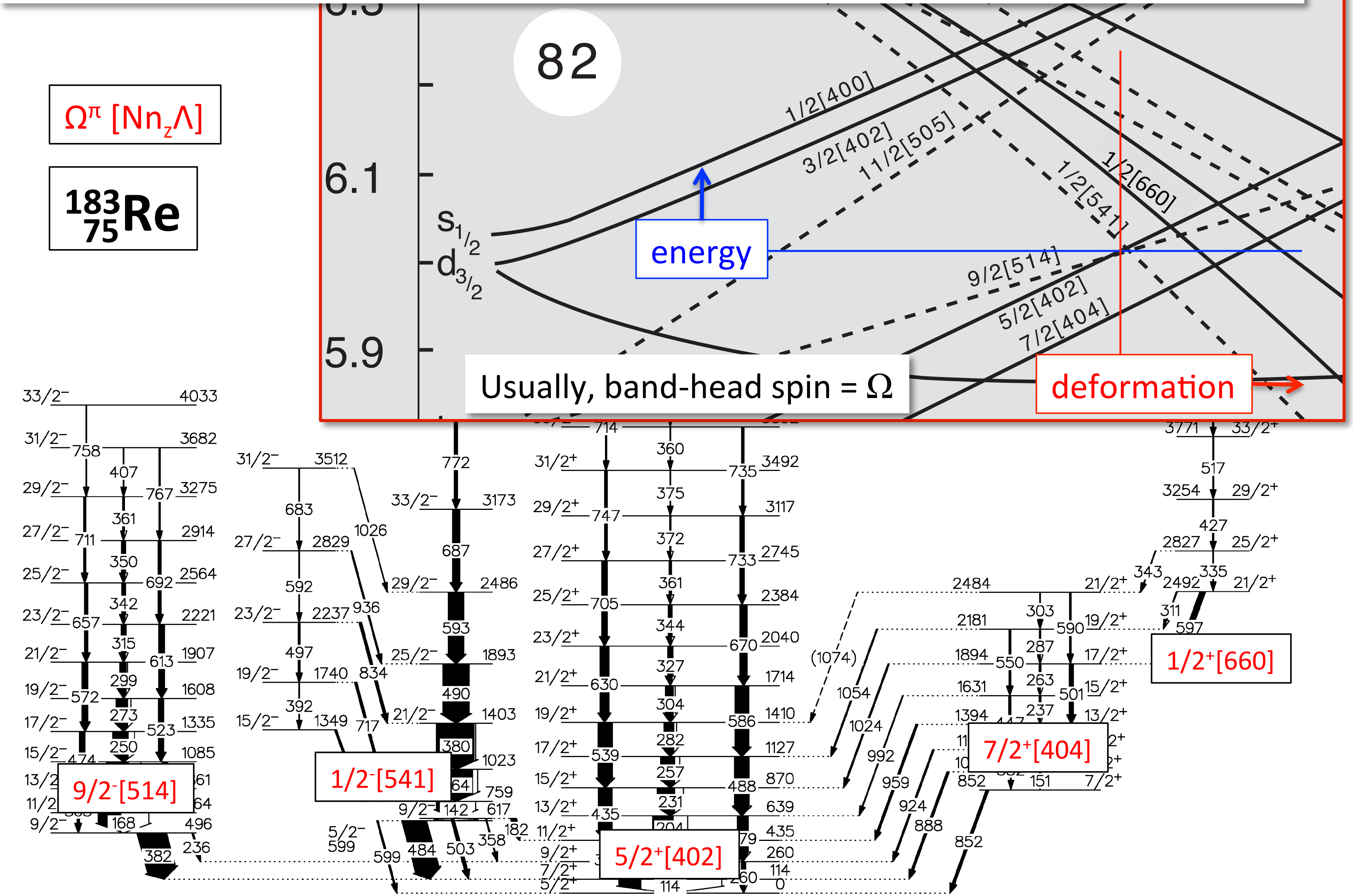


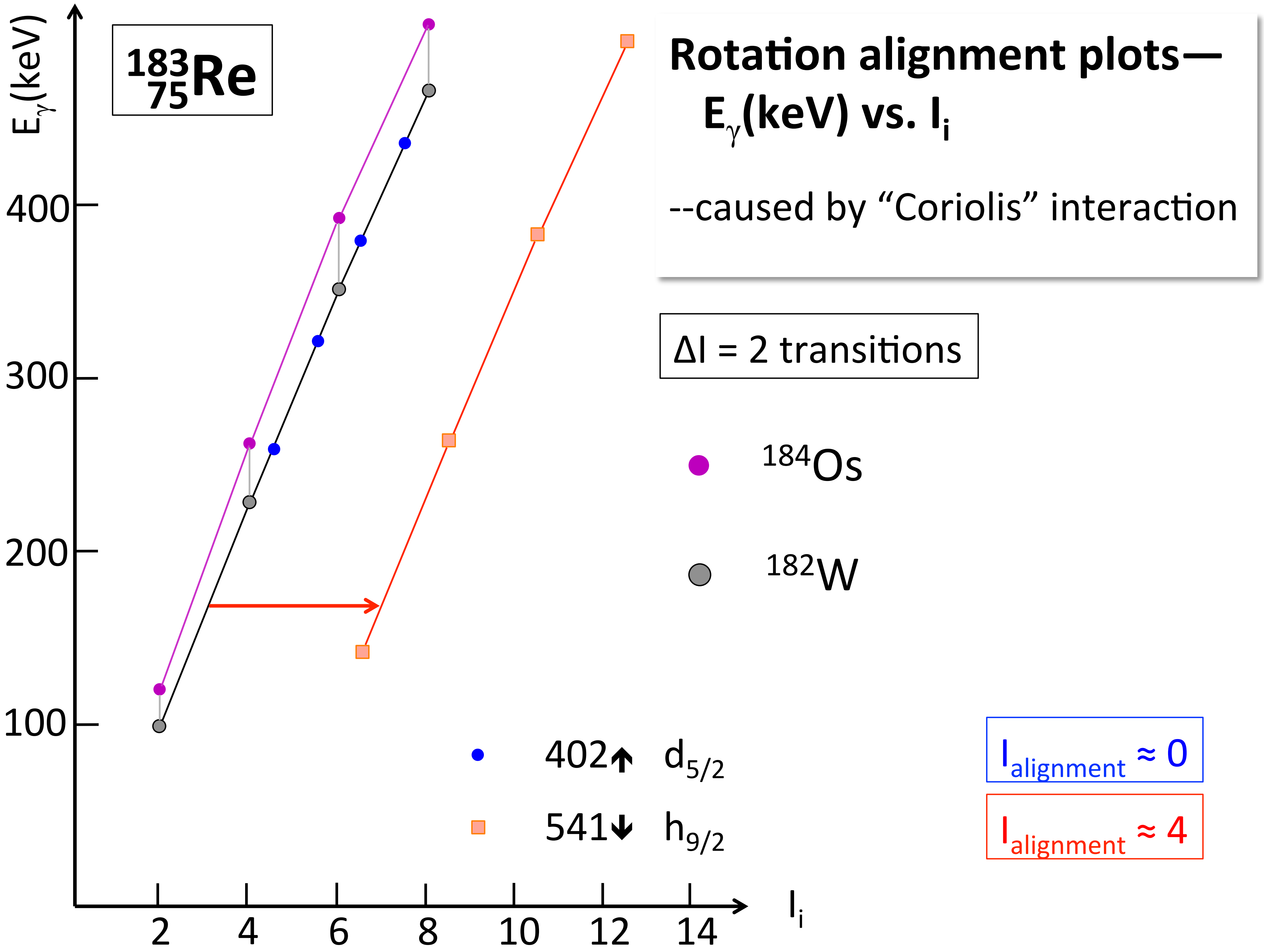

(17/2) 1023
(7/2) 892
(3/2) 829
(13/2) 759
(1/2) 701
(9/2) 618
(5/2) 599
15/2 1085
13/2 861
11/2 664
9/2 496
17/2 1127
15/2 871
13/2 639
11/2 435
9/2 260
7/2 115
5/2 0
(11/2) 1184
(9/2) 1003
(7/2) 852
(5/2) 1000
(3/2) 955
1/2 879
183Re
NnzΛΣ
541↓
514↑
402↑
404↓
400↑

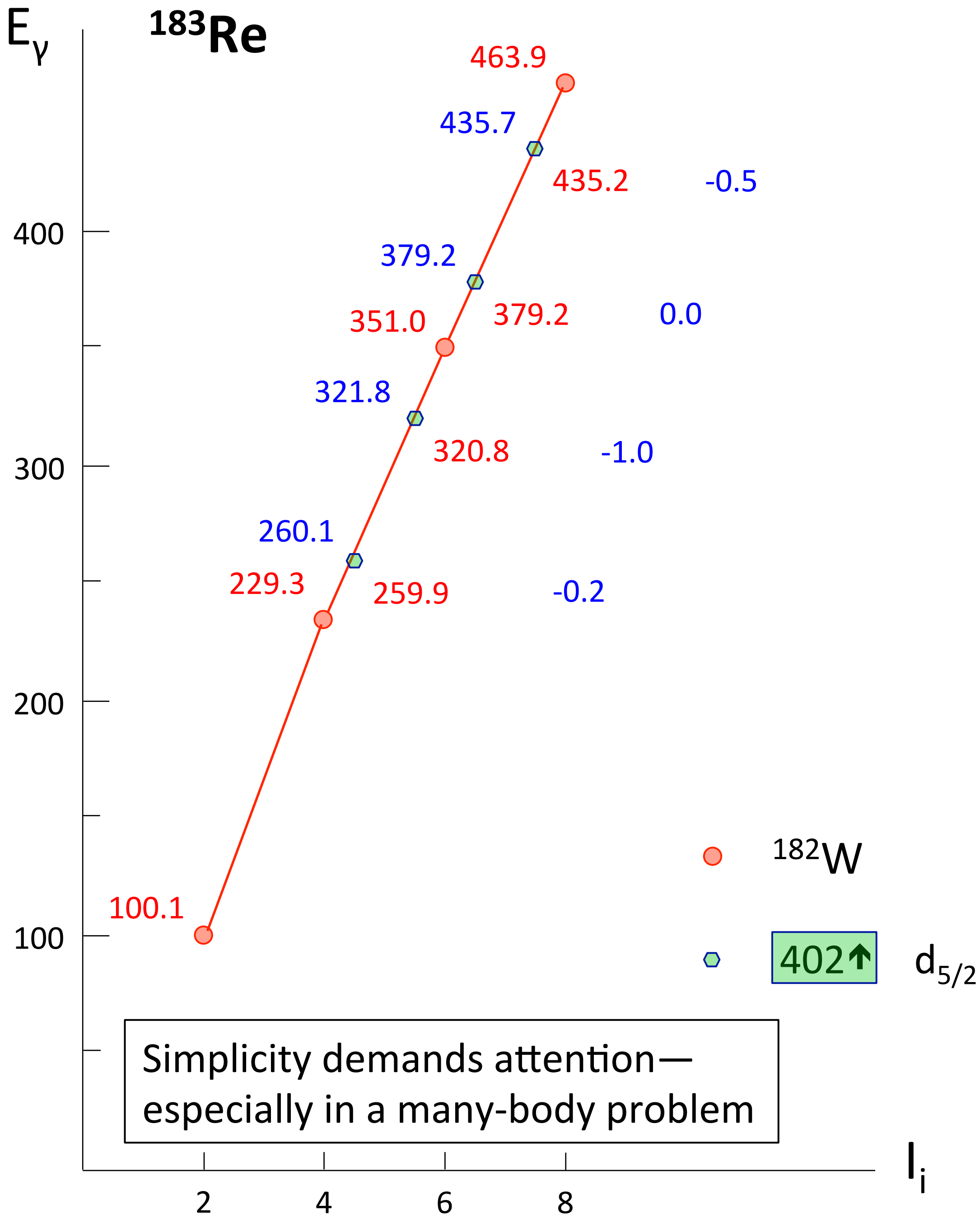
“Strong coupling” of single nucleon to deformed core: Nilsson model plus symmetric-top model



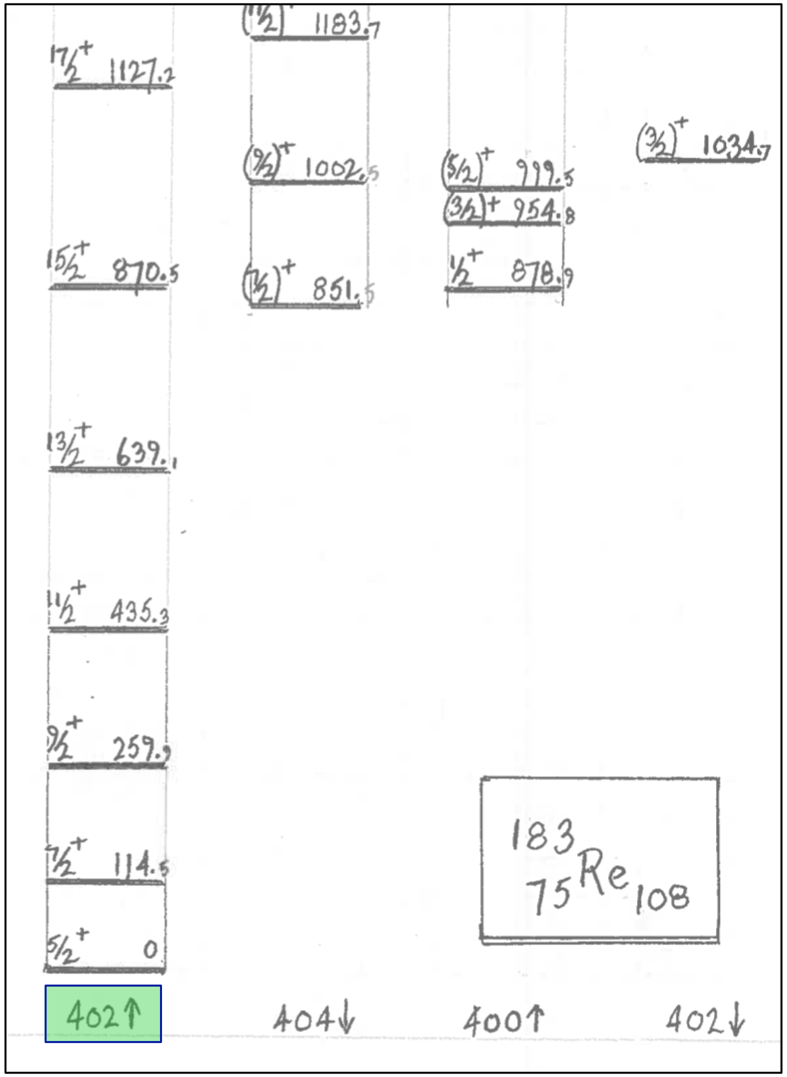
“Strong coupling” of single nucleon to deformed core: Nilsson model plus symmetric-top model



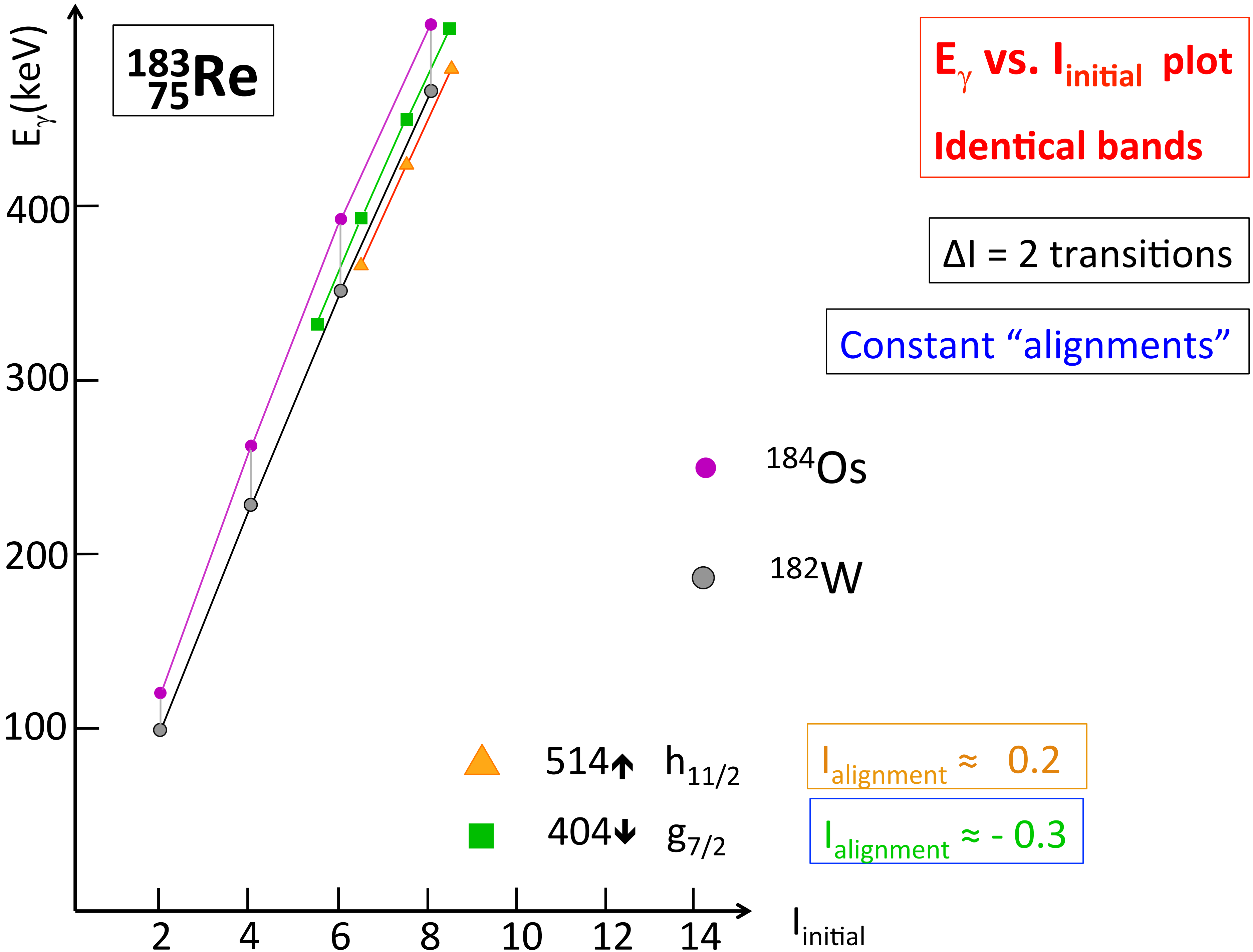

Rotation alignment plots—
Eγ(keV) vs. Ii
--caused by "Coriolis" interaction
183 75 Re
Eγ(keV)
400
300
200
100
2
4
6
8
10
12
14
Ii
ΔI = 2 transitions
184Os
182W
402↑ d5/2
541↓ h9/2
Ialignment ≈ 0
Ialignment ≈ 4



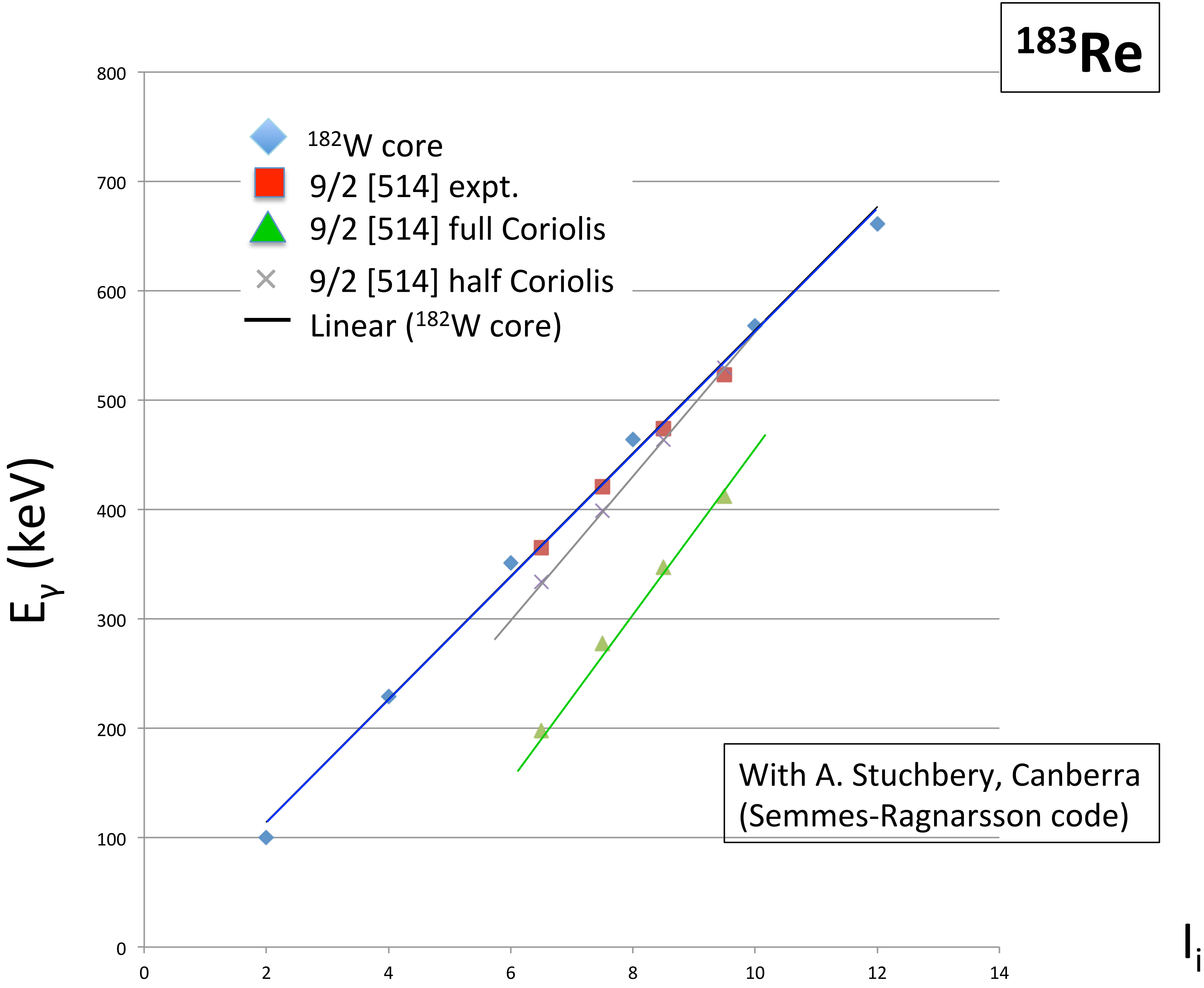
E_γ
^{183}Re
463.9
435.7
435.2
-0.5
400
379.2
351.0
379.2
0.0
321.8
320.8
-1.0
300
260.1
229.3
259.9
-0.2
200
^{182}W
100.1
100
402↑
$d_{5/2}$
Simplicity demands attention—
especially in a many-body problem
I_i
2
4
6
8



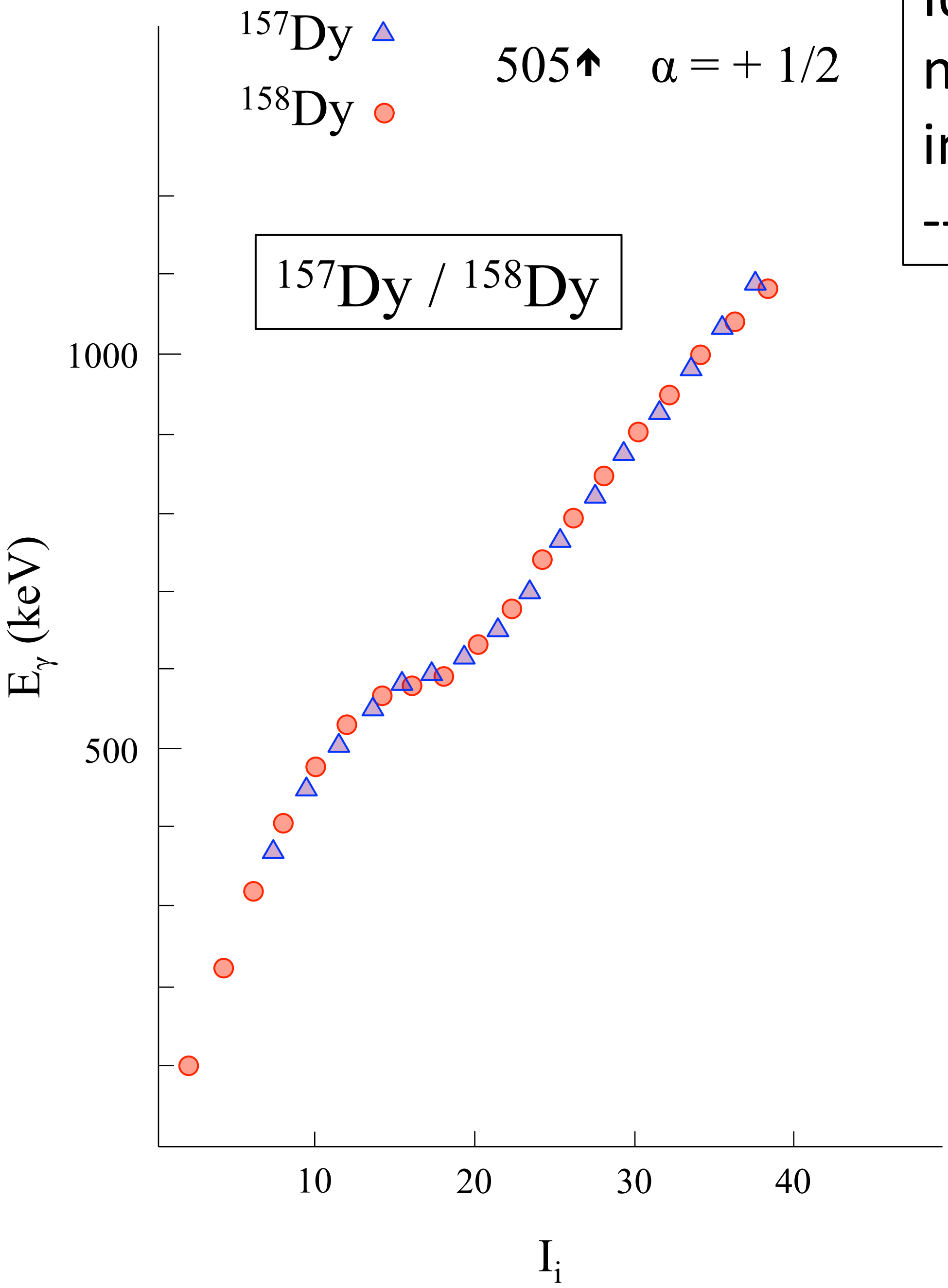
1183.7
17/2+ 1127.2
(9/2)+ 1002.5
(5/2)+ 999.5
(3/2)+ 954.8
(3/2)+ 1034.7
15/2+ 870.5
(7/2)+ 851.5
1/2+ 878.9
13/2+ 639.1
11/2+ 435.3
9/2+ 257.9
7/2+ 114.5
5/2+ 0
183 75 Re 108
402↑
404↓
400↑
402↓



$^{183}_{75}Re$
E_γ(keV)
400
300
200
100
2
4
6
8
10
12
14
$I_{initial}$
E_γ vs. $I_{initial}$ plot
Identical bands
ΔI = 2 transitions
Constant "alignments"
^{184}Os
^{182}W
514↑ $h_{11/2}$
404↓ $g_{7/2}$
$I_{alignment} \approx 0.2$
$I_{alignment} \approx -0.3$



183Re
182W core
9/2 [514] expt.
9/2 [514] full Coriolis
9/2 [514] half Coriolis
Linear (182W core)
Eγ (keV)
Ii
0
100
200
300
400
500
600
700
800
0
2
4
6
8
10
12
14
With A. Stuchbery, Canberra
(Semmes-Ragnarsson code)

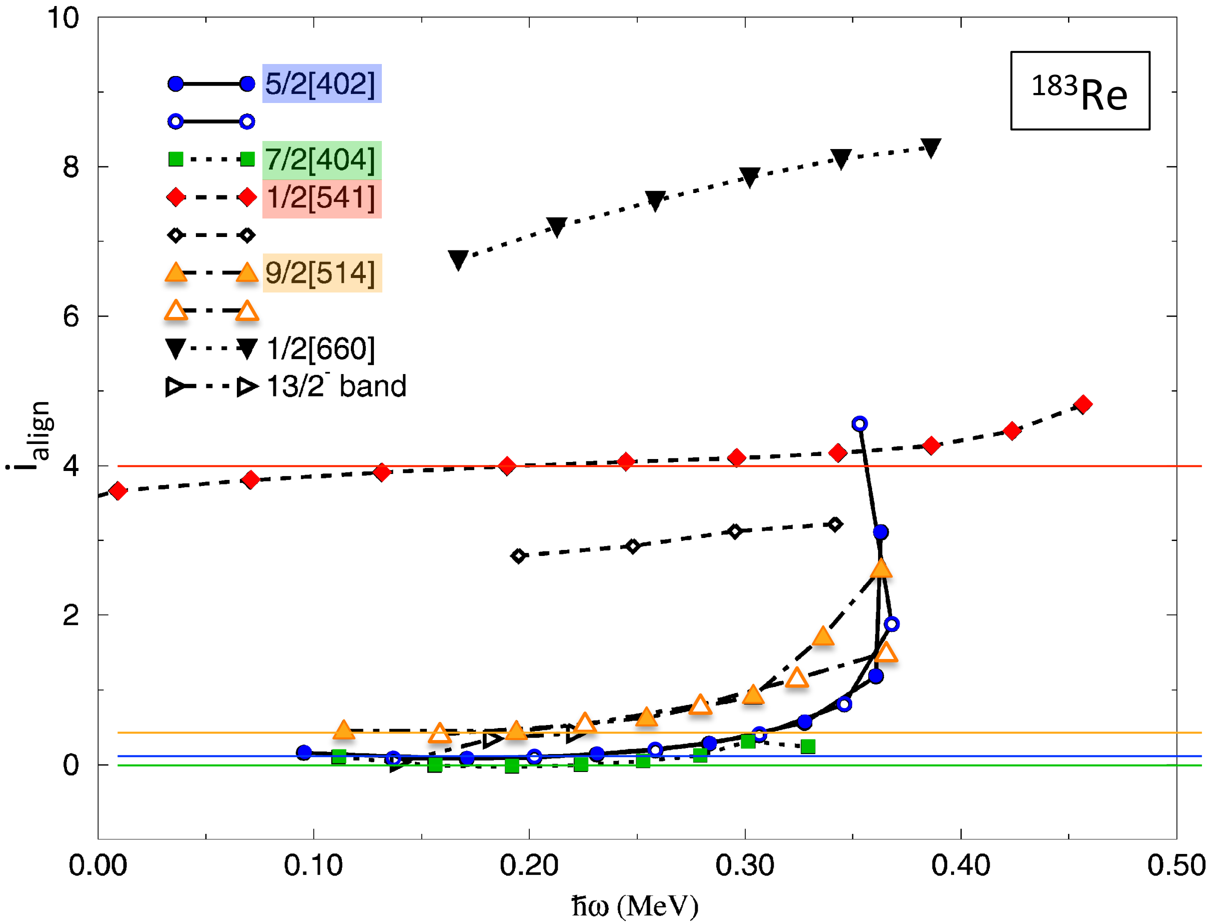


Identical bands (odd and even neighbors) can persist independently of “backbending” --not recognized

“Standard” alignment plots (for ^{183}Re): obscure

$$i_{Harris} = (\mathscr{I}_0 + \omega^2 \mathscr{I}_1)\,\omega$$

Harris formula fit to even-even core; then subtract i_{Harris} from I to get i_{align}

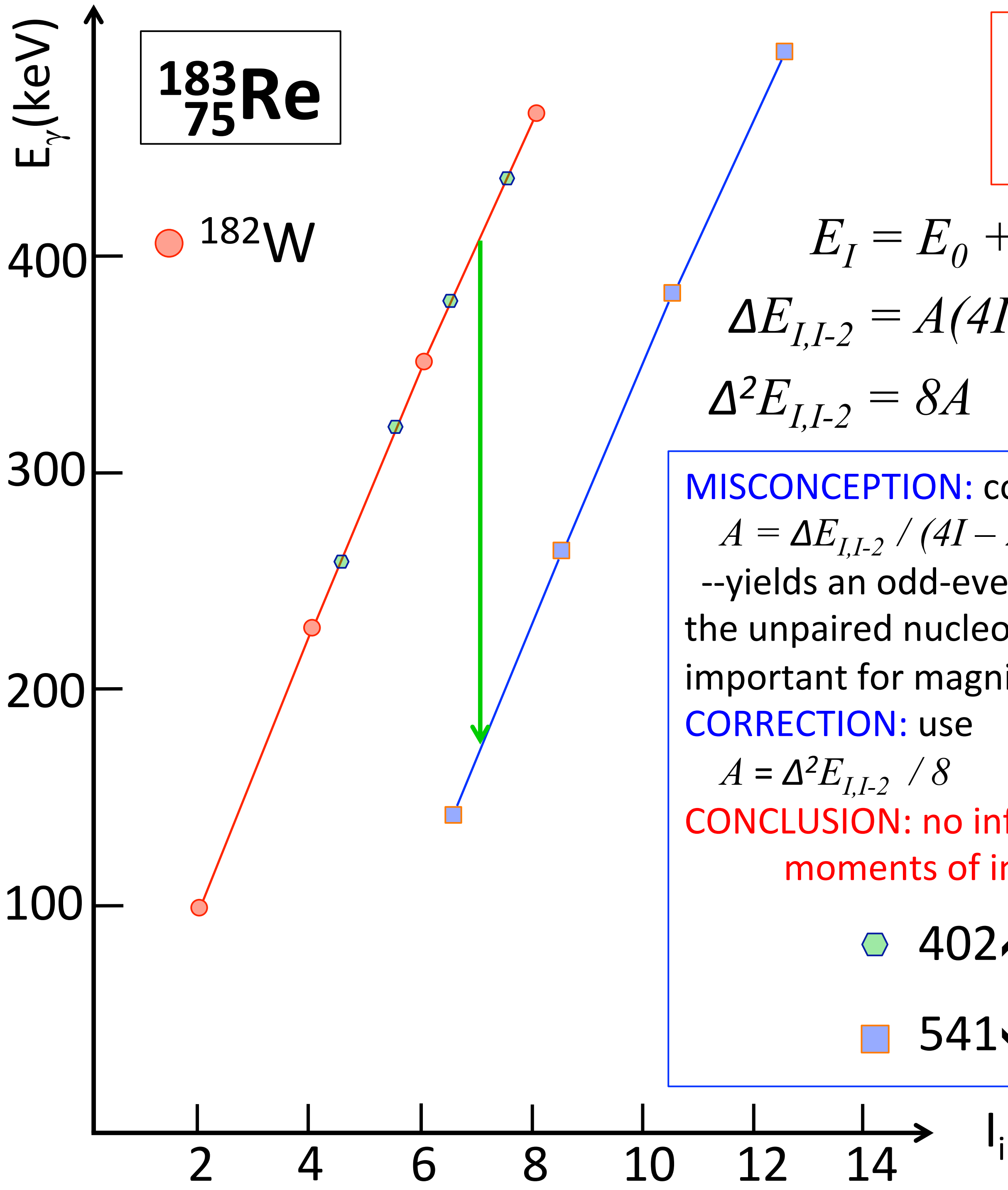


Recall:
$p = mv$

$L = \mathscr{I}\omega$

Purry
NP A672 54

E_γ vs. $I_{initial}$ as a rotation alignment plot



$$E_I = E_0 + AI(I+1) - 2A\langle I \bullet J \rangle$$

$$\Delta E_{I,I-2} = A(4I - 2) - const.$$

$$\Delta^2 E_{I,I-2} = 8A$$

MISCONCEPTION: commonly, use

$$A = \Delta E_{I,I-2} / (4I - 2)$$

--yields an odd-even staggering and conclusion that the unpaired nucleon blocks pairing which is important for magnitudes of moments of inertia.

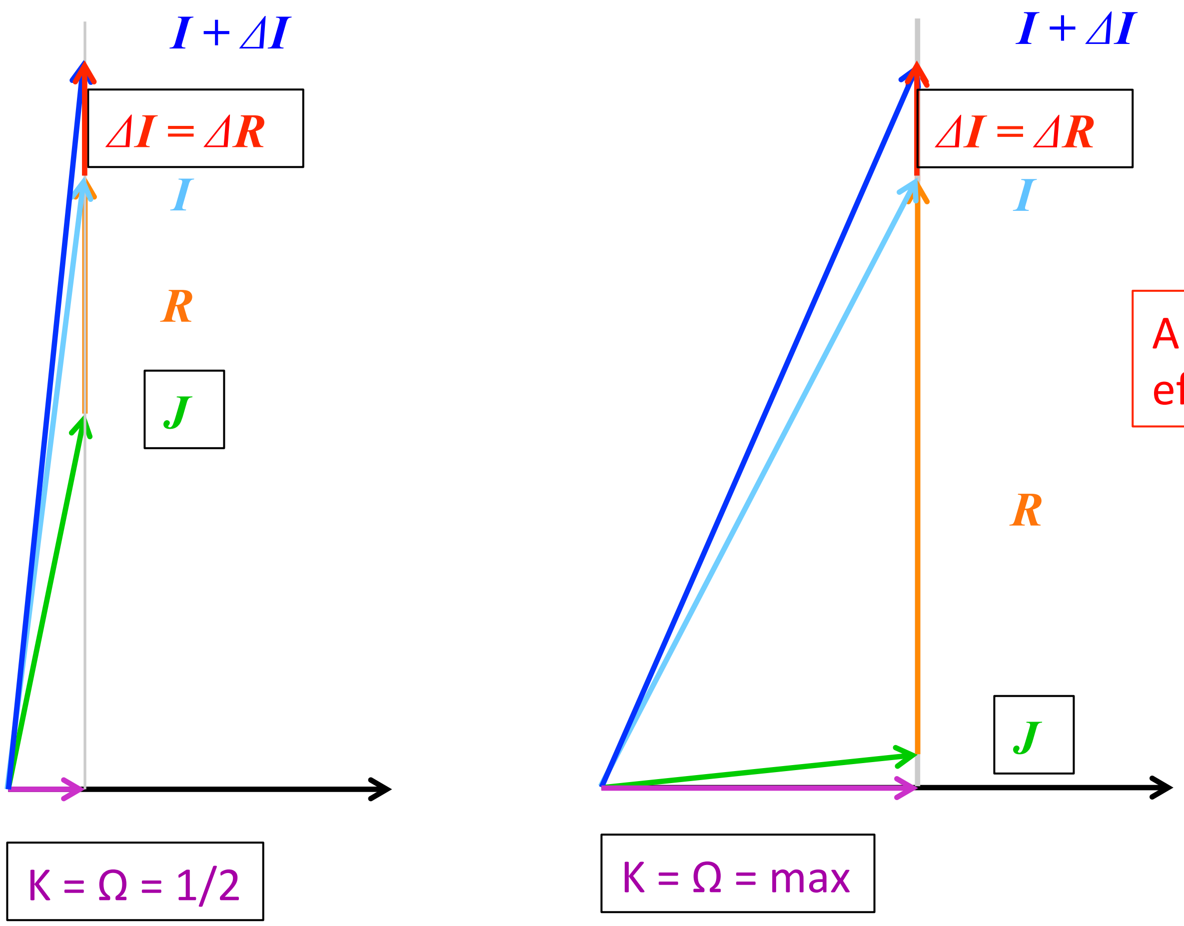
CORRECTION: use

$$A = \Delta^2 E_{I,I-2} / 8$$

CONCLUSION: no influence of pairing on moments of inertia

⬡	402↑	$d_{5/2}$	$const. = 0$ keV
■	541↓	$h_{9/2}$	$const. = -230$ keV

"Coriolis" contribution to energy *differences*



A vectorial difference effect

$<\Delta I \bullet J>$ *large*

$<\Delta I \bullet J> \approx 0$

Philosophical point: simple effects necessitate simple explanations

CONCLUSIONS

- Odd-mass nuclei exhibit bands that differ from the neighboring even-even nucleus ground-state bands by an "alignment" term described by **I•j**.

  Except for this difference, the bands are *identical**.

  CONSEQUENTLY, there is *no evidence* for:

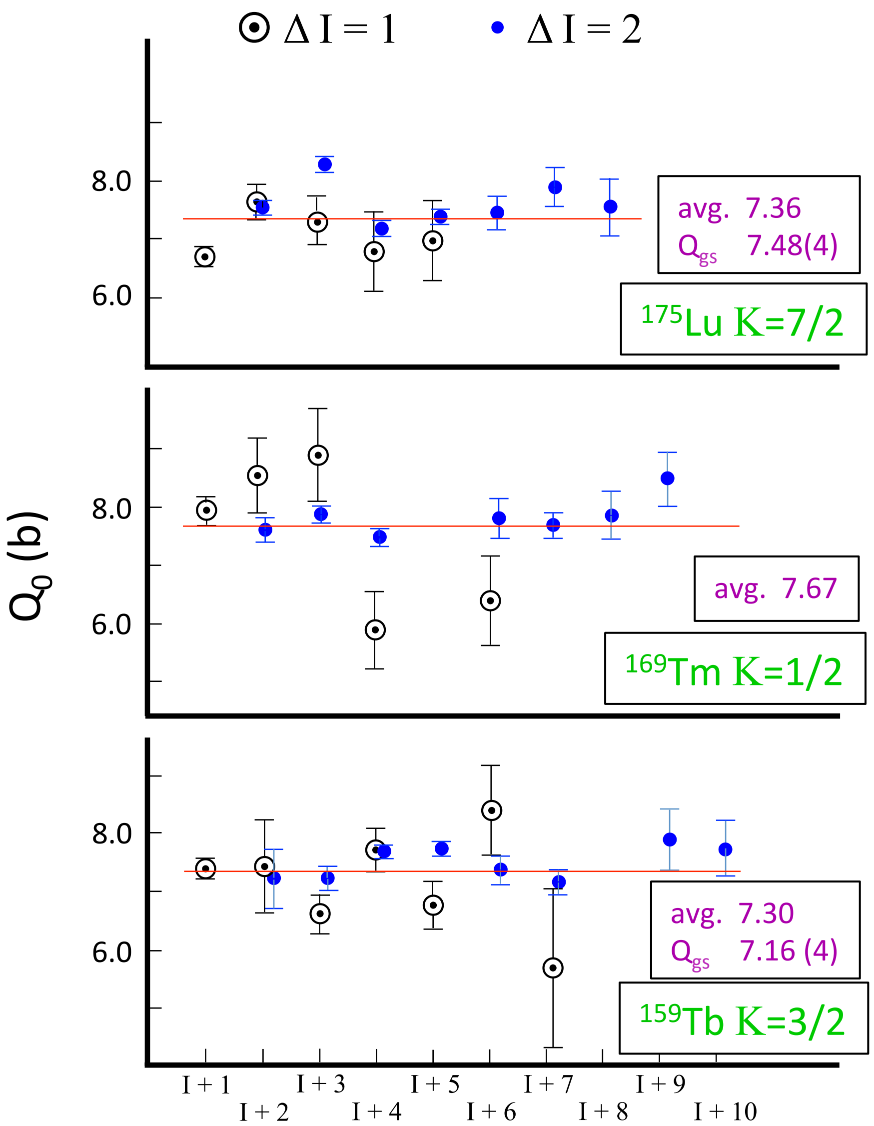
- odd-particle "blocking" of correlations involved in the even-even core collectivity;
- "deformation-driving" effects caused by the odd particle;
- "Coriolis" alignment effects (which should scale with increasing rotational frequency).

*There are small differences which can probably be attributed to band mixing.

So why are “moments of inertia” varying with spin?

Rotor Model in odd-mass nuclei:

No variation of intrinsic quadrupole moments with spin



$$B(E2; I_i \rightarrow I_f) = \langle I_f \| M(E2) \| I_i \rangle^2 / (2I_i + 1)$$

$$\langle I_f \| M(E2) \| I_i \rangle = (2I_i + 1)^{1/2} (5/16\pi)^{1/2} \langle I_i\, K\, 2\, 0 \mid I_f\, K \rangle\, eQ_0$$

Data:

^{175}Lu Skensved et al., NP **A366** 125 1981

^{169}Tm Ward et al., NP **A289** 165 1977

^{159}Tb Chapman et al., NP **A397** 296 1983

Failure of a simple model

- Rotor models require moment(s) of inertia
- "Moments of inertia" change with spin--data
- Intrinsic quadrupole moments do not change with spin--data
- Pairing does not play a role in magnitudes of moments of inertia—data (present message)
- SO WHAT IS CHANGING?

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Nuclear Physics A 662 (2000) 125–147

PHYSICS A

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SU(3) quasi-dynamical symmetry as an organizational mechanism for generating nuclear rotational motions

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"A particularly interesting challenge was to learn how a model, without pair correlations, could give correct moments of inertia when it is known that the cranking model is only successful when pairing correlations are included. The early calculations of Park et al. indicated that the dominant contribution to rotational energies came from the potential energy part of the Hamiltonian, thus calling into question the very concept of the moment of inertia as an inverse coefficient of the L2 term in the kinetic energy. The results of the present calculation indicate that the inclusion of only stretched states, as in the calculation of Park et al.,[#] tends to exaggerate this effect. Nevertheless, it confirms that the dominant component of the rotational energies comes from the potential energy; for the self-consistent value of x only about 20% of the rotational energy comes from the kinetic energy in the present calculation." #--1984

Models with $I(I+1)$ spectra, but without rotational kinematics

- Elliott model:

  $H_{Elliott} = H_{shell\ model} + \kappa\, Q \bullet Q$
- Interacting boson model:

  $H_{boson} = H_{one\text{-}body\ boson} + V_{boson\text{-}boson}$ $(= Q \bullet Q)$
- Symplectic shell model:

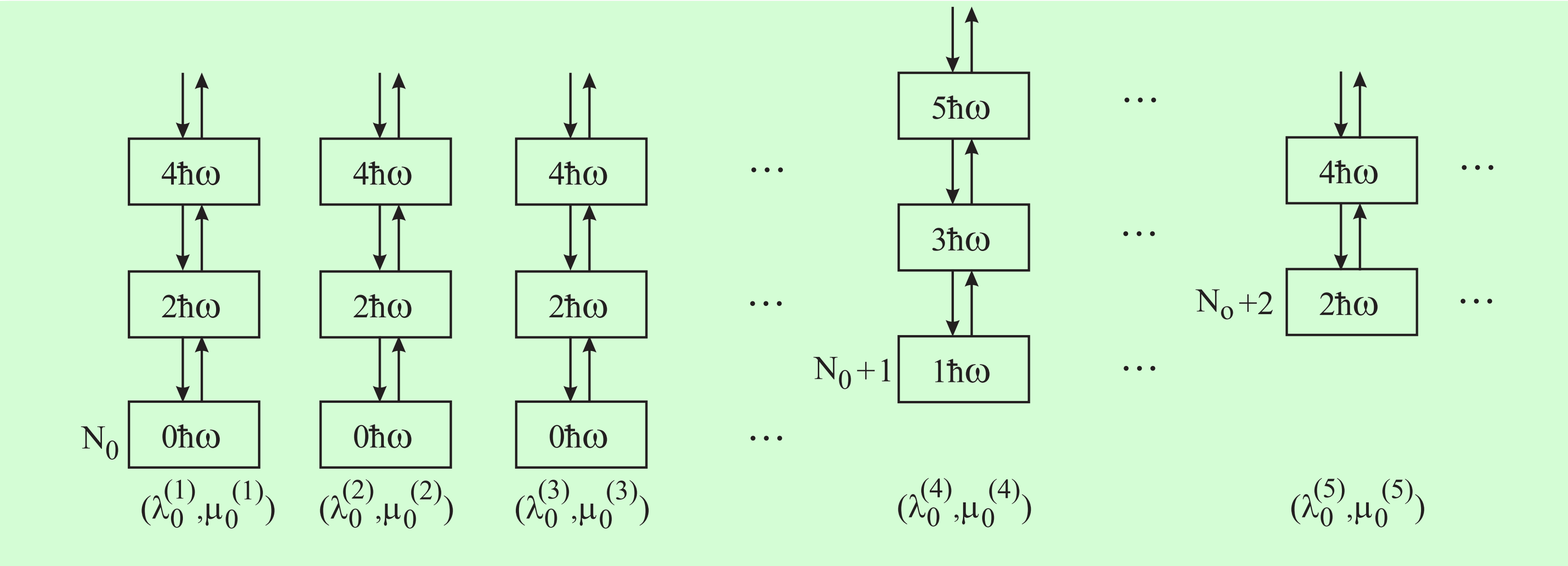
  $H_{symplectic} = H_{SU(3)} + \kappa\, Q \bullet Q + V_{\Delta N=2}$ (GMR; GQR)

  GMR= giant monopole resonance

  GQR= giant quadrupole resonance

A new universal model perspective: shell, SU(3) and multi-shell, Sp(3,R) structure

D.J. Rowe, A.E. McCoy, and M.A. Caprio, Phys. Scr. **91** (2016) 033003



Work in a Cartesian harmonic oscillator basis: $\{n_x\ n_y\ n_z\}$

$N = n_x + n_y + n_z$, $\lambda \sim 2n_z - n_x - n_y$, $\mu \sim n_x - n_y$ - - SU(3) shell "cores".

Allow 2 ħω admixtures of L = 0, 2 [GMR, GQR] configurations.

The spin-orbit and pairing interactions are perturbations.

CONCLUSION

- We are at the end of the beginning
- Now begins the middle

(of the study of the nuclear many-body problem)