Quadrupole Triaxiality Softness - collective models. (In Memory of S.G. Rohoziński)

From simple phenomenological model to fully microscopic General Bohr Hamiltonian

OUTLINE

1. Long lasting experimental interest . nuclei region 50 < Z, N < 82
   Dubna 1966 - Swierk LINIAC 1975- LBNL SUPERHILAC 1979 - HIL UW 2021

2. First approach - Warsaw model: γ-independent potential, one inertia constant $B = B_{\gamma\gamma} = B_{\beta\beta} = B_x = B_y = B_z = \text{const}$
   Rohozinski, et al. ***Z. Physik 268, 401 (1974)***, parameters far away of microscopic

3. γ- dependent Inertial functions and γ-independent Potential
   J. Dobaczewski et al. ***Z. Physik A 282, 203 (1977)***, *still parameters far away of microscopic*

4. full 5-dimensional General Bohr Hamiltonian(GBH) with totally microscopic Potential and 6 Inertial functions
   G. Rohozinski, J.Dobaczewski, B. Nerlo-Pomorska, K. PomorskI, J. Srebrny **Nuclear Physics A292 (1977)66**
   Important 20% decrease of G pairing strength to get closer to experiment

5. *J. Srebrny et al.* ***Nuclear Physics A 766 (2006) 25*** *GBH with* pairing vibration included
   ^{104}Ru case very good reproduction of rich COULEX experimental data

6. ^{140}Sm phenomenological Warsaw model and full microscopic GBH

Julian Srebrny, Heavy Ion Laboratory University of Warsaw

Warsaw Nuclear Physics Seminar - November 25, 2021

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Ch. Droste group (Warsaw) experiments (> 40) at 50 < Z, N < 82 region

1966-1973 **JINR Dubna heavy ions ,** electron spektrometer
^{120,122,123,124}Cs, ^{130m}Ba, ^{129m}La, ^{135m}Ce, ^{137m}Nd, ^{143}Sm

1973-1977 **IBJ Świerk LINIAC, 10 MeV p**
^{124,126,128}**Xe ,** ^{130} **Ba** (p,n), after beta–decay, Ge-Ge

1977-1980 **IFJ PAN Krakow , cyclotron 28 MeV α**
^{127,128,130}Xe Ge- Ge, polarimeter , timing

1978- 1986 **KFA Julich cyclotron p,d,3He, 4He 22-45 MeV /A**
^{134}Ba, ^{134,136}La, ^{141}Eu, ^{142,143}Gd, ..

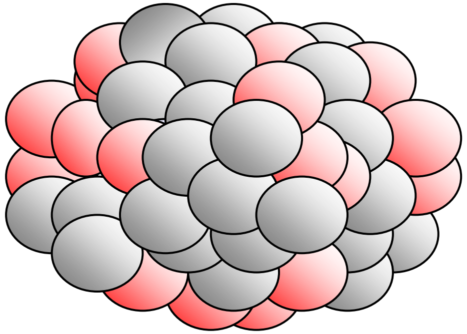
1985 **GSI Darmstadt LINIAC heavy ions**
COULEX of ^{128}Xe beam

1988 **H-M Institut Berlin**
RDM Plunger ^{121,123}Cs

1985-1993 **NBI Riso Tandem VdG** NORDBALL
Plunger, DSAM, COULEX, ^{119}I, ^{118}Te, ^{129}Xe

1988 **SUNY Stony Brook**
Ge-Ge, polarisation, RDM Plunger ^{125,127}La, ^{126}Ce, ^{127} Pr

1997-2021 **HIL UW cyclotron U-200P,**
OSIRIS – EAGLE, ULESE, FATIMA, GAMMAPOOL 6-16 Ge, electron conversion, DSAM, RDM PLUNGER, timing, Ge-Ge – LaBr3
^{124,126,128}Cs, ^{129,130,131,132}La, ^{130m}Ba,^{132m}Ce, $^{134m,\ 136m}$Nd, ^{140}Sm



Nuclei are aggregates consisting of two types of particles , protons and neutrons , together referred to as the nucleons.

Aage Bohr , Ben R.Mottelson
Nuclear Structure

a finite number of fermions quantum system

Fundamental problem for nuclear theory is coupling of motion of nucleus

as a whole(**collective**)

to single protons and neutrons(**microscopic**)

ß- deformation: quadrupole deviation from spherical shape

γ– deformation : quadrupole deviation from axially symetrical shape

IOP PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **36** (2009) 123101 (46pp) doi:10.1088/0954-3899/36/12/123101

TOPICAL REVIEW

Quadrupole collective states within the Bohr collective Hamiltonian

L Próchniak[1] **and S G Rohoziński**[2]

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[2] Institute of Theoretical Physics, University of Warsaw, Hoża 69, 00-681 Warsaw, Poland

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$H = T + V$

for classic point object $T = mV^2/2 = m(V^2x + V^2y + V^2z)/2$

---

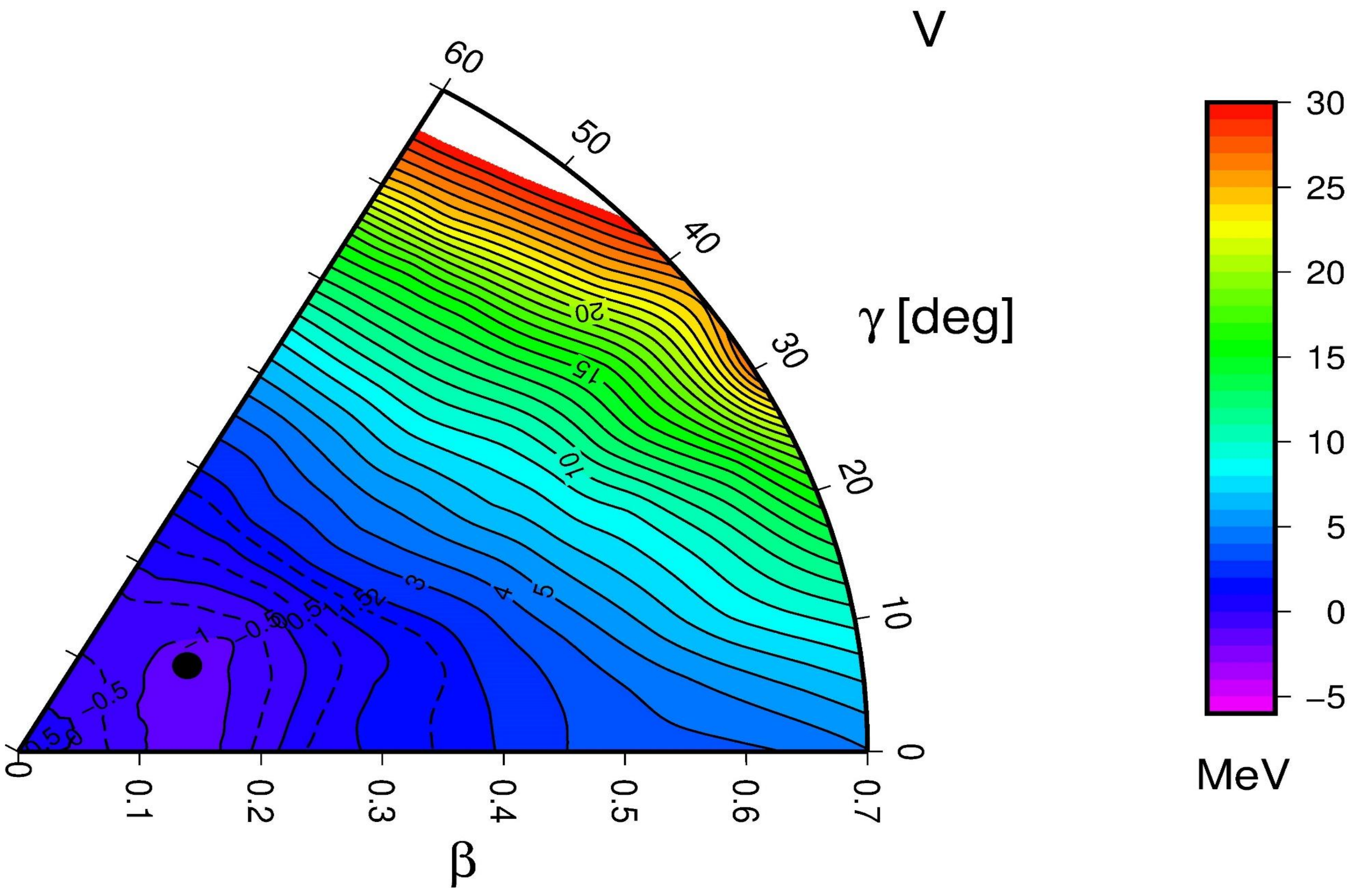
for complex object

$$H = 1/2[B_{\beta\beta}(\beta,\gamma)\dot{\beta}^2 + 2B_{\beta\gamma}(\beta,\gamma)\dot{\beta}\beta\dot{\gamma} + B_{\gamma\gamma}(\beta,\gamma)\beta^2\dot{\gamma}^2]$$
$$+ 1/(8\beta^2) \sum_{\kappa=x,y,z} I_\kappa^2/(B_\kappa(\beta,\gamma)\sin^2\gamma_\kappa) + V(\beta,\gamma), \qquad (1)$$

where β, γ are the Bohr deformation parameters, $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$ and B_x, B_y, B_z are the vibrational and rotational inertial functions, respectively, V is the potential, I_x, I_y, I_z are components of the total angular momentum in the intrinsic system, and $\gamma_x = \gamma - 2\pi/3$, $\gamma_y = \gamma + 2\pi/3$, $\gamma_z = \gamma$.

for a quantum system of a finite number of fermions full 5-dimensional General Bohr Hamiltonian(GBH) :
potential $V(\beta,\gamma)$ and 6 inertial functions $B_i(\beta,\gamma)$ have to be calculated microscopicaly,
it means: to take into account influence of individual nucleons for collective motion

^{140}Sm, SLy4 sen



Leszek Próchniak

Nuclei from the Region 52 < Z, N < 80 as Susceptible to the Gamma-Deformations

S.G. Rohozinski, J. Srebrny and K. Horbaczewska *Z. Physik 268, 401 (**1974**)* *(Warsaw model)*

γ-independent potential, one inertia constant $B = B_{\gamma\gamma} = B_{\beta\beta} = B_x = B_y = B_z = const$ $B_{\beta\gamma} = 0$

In the computations we have used the potentials in the form suggested by Myers and Swiatecki 1966

$$V(\beta) = 1/2\, V_c\, \beta^2 + G[e^{-(\beta/a)} - 1]$$

Ground and excited state deformations in the 50 < Z, N < 82 region

D. A. Arseniev, A. Sobiczewski, V. G. Soloviev Nucl. Phys A126 (1969) 15

– important role of triaxialty and γ-softness

Ground state moments of inertia of deformed nuclei around barium

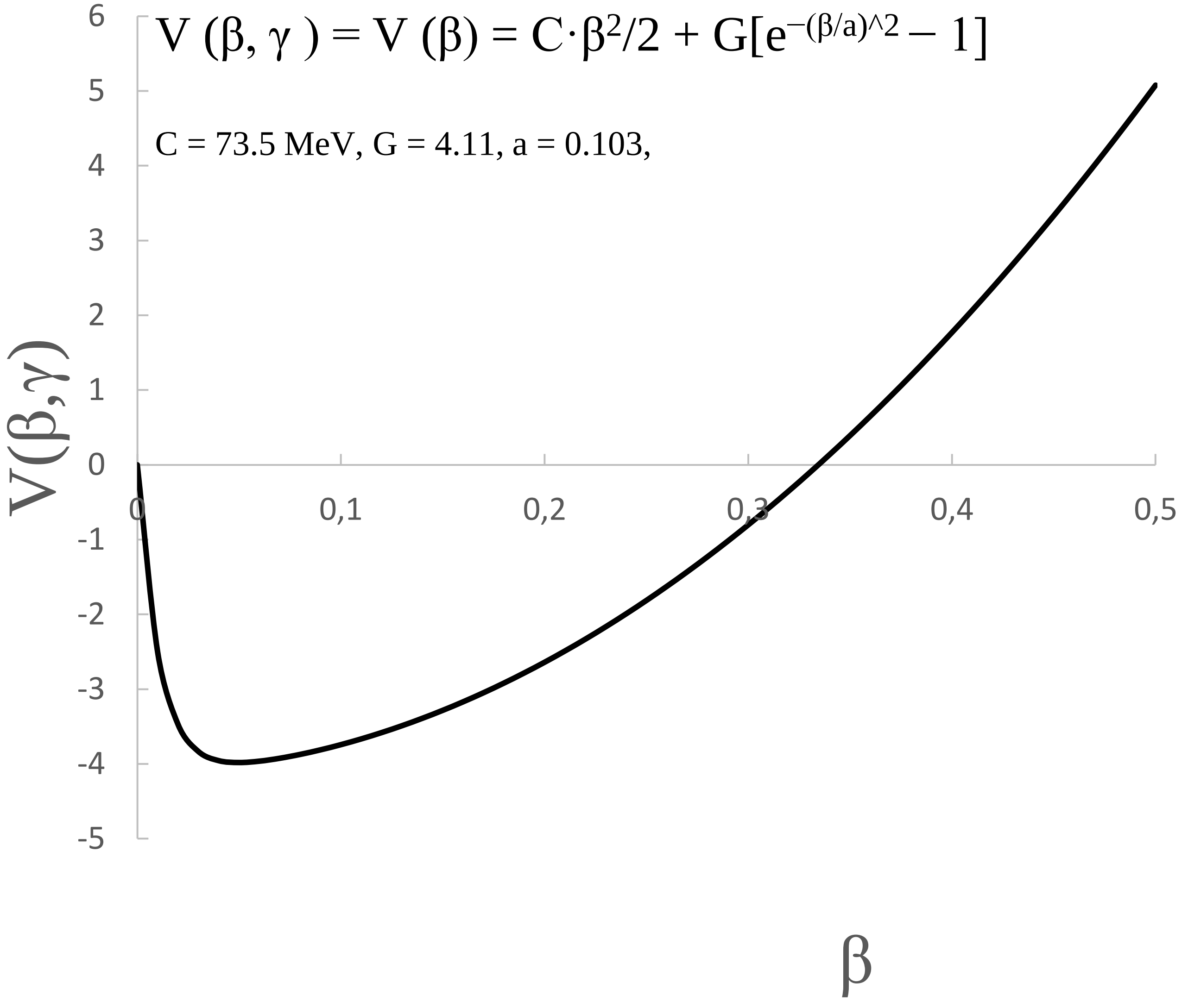
Pomorski, K., Nerlo- Pomorska, B., Ragnarsson, I., Sheline, R. K, Sobiczewski, A.:

Nucl. Phys. A205, 433 (1973)

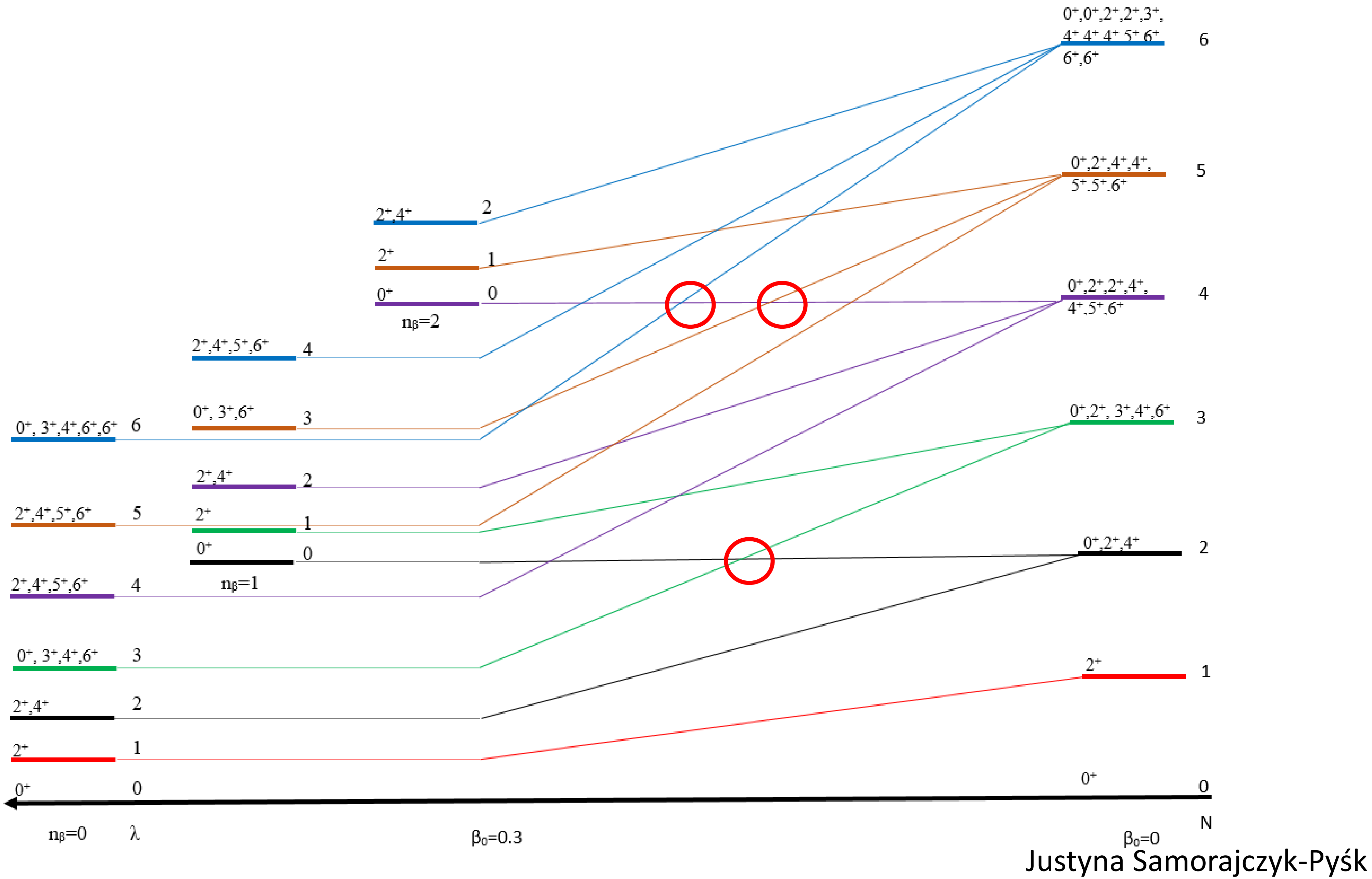
- microscopic moments of inertia by cranking methods

It was found that mass parameters B are a several times larger than the microscopic values. The stiffnesses are also larger than those calculated microscopically.

Warsaw model - potential energy



Justyna Samorajczyk-Pyśk


0+,0+,2+,2+,3+,
4+,4+,4+,5+,6+,
6+,6+
6
0+,2+,4+,4+,
5+,5+,6+
5
0+,2+,2+,4+,
4+,5+,6+
4
0+,2+, 3+,4+,6+
3
0+,2+,4+
2
2+
1
0+
0
N
β0=0
2+,4+
2
2+
1
0+
0
nβ=2
2+,4+,5+,6+
4
0+, 3+,6+
3
2+,4+
2
2+
1
0+
0
nβ=1
0+, 3+,4+,6+,6+
6
2+,4+,5+,6+
5
2+,4+,5+,6+
4
0+, 3+,4+,6+
3
2+,4+
2
2+
1
0+
0
nβ=0
λ
β0=0.3
Justyna Samorajczyk-Pyśk

Nuclei from the Barium Region: Nonaxial or Gamma-Soft ?

*J. Dobaczewski , S.G. Rohozinski, J. Srebrny   Z. Physik A 282, 203(**1977**)*

We assume the γ-independent potential in the form

$$V(\beta) = 1/2\, C_2 \beta^2 + C_8 \beta^8 + G[\exp(-\beta^2/a^2) - 1]. \qquad (2)$$

In this case the dynamic effects in the collective motion in γ-direction have to be taken into account. Our approximation consists in that these effects are not taken into account in the β-vibrations. We put

$$B_{\beta\beta}(\beta, \gamma) \equiv B = \text{const},$$
$$B_{\beta\gamma}(\beta, \gamma) = 0,$$
$$B_\alpha(\beta, \gamma) = B_\alpha(\tilde\beta, \gamma) \equiv B_\alpha(\gamma), \qquad (3)$$

where α stands for each of the subscripts $\gamma\gamma$, x, y, z, and $\tilde\beta$ is some fixed value of β around which we believe the wave functions are localized.

$V^{micr}(ß)$-   **B. Pomorska**

In the calculations we use the inertial functions in the form

$$B_\alpha(\gamma) = f\, B_\alpha^{micr}(\gamma) + b$$

where B_α^{micr} are taken from the microscopic calculations
Kaniowska, T., Sobiczewski, A., Pomorski, K., Rohozinski, S.G. *Nucl. Phys. A274, 151 (1976)*

When both PES and the inertial functions are assumed to be purely microscopic f= 1, b = 0, a large discrepancy appears between the experimental and theoretical energy levels

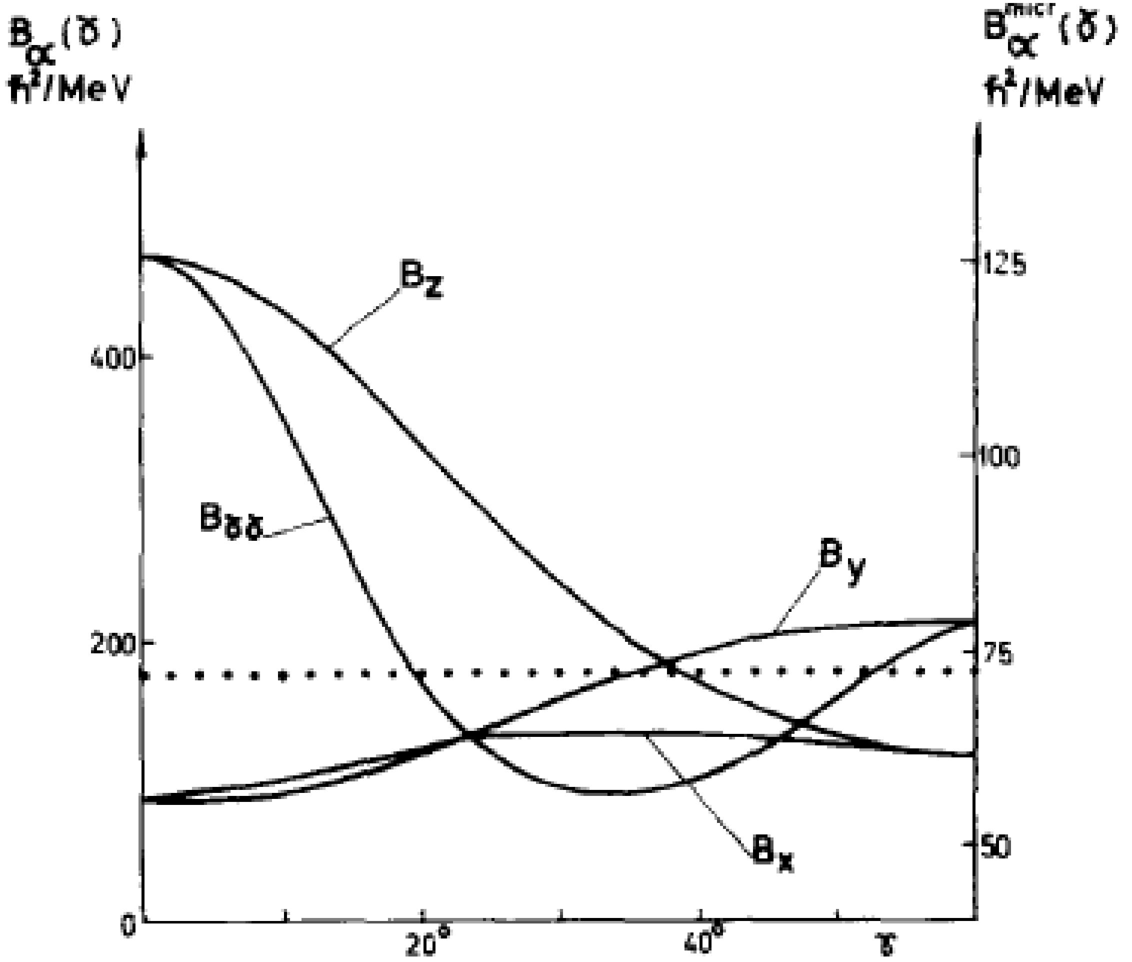


Fig. 2. The γ-dependence of the inertial functions B_x, B_y, B_z, $B_{\gamma\gamma}$. The left hand scale refers to the values $f = 5.53$, $b = -226\hbar^2$/MeV (Eq. (7)) obtained from the adjustment to the experimental data. The right hand scale refers to the microscopic calculations [9] for $\bar{\beta} = 0.2$ ($f = 1$, $b = 0$). The dotted line indicates the mean value of the inertial functions

$$B_\alpha(\gamma) = f\, B_\alpha^{micr}(\gamma) + b$$

^{134}Ba

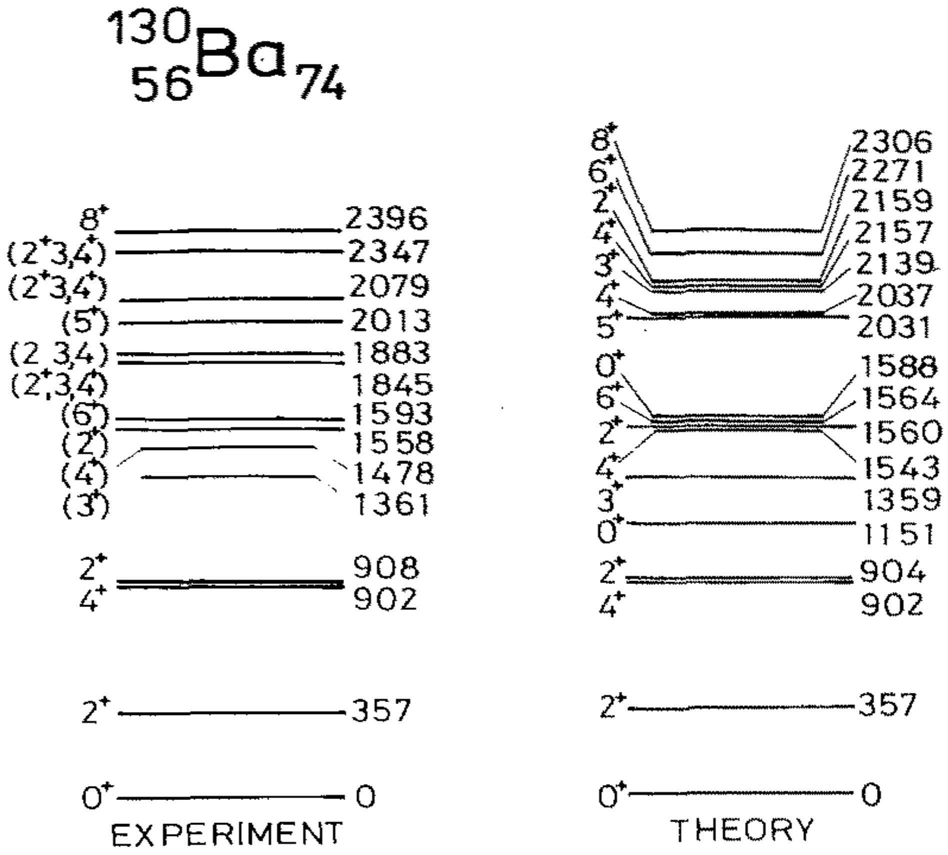
When both PES and the inertial functions are assumed to be purely microscopic f= 1, b = 0, a large discrepancy appears between the experimental and theoretical energy levels

Z. Phys. A 305 (**1982**)335 **The Decay of ^{130}La to Levels in ^{130}Ba**

W. Urban, T. Rząca, Ch. Droste, L. Goettig, T. Morek, and J. Srebrny

2 Ge(Li)

f=4, b = - 140.3 h^2/MeV



Nuclear Physics A 587 (**1995**)211 **Low spin states in ^{130}Ba**

K. Kirch, G. Siems , M. Eschenauer , A. Gelberg , R. Ktihn , A. Mertens , U. Neuneyer , O. Vogel , I. Wiedenhover , P. von Brentano , T. Otsuka

^{120}Sn(14N,4n)^{130}La → ^{130}Ba   γ – γ   6 HPGe ACS

0^+ 1179 keV

Full 5-dimensional General Bohr Hamiltonian(GBH)

6 inertial functions B_i (β,γ) and potential V(β,γ)

$$H = 1/2[B_{\beta\beta}(\beta,\gamma)\dot{\beta}^2 + 2B_{\beta\gamma}(\beta,\gamma)\dot{\beta}\beta\dot{\gamma} + B_{\gamma\gamma}(\beta,\gamma)\beta^2\dot{\gamma}^2] + 1/(8\beta^2)\sum_{\kappa=x,y,z} I_\kappa^2/(B_\kappa(\beta,\gamma)\sin^2\gamma_\kappa) + V(\beta,\gamma), \qquad (1)$$

where β, γ are the Bohr deformation parameters, $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$ and B_x, B_y, B_z are the vibrational and rotational inertial functions, respectively, V is the potential, I_x, I_y, I_z are components of the total angular momentum in the intrinsic system, and $\gamma_x = \gamma - 2\pi/3$, $\gamma_y = \gamma + 2\pi/3$, $\gamma_z = \gamma$.

Microscopic Dynamic Calculations of Collective States in Xenon and Barium Region

S. Rohozinski, J. Dobaczewski, B. Nerlo-Pomorska, K. Pomorski, J. Srebrny, *Nucl. Phys. A 292 (**1977**) 66*

7 microscopicaly calculated functions : Potential V(ß,γ), and 6 Inertial Functions

B_α*(ß,γ) - 3 Vibrational Inertia and 3 Rotational Moments of Inertia*

Calculated with Nilsson single particle levels and cranking

Still problem in comparison to experiment.

Only 20% decrease of G pairing strength allowed to get closer to experiment.

It enlarge Inertial functions few times and gave proper energy scale

Experimental and theoretical investigations of quadrupole collective degrees of freedom in ^{104}Ru COULEX- Rochester, Berkeley, Brookhaven

J. Srebrny , T. Czosnyka , Ch. Droste , S.G. Rohozinski , L. Próchniak , K. Zając , K. Pomorski , D. Cline, C.Y. Wu, A. Bäcklin , L. Hasselgren , R.M. Diamond , D. Habs , H.J. Körner , F.S. Stephens , C. Baktash, R.P. Kostecki *Nuclear Physics A 766 (**2006**) 25*

Everything calculated microscopicaly, no parameters fitted to ^{104}**Ru** experimental data
And only adding microscopic pairing vibration allowed us to get good agreement with rich experimental data without any fitted parameters.

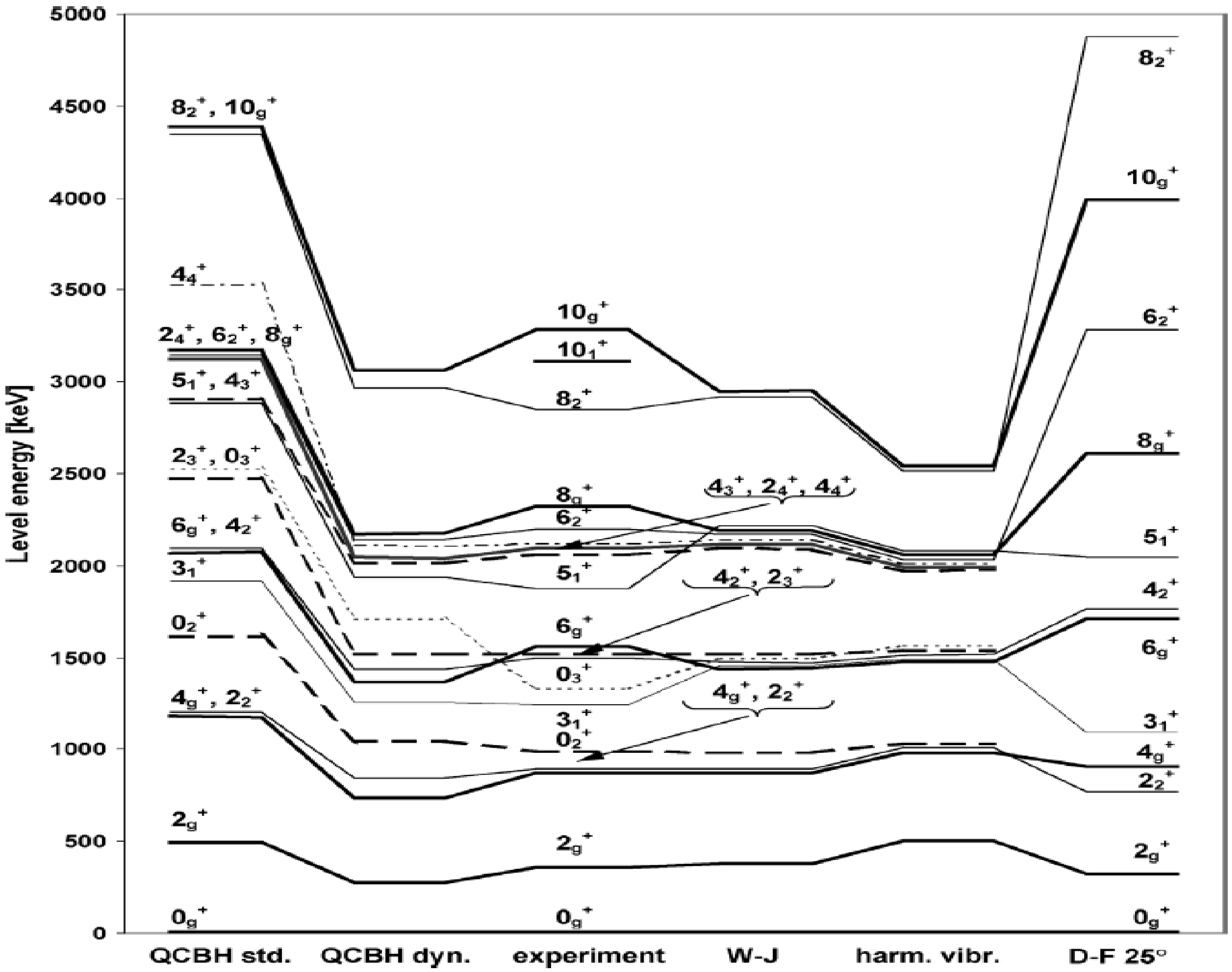


Fig. 6. Comparison of experimental and theoretical energy levels values. Ground state band levels are marked by thick continous lines, 2_2^+ band levels are marked by thin continous lines, 0_2^+ band levels are marked by thick dashed lines, 0_3^+ band levels are marked by dotted lines.

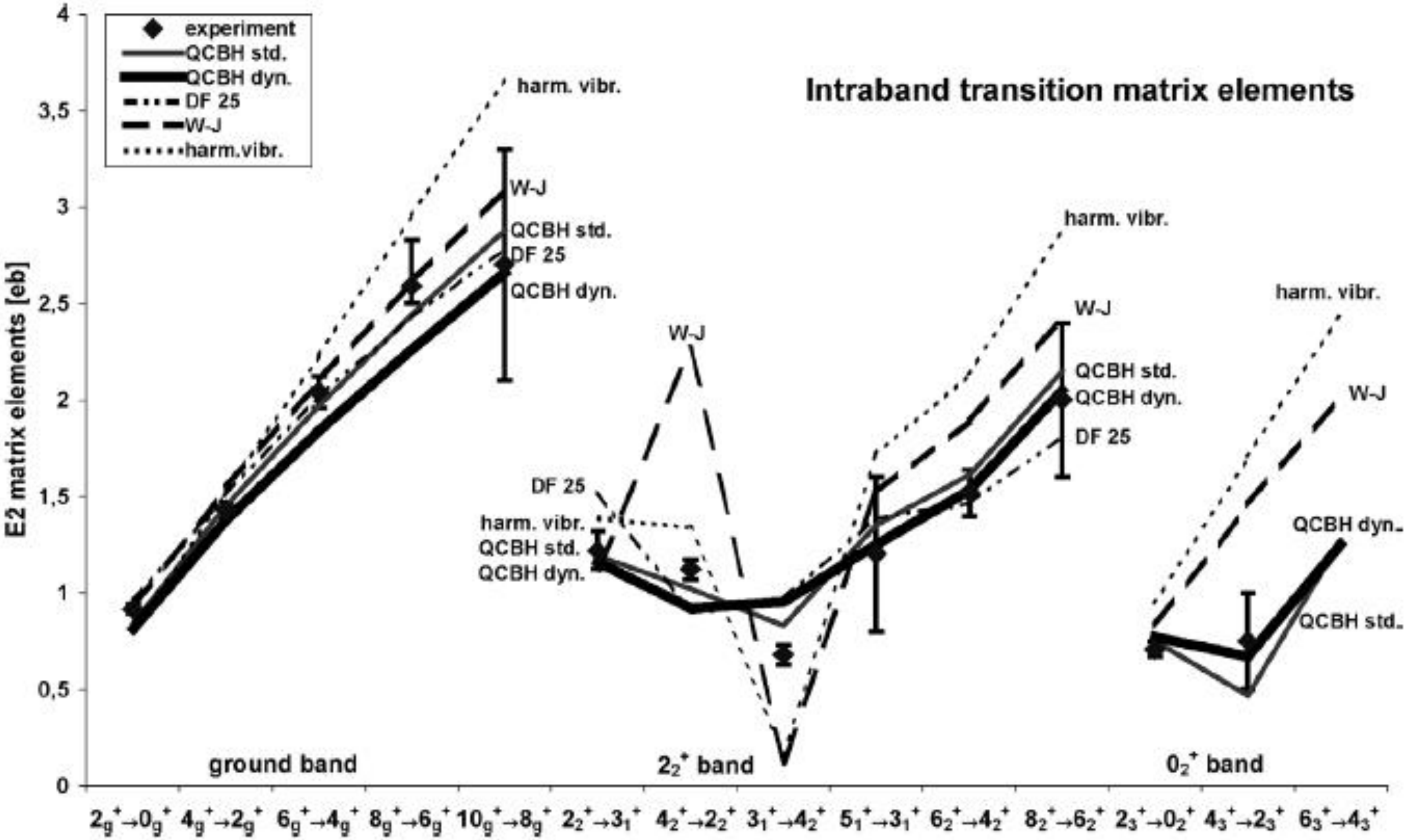


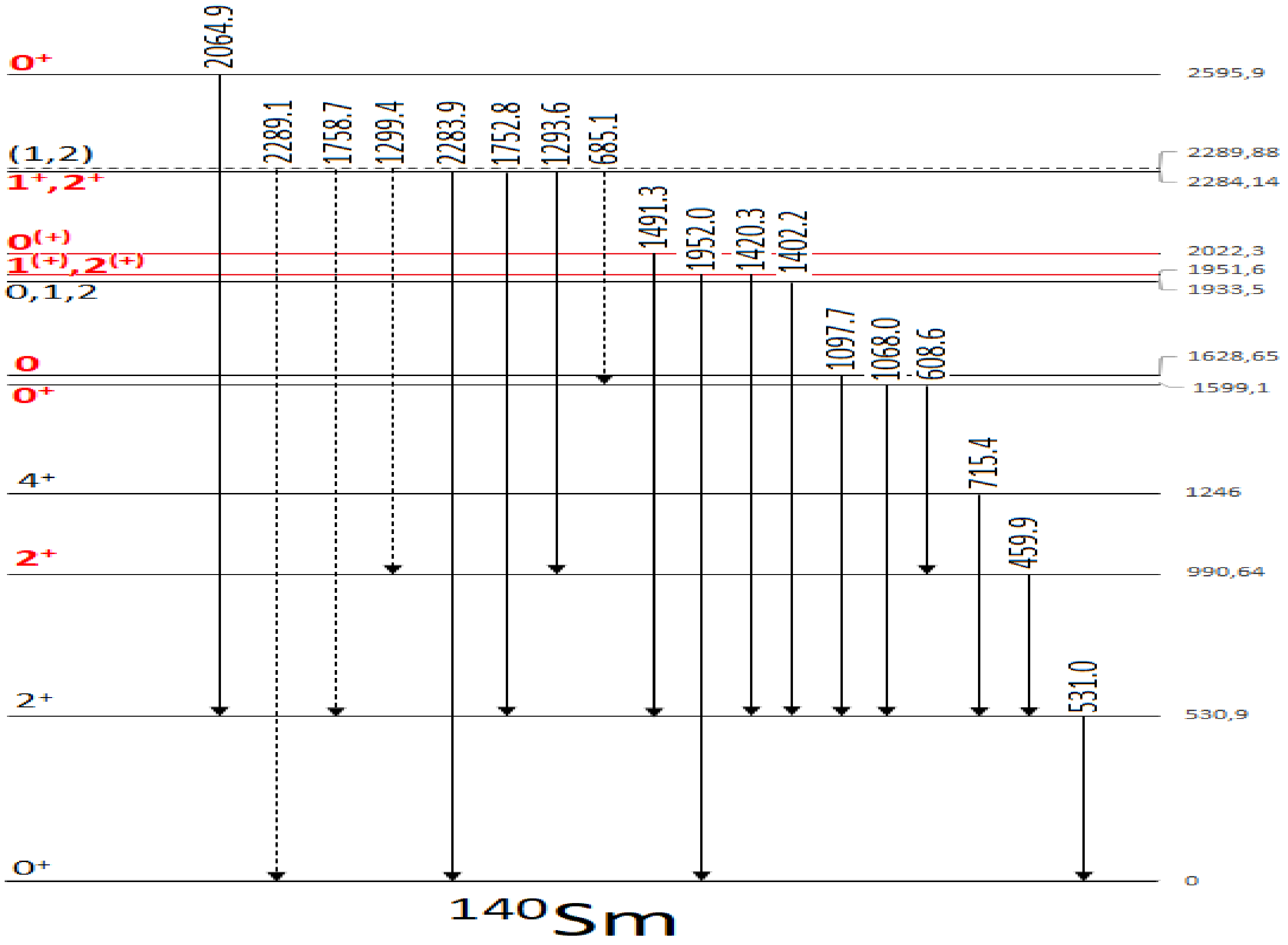
Fig. 7. Intraband transition E2 matrix elements for the ground, 2_2^+ and 0_2^+ bands.

Low-spin levels in 140Sm: Five 0+ states and the question of softness against nonaxial deformation

J. Samorajczyk-Pysk , Ch. Droste, L. Próchniak , J. Srebrny , S. G. Rohozinski, J. Andrzejewski, S. Dutt , A. Gawlik , K. Hadynska-Klęk , Ł. Janiak , M. Klintefjord, M. Kowalczyk , J. Kowalska, R. Kumar, T. Marchlewski , P. J. Napiorkowski , J. Perkowski, W. Piatek, M. Piersa-Siłkowska , T. Roginski, M. Saxena, A. Stolarz and A. Tucholski

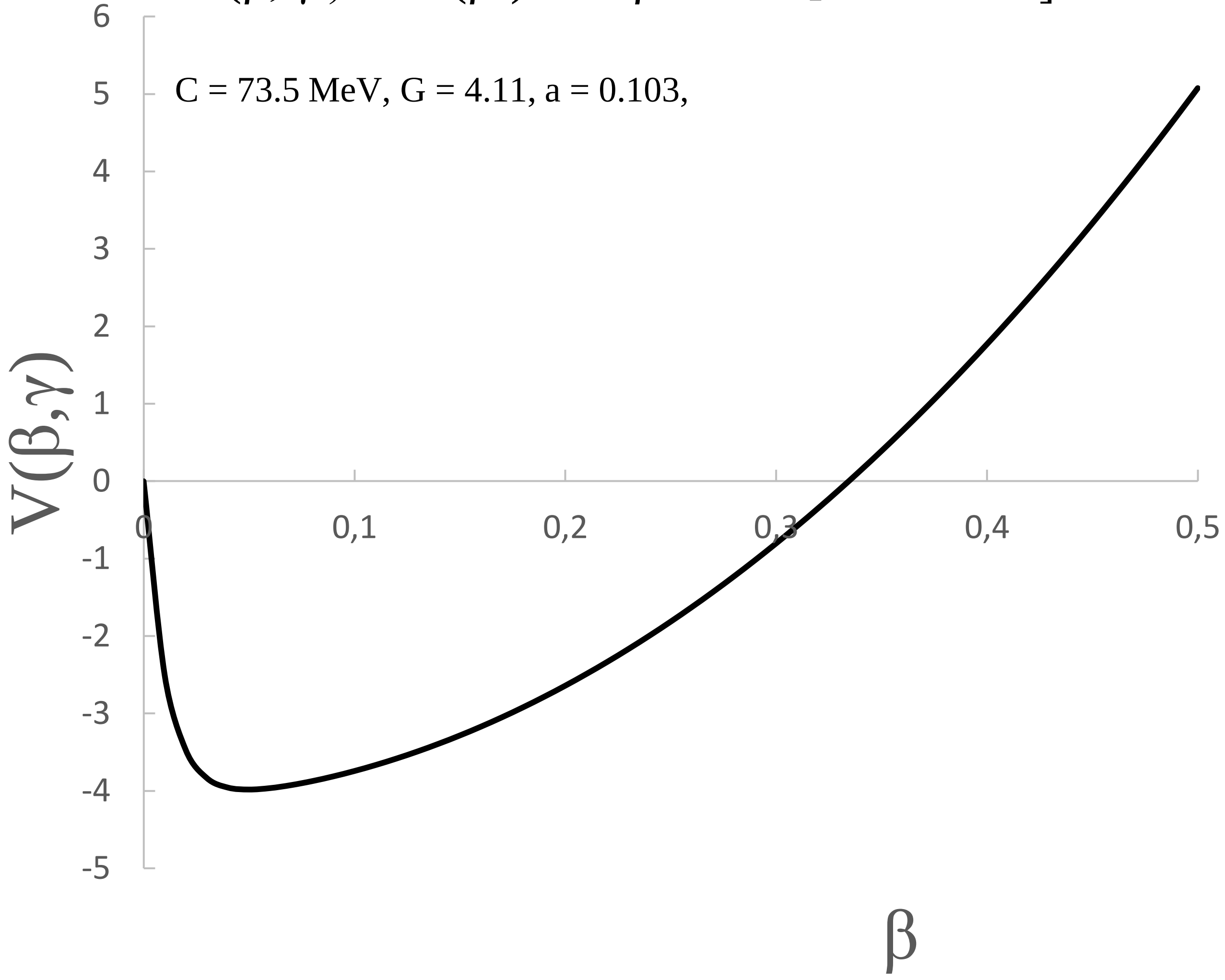
PHYSICAL REVIEW C 104, 024322 (**2021**)

EAGLE on the Warsaw cyclotron
γ – γ correlations



Warsaw model - potential energy

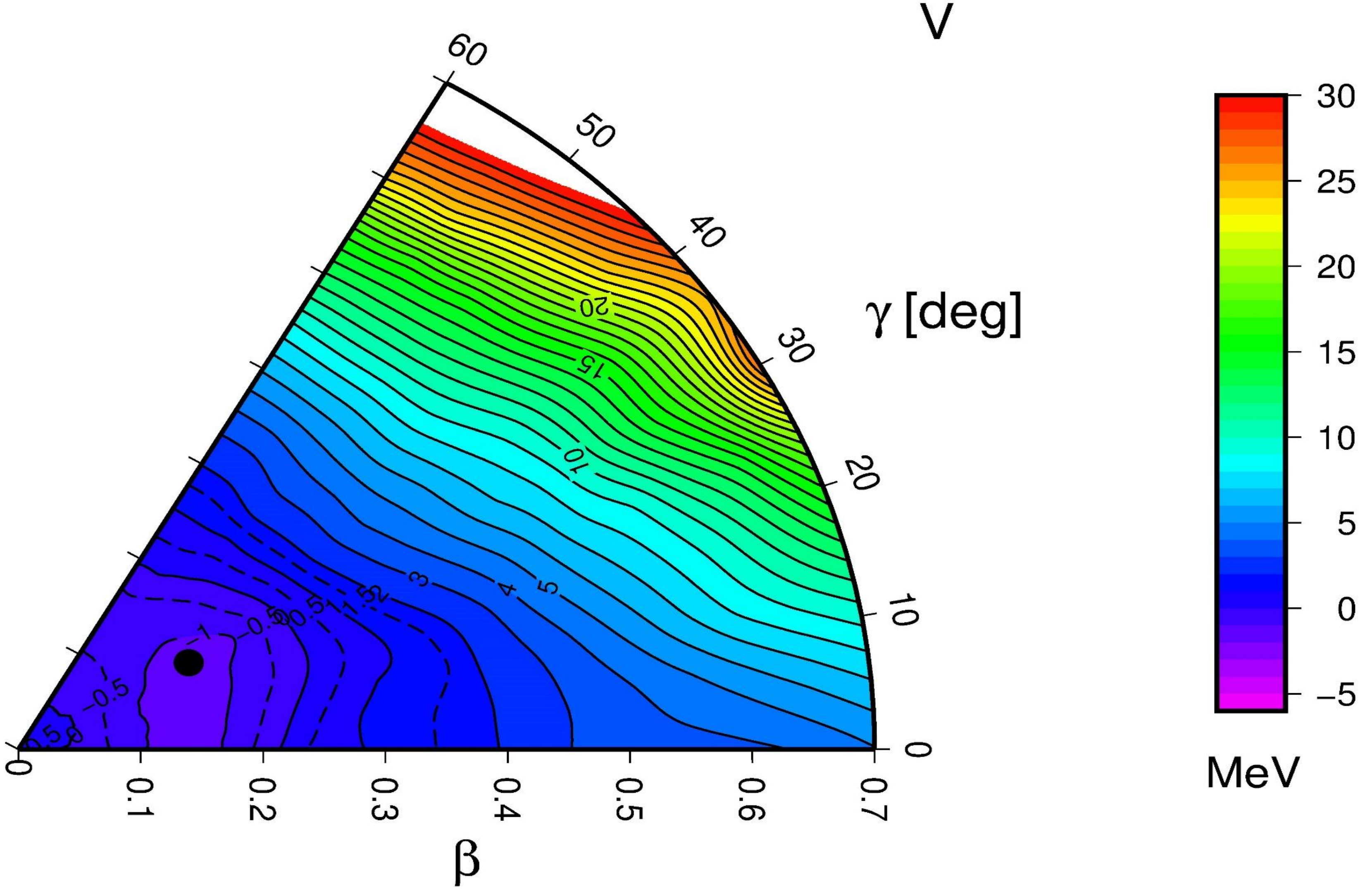
$$V(\beta, \gamma) = V(\beta^2) = C\cdot\beta^2/2 + G[e^{-(\beta/a)^2} - 1]$$



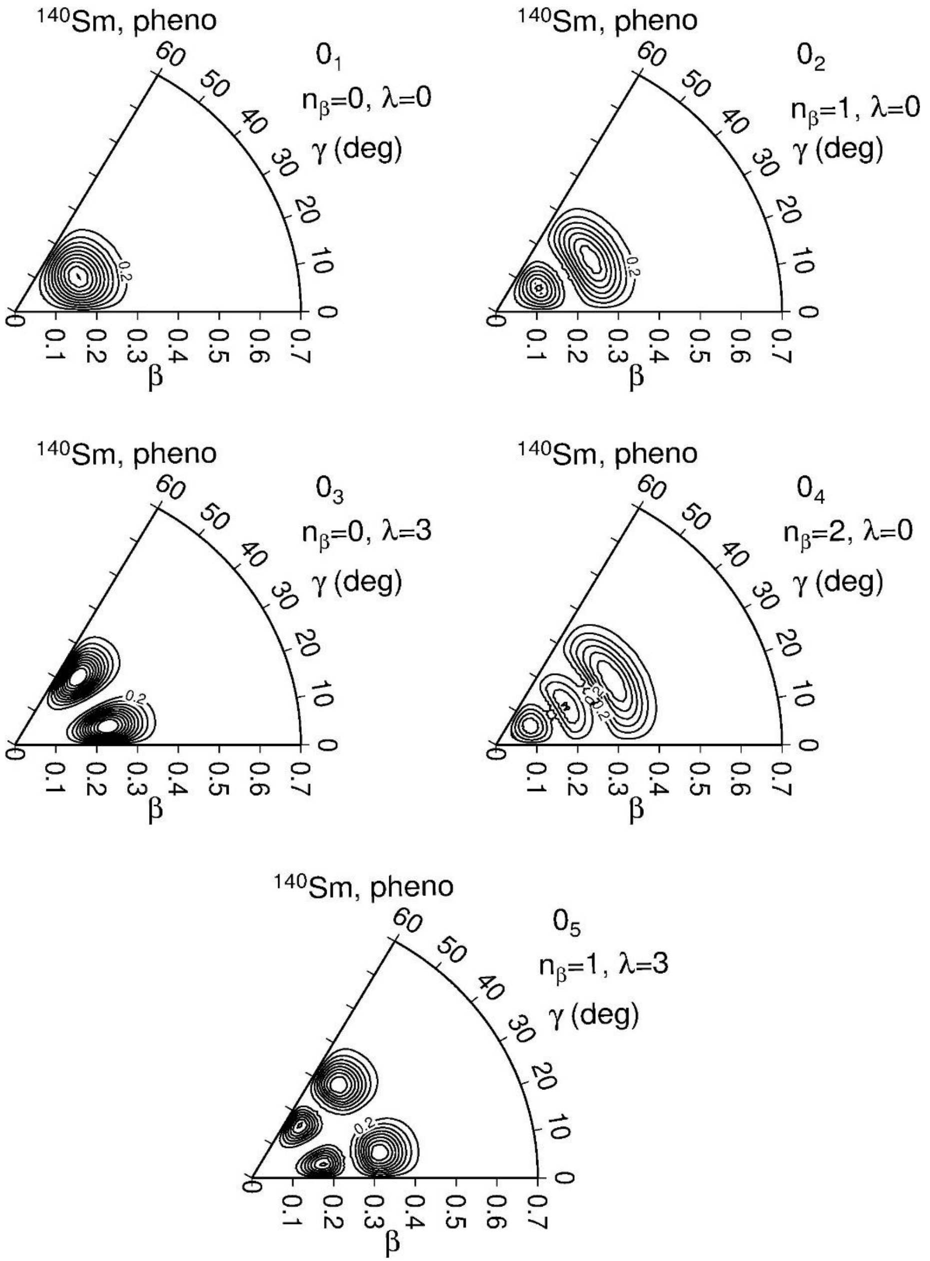
γ- independent

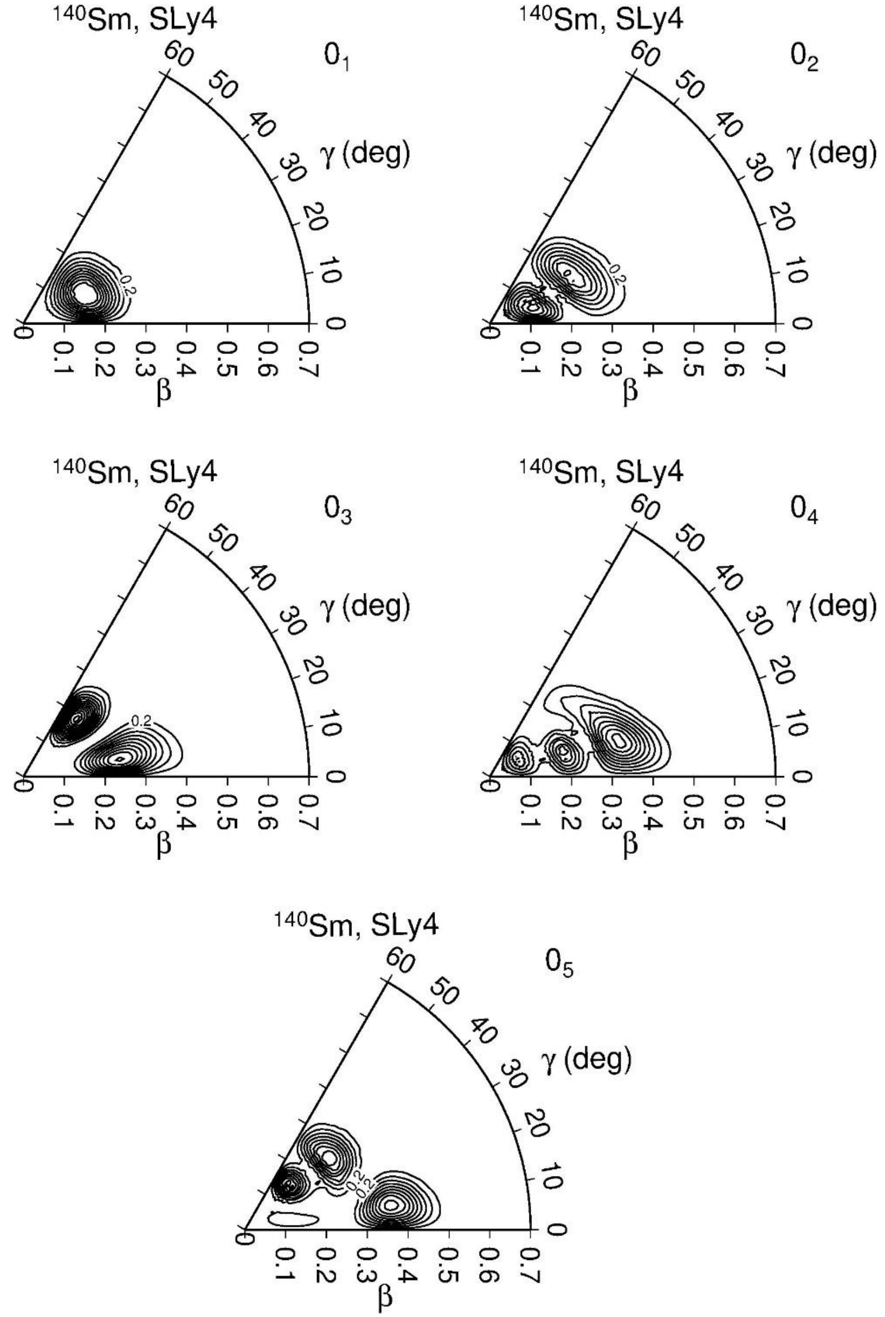
Justyna Samorajczyk-Pyśk

^{140}Sm, SLy4 sen



Leszek Próchniak





Justyna Samorajczyk-Pyśk

TABLE VI. Theoretical quadrupole invariants and the β_p, γ_p values (see text) for chosen states in ^{140}Sm. The phenomenological model.

State	$\langle Q^2 \rangle$ [e^2b^2]	$\sigma(Q^2)$ [e^2b^2]	$\langle Q^3 \cos 3\gamma \rangle$ [e^3b^3]	$\sigma(Q^3 \cos 3\gamma)$ [e^3b^3]	β_p	$\sigma(\beta_p)$	γ_p [deg]	$\sigma(\gamma_p)$ [deg]
0_1^+	1.20	0.63	0.15	1.83	0.19	0.05	28	11
0_2^+	1.78	1.19	0.37	3.68	0.23	0.08	27	11
0_3^+	1.94	0.80	0.36	3.37	0.24	0.05	28	15
0_4^+	2.38	1.69	0.69	5.87	0.27	0.10	27	11
0_5^+	2.63	1.48	0.74	5.96	0.28	0.08	27	15
2_1^+	1.42	0.68	0.20	2.24	0.21	0.05	28	11
2_2^+	1.66	0.73	0.27	2.76	0.22	0.05	28	11

TABLE VII. Same as Table VI, but for the microscopic model.

State	$\langle Q^2 \rangle$ [e^2b^2]	$\sigma(Q^2)$ [e^2b^2]	$\langle Q^3 \cos 3\gamma \rangle$ [e^3b^3]	$\sigma(Q^3 \cos 3\gamma)$ [e^3b^3]	β_p	$\sigma(\beta_p)$	γ_p [deg]	$\sigma(\gamma_p)$ [deg]
0_1^+	1.12	0.55	0.27	1.58	0.18	0.05	26	11
0_2^+	1.43	1.03	0.31	2.84	0.21	0.07	27	11
0_3^+	1.95	0.99	1.09	3.64	0.24	0.06	23	15
0_4^+	2.37	1.70	2.34	5.49	0.27	0.10	20	12
0_5^+	2.77	1.78	2.25	6.76	0.29	0.09	22	15
2_1^+	1.28	0.57	0.36	1.85	0.20	0.04	26	11
2_2^+	1.44	0.59	0.23	2.14	0.21	0.04	28	10

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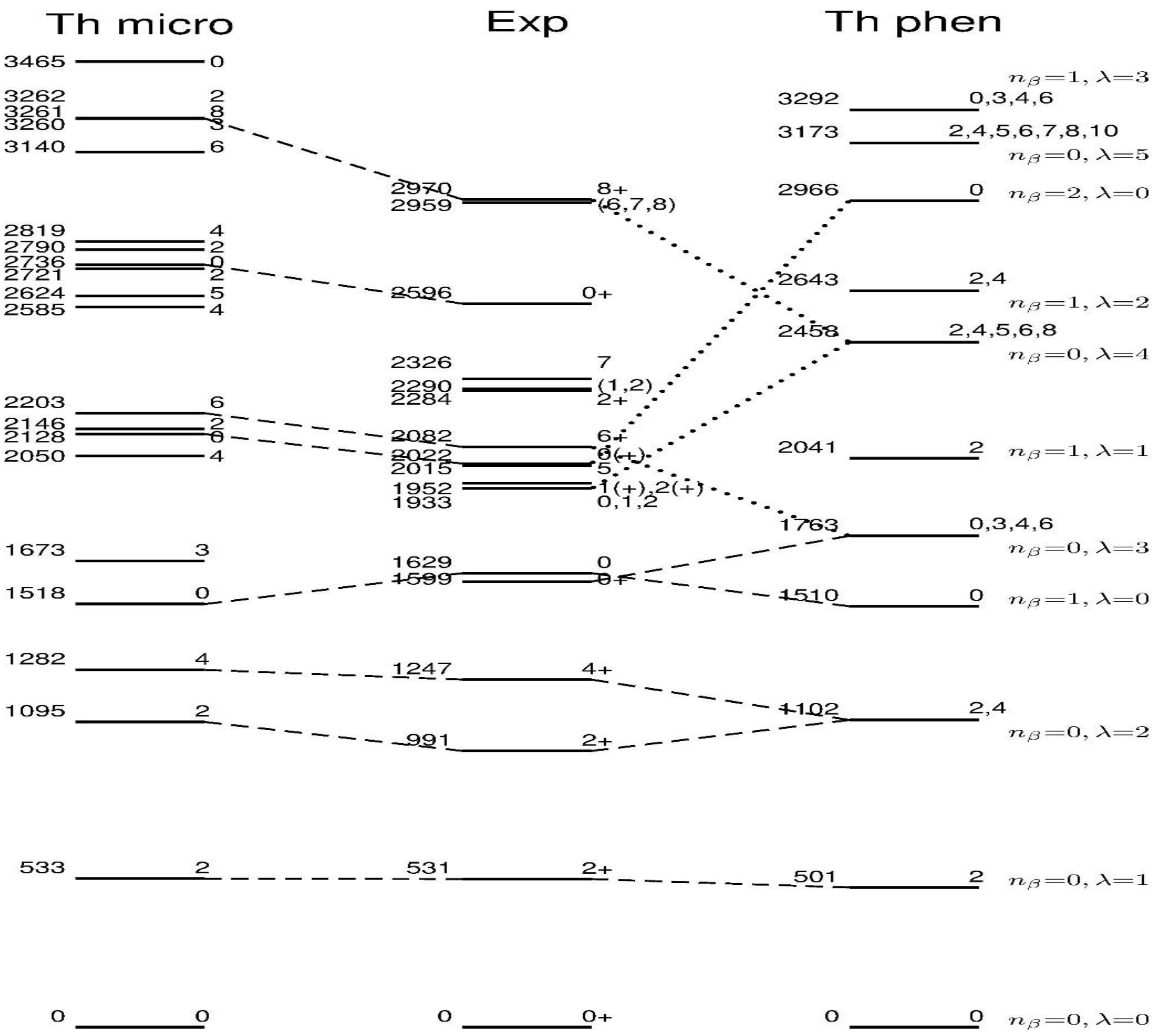

^{140}Sm

Justyna Samorajczyk-Pysk

50 < Z, N < 82 collective quadrupole

CONCLUSIONS

1. Simple phenomenological Warsaw model of γ-independent potential gave reasonable general pictures .
   However V and B far away from microscopic calculations 1974

2. γ-dependent B(ß,γ) help a lot. However still V and B far away from microscopic calculations. 1977

3. Only microscopic dynamic pairing and GBH gave proper energy scale of energy levels
   and good E2 matrix elements, ilustrated by ^{104}Ru case. 1977 - 2006

4. For ^{140}Sm similar wave functions and mean deformations for simple phenomenological Warsaw model
   and full microscopic GBH . 2021 experimental 0^+_3 - ?

5. G. Rohozinski, L. Prochniak, K. Pomorski, B. Pomorska, K. Zając, Ch. Droste, J. Samorajczyk-Pyśk, …..