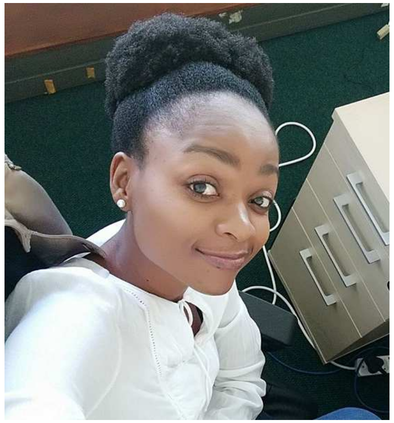
Nuclear Collective Excitations and Realistic Models

John Sharpey-Schafer et al.



Dr. Rob Bark
iThemba LABS

Dr. Suzan Bvumbi
NRWDI, Pelendaba

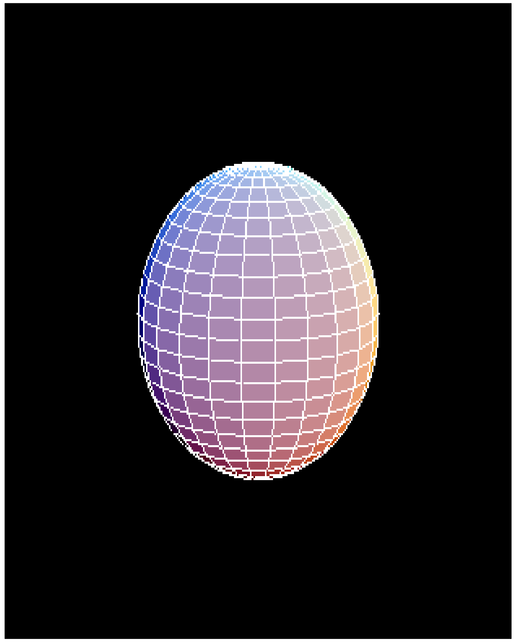
Dr. Tshepo Dinoko
NMISA, Pretoria

Dr. Siyabonga Majola
Uni. Johannesberg

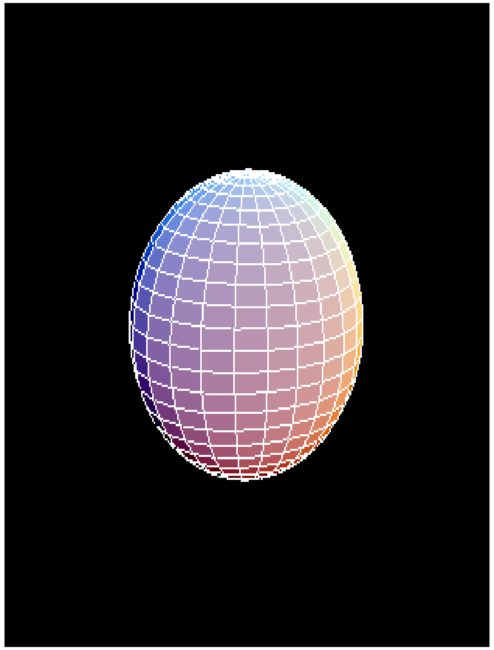
*Review Article; EPJA**55** (2019) 15*

"Stiff" Deformed Nuclei, Configuration Dependent Pairing and the β and γ Degrees of Freedom.

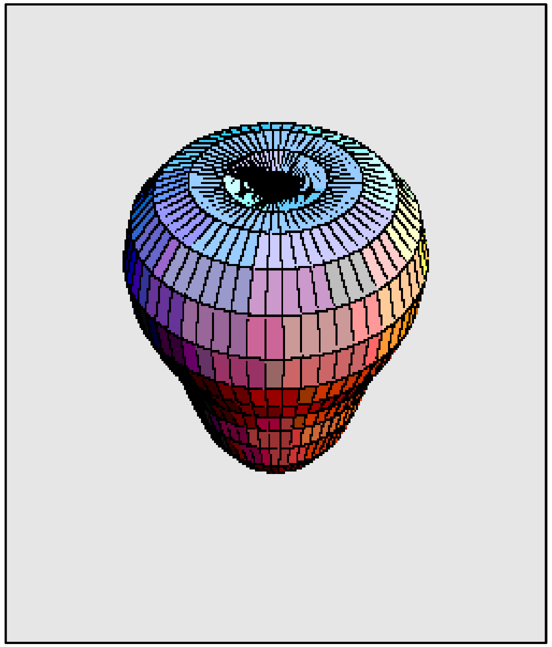
Examples of collective Classical time-dependent Vibrations of the Mean-field



$\lambda = 2$, $\mathbf{a}_{2,0}$
Quadrupole β vibration



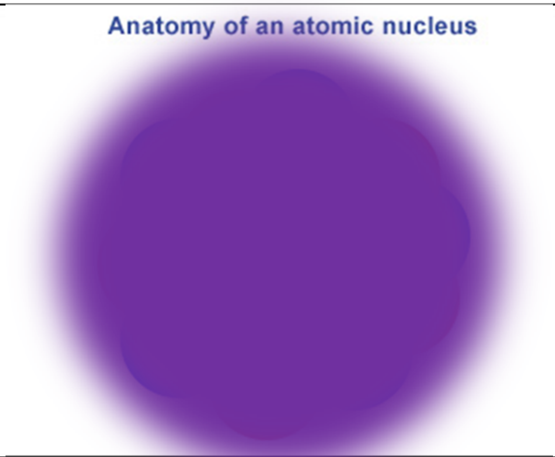
$\lambda = 2$, $\mathbf{a}_{2,2}$
Quadrupole γ vibration



$\lambda = 3$, $\mathbf{a}_{3,0}$
Octupole vibration

web-docs.gsi.de/~wolle/TELEKOLLEG/KERN/index-s.html

However !!   *Simple pictures can be Misleading*



An atomic nucleus is a compact bunch of postively charged protons (red) and uncharged neutrons (blue). The neutrons are the glue that holds the nucleus together, but if there's an excess of neutrons, the nucleus is unstable and will undergo radioactive decay to a more stable form. Image: Cam-Ann/Wikimedia Commons

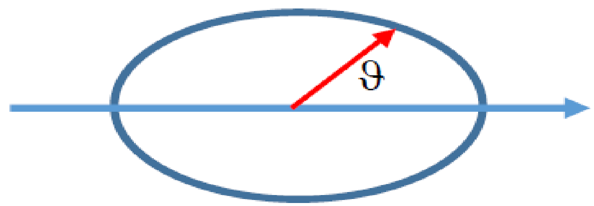
the **Heisenberg Uncertainty Principle**
tells us that the Nucleon-Nucleon Interaction
is not *Strong Enough to Localise the Nucleons*

You can only Measure <R^2>
for instance with electron scattering (e,e)

Assume axially symmetric (m=0) Shape

Expand shape in Legendre Polynomials

CLASSICALLY USE DIMENSIONAL ANALYSIS



$\mathbf{R}(r,\vartheta) = R\{1 + a_2P(\cos\vartheta) + a_3P_3(\cos\vartheta) \;\ldots.\} = R\{1 + \sum a_\lambda P_\lambda(\cos\vartheta)\}$

Consider the Dimensions supposing that $\omega^2 \propto \gamma^x \rho^y R^z$

ω^2 = frequency^2 of vibration with dimensions $[T]^{-2}$

γ = surface energy/unit area with dimensions $[M]\,[L]^2\,[T]^{-2}\,[L]^{-2}$

ρ = density with dimensions $[M]\,[L]^{-3}$

R = radius with dimensions $[L]$

Therefore $[T]^{-2} = [M]^x\,[T]^{-2x}\;[M]^y\,[L]^{-3y}\;[L]^z$

So that
$-2 = -2x$ ie. $x = 1$
$0 = x+y$ ie. $y = -1$
$0 = -3y + z$ ie. $z = -3$

$$\omega^2 \propto \gamma / \rho R^3$$

Classical Vibrations of a Liquid Drop

By considering a superfluid incompressible liquid sphere

Lord Rayleigh (John William Strutt) Proc. Roy. Soc. 29,71 (1879) Appendix II Equ. 40, got:

$$\omega^2 = \frac{(\lambda-1)\lambda(\lambda+2)\gamma}{\rho R^3}$$

Also See:
S Flügge, Ann Phys Lpz 431 (1941) 373

For a charged spherical nucleus this becomes :
www.eng.fsu.edu/~dommelen/quantum/style_a/nt_liqdrop.html

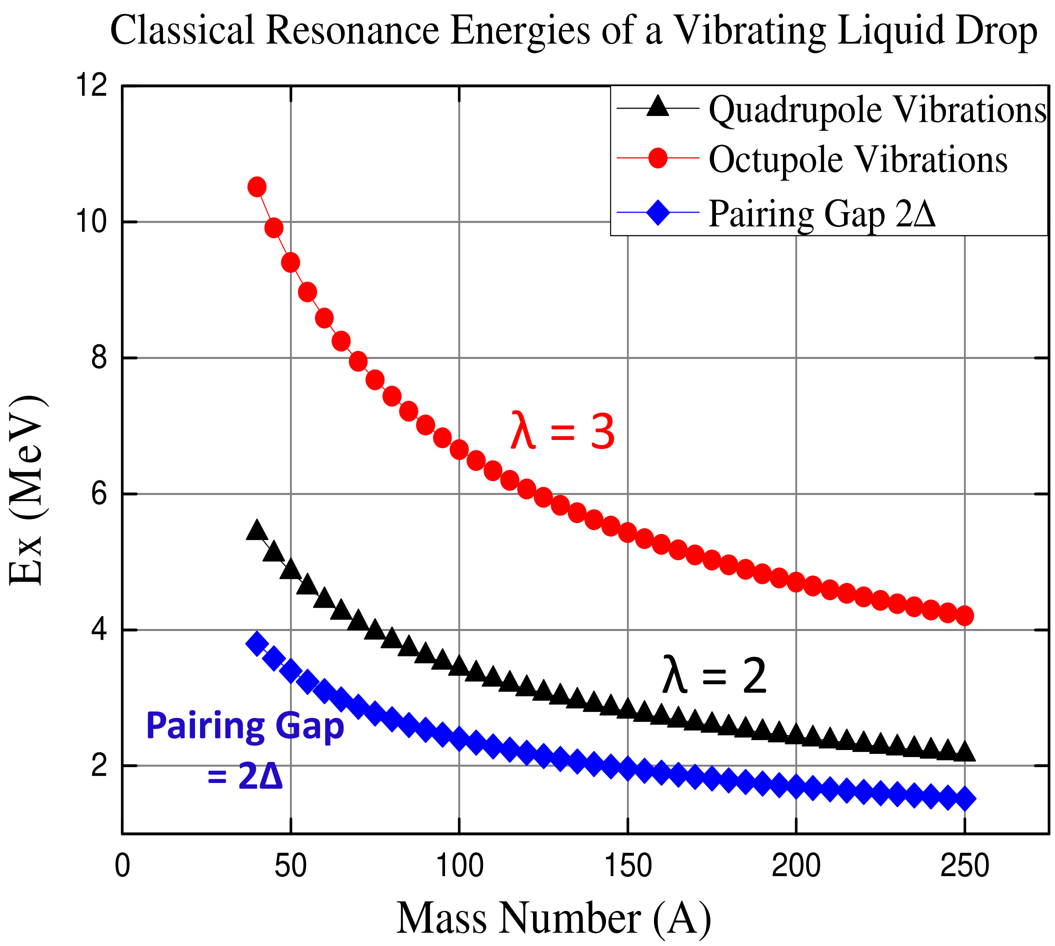
$$\omega^2 = \frac{(\lambda-1)\lambda(\lambda+2)}{3}\frac{C_s}{R_A^2 mA} - \frac{2(\lambda-1)\lambda}{(2\lambda+1)}\frac{e^2Z^2}{4\pi\epsilon_0 R_A^3 m_p A^2}$$

Where C_s is the SURFACE term in the Weizsäcker Binding Energy formula

$$E_b = C_V - C_S A^{2/3} - C_C Z^2 A^{-1/3} - C_A (N-Z)^2 A^{-1} \pm \delta$$

and the second term has little effect for Z<80.

Quantization of the Vibrations of a Liquid Drop



Quantize Using $E_x = \hbar\omega$
and

$$\omega^2 = (\lambda - 1)\lambda(\lambda + 2)C_s/3{R_A}^2 mA$$

Where $C_S \sim 18$ ***MeV and*** $R_A = 1.3$ ***fm***

The Classical Result is that

Vibrations are WELL above the Pairing Gap

Pairing Energy $\Delta \approx 12/A^{1/2}$ **MeV**
From Bohr and Mottelson

Other Factors Affecting Vibrations

1. Moments–of-Inertia not superfluid ➡ ***will put vibrational energy UP*** ⬆

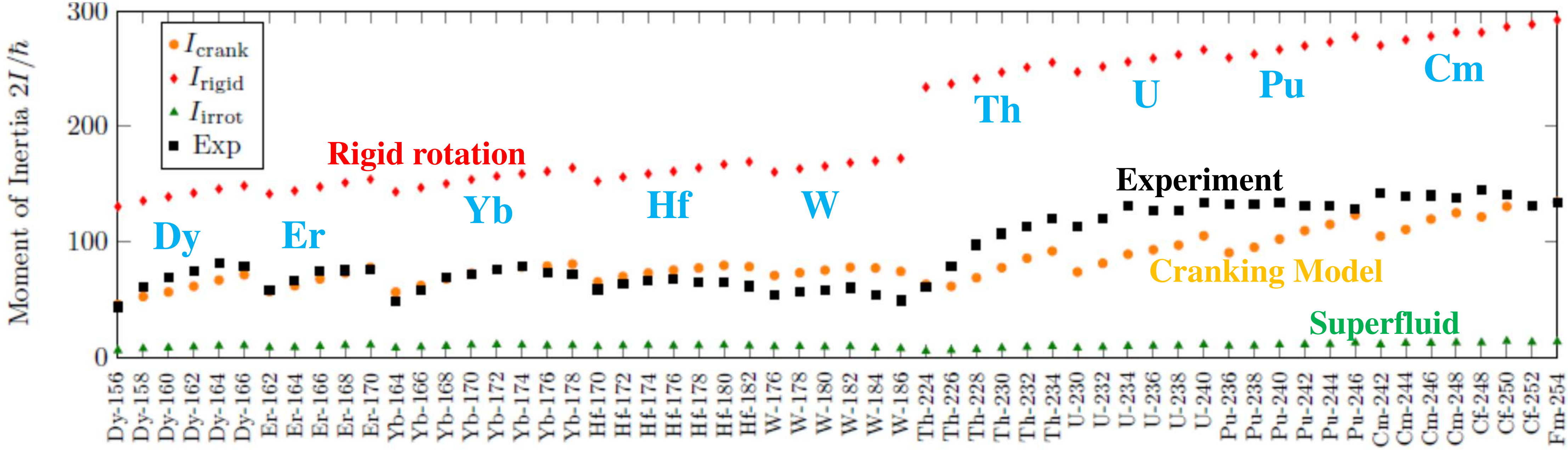


Fig. 1. Comparison of the different models for moment of inertia with experimental data.

P Tamagno & O Litaize, EPJ Web of Conf., **193**,01004 (2018)
Inglis-Belyaev Cranking code CONRAD

2. Shell Corrections → *will put vibrational energy UP*

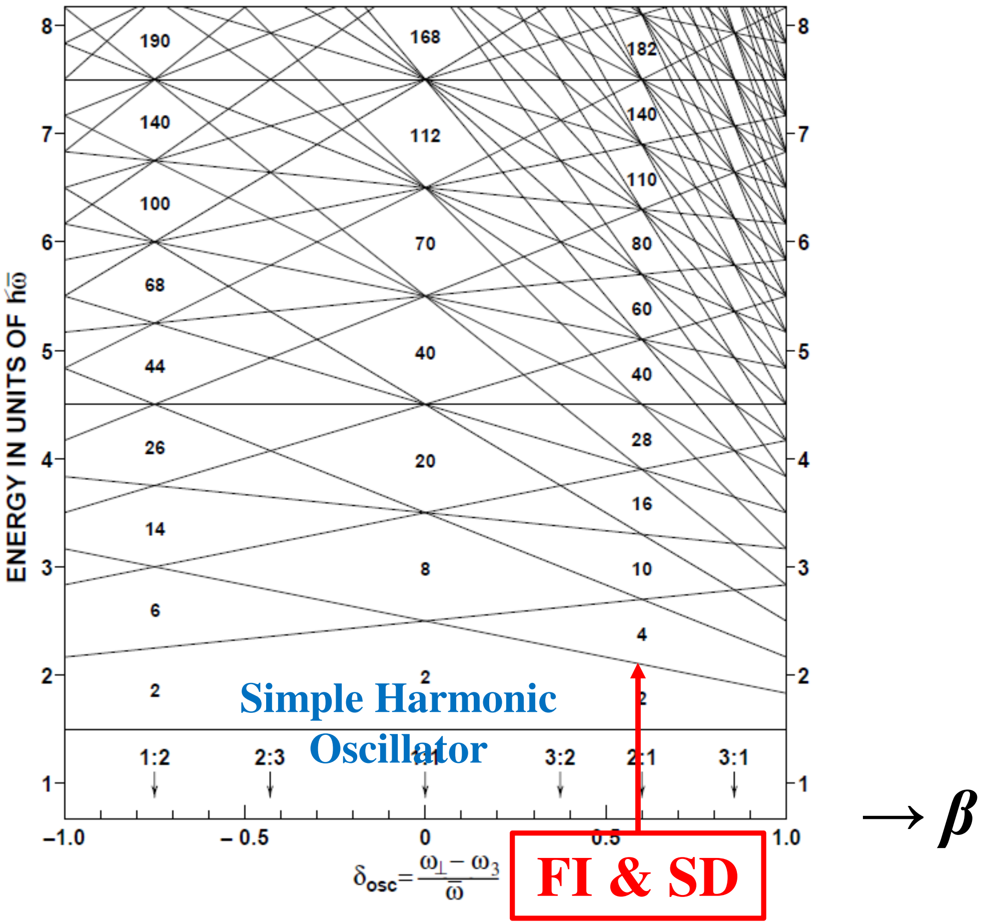
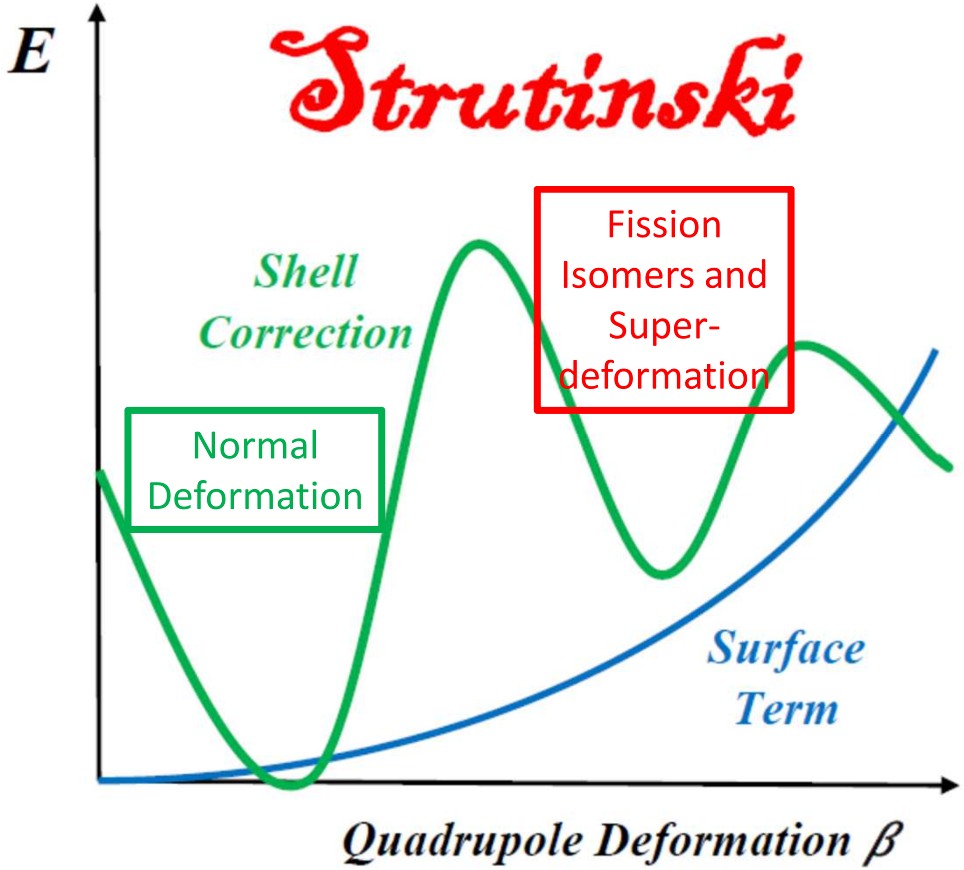
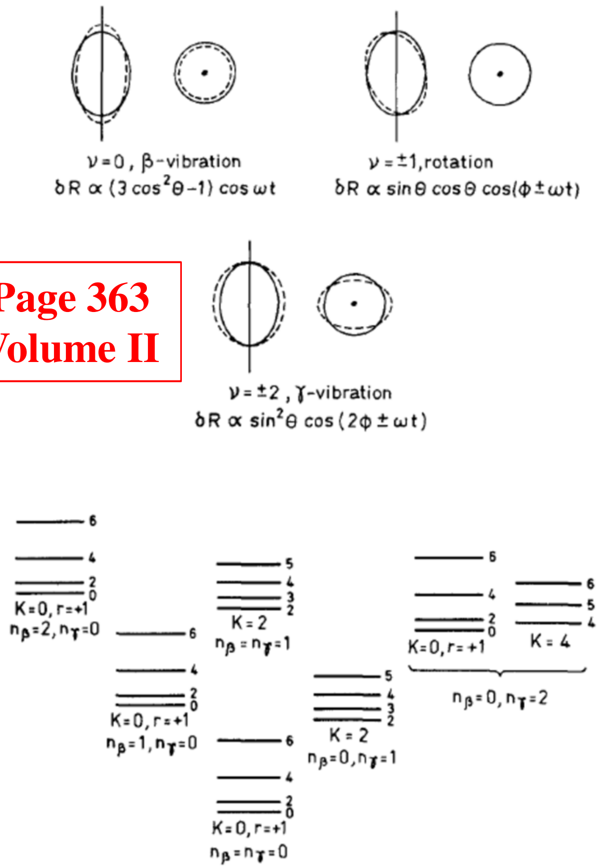


Figure 17. Single-particle level energies calculated for an axially symmetric harmonic oscillator (from reference 18).



The Bohr and Mottelson Approach

MODES OF NUCLEAR VIBRATION



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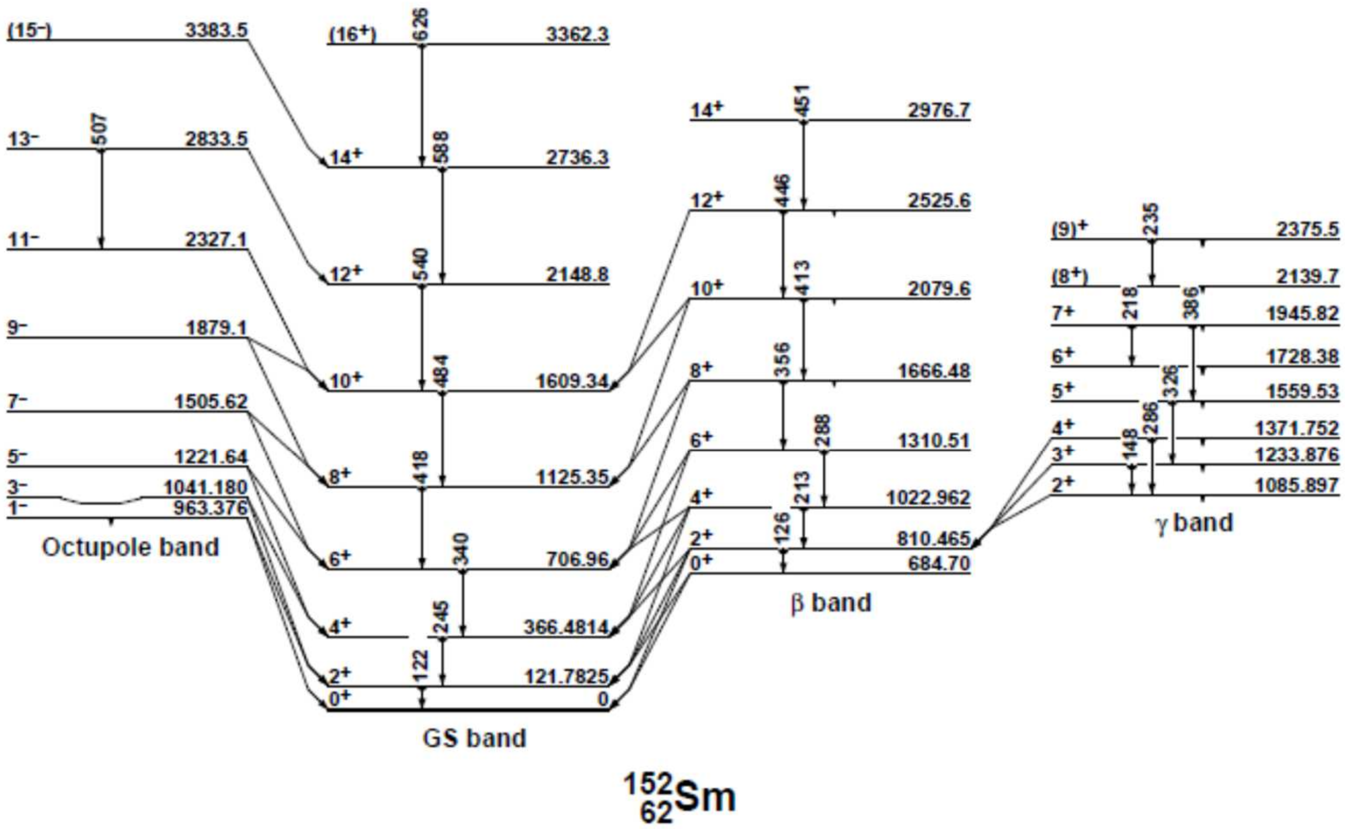
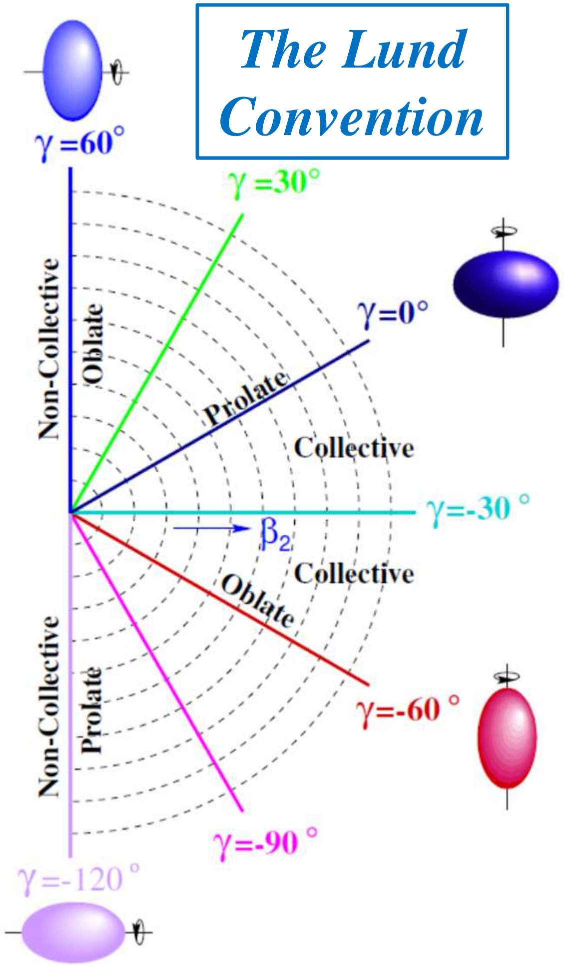
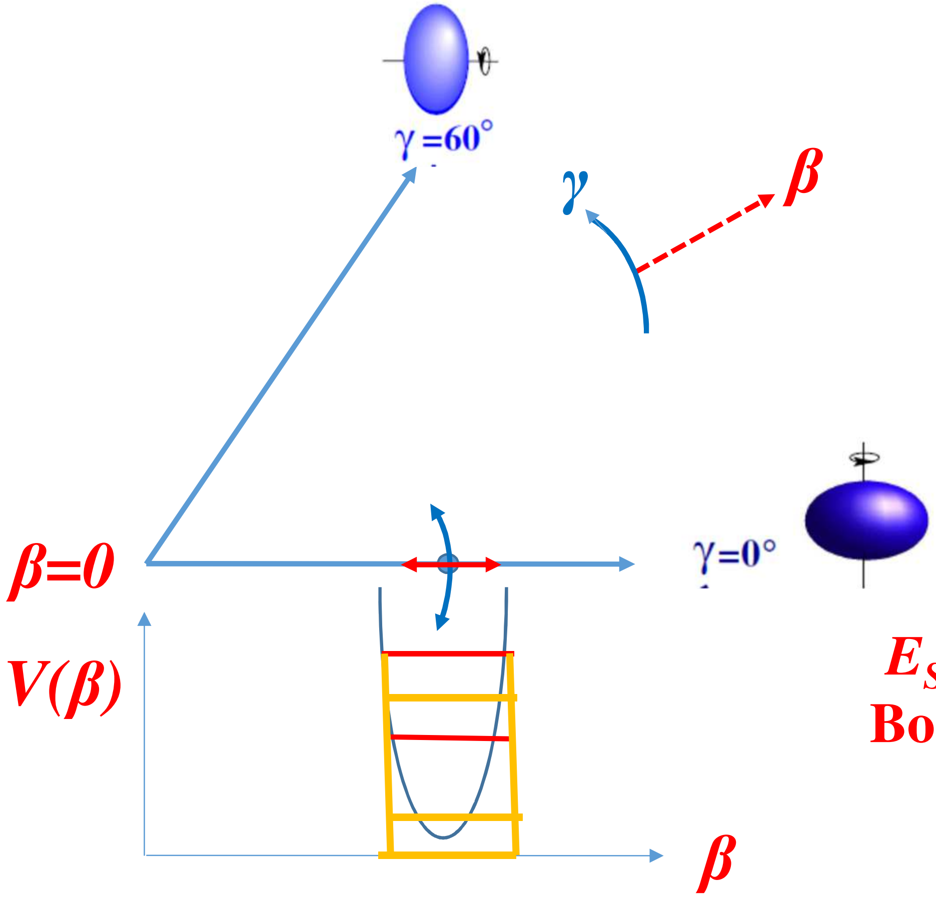


Figure 3. Ground-state, and β-, γ-, and octupole-vibrational bands in ^{152}Sm.

The β and γ Quadrupole Degrees of Freedom





E_{SHO} = (n+½) ħω
Bohr & Mottelson

$E_{SqW} \propto n^2$
Iachello X(5)

UNEXPECTED STRONG PAIR CORRELATIONS IN EXCITED 0^+ STATES OF ACTINIDE NUCLEI*

J. V. Maher, J. R. Erskine, A. M. Friedman, J. P. Schiffer,† and R. H. Siemssen
Argonne National Laboratory, Argonne, Illinois 60439
(Received 1 June 1970)

The (p,t) reaction has been studied with 17-MeV protons on targets of Th^{230}, $U^{234,236,238}$, and $Pu^{242,244}$. The results indicate unexpectedly strong $l=0$ transitions to states at about 900-keV excitation. Their cross sections are approximately 15 % of the ground-state transitions; this percentage does not change appreciably with neutron number. This result, together with other available evidence, seems to suggest a simple and rather stable collective mode which has not yet emerged from any theoretical calculations.

J V Maher et al.
PRL 25 (1970) 302

VOLUME 25, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1970

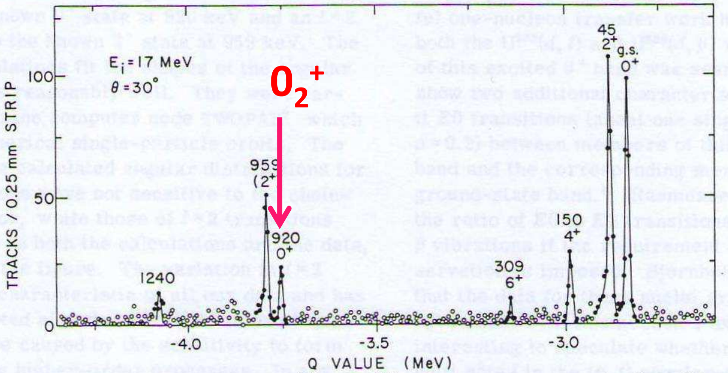


FIG. 1. Spectrum of tritons from the reaction $U^{238}(p,t)U^{236}$. The target was 35 $\mu g/cm^2$ of U^{238} evaporated onto a carbon foil. The peaks are labeled by the excitation energies (keV) and spins of the corresponding states in U^{236}.

$^{238}U(p,t)^{236}U$
17 MeV
$\theta = 30^o$

0_2^+ ***NOT*** a β-vibration
NOR a pairing vibration

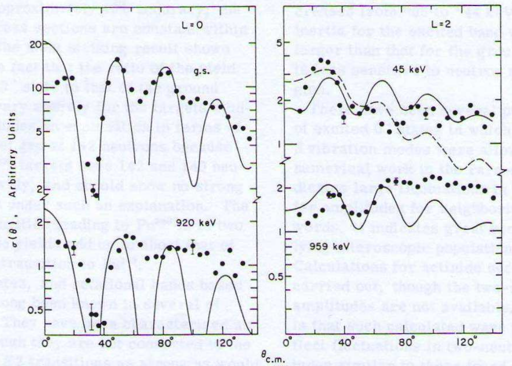


FIG. 2. Angular distributions for the reaction $U^{238}(p,t)U^{236}$. The relative yields for the various experimental data sets are correctly shown. The cross section at ~60° for the ground-state transition is 220 ± 80 μb/sr. The DWBA curves were calculated with a spherical $3d_{5/2}$ form factor for the solid curves and $1j_{15/2}$ for the dashed curve. Relative error bars are shown on a few representative points.

Two Neutron Transfer to ^{154}Gd (N=90)

Shiro Yoshida, Nucl. Phys. 33, 685 (1962)

Showed that with Monopole Pairing ALL the TWO neutron Transfer strength will be Decanted into the Residual Ground State

SEE ALSO

R.J. Ascuitto, B. Sorensen, Nucl. Phys. A 190, 297 (1972)

*Shahabuddin et al; NP **A340** (1980) 109*

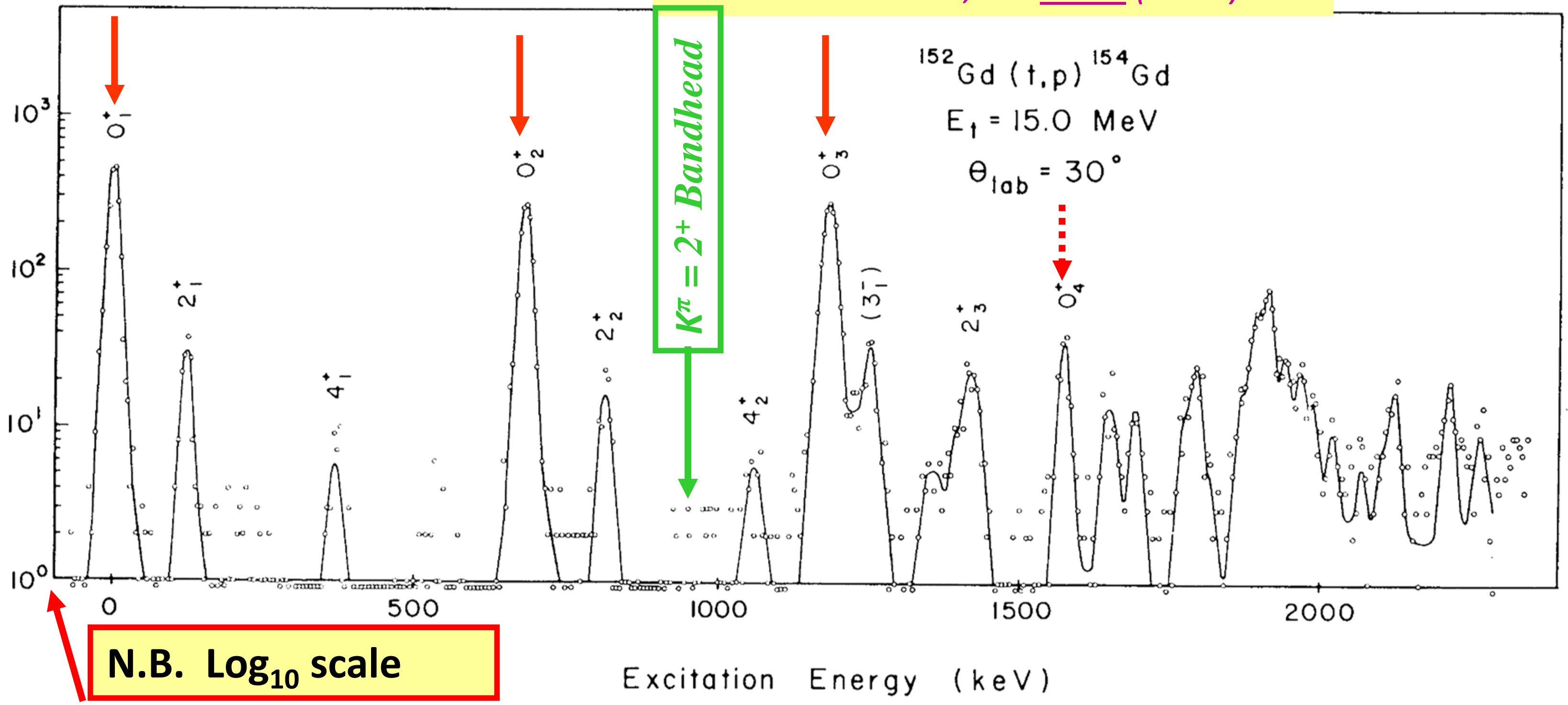
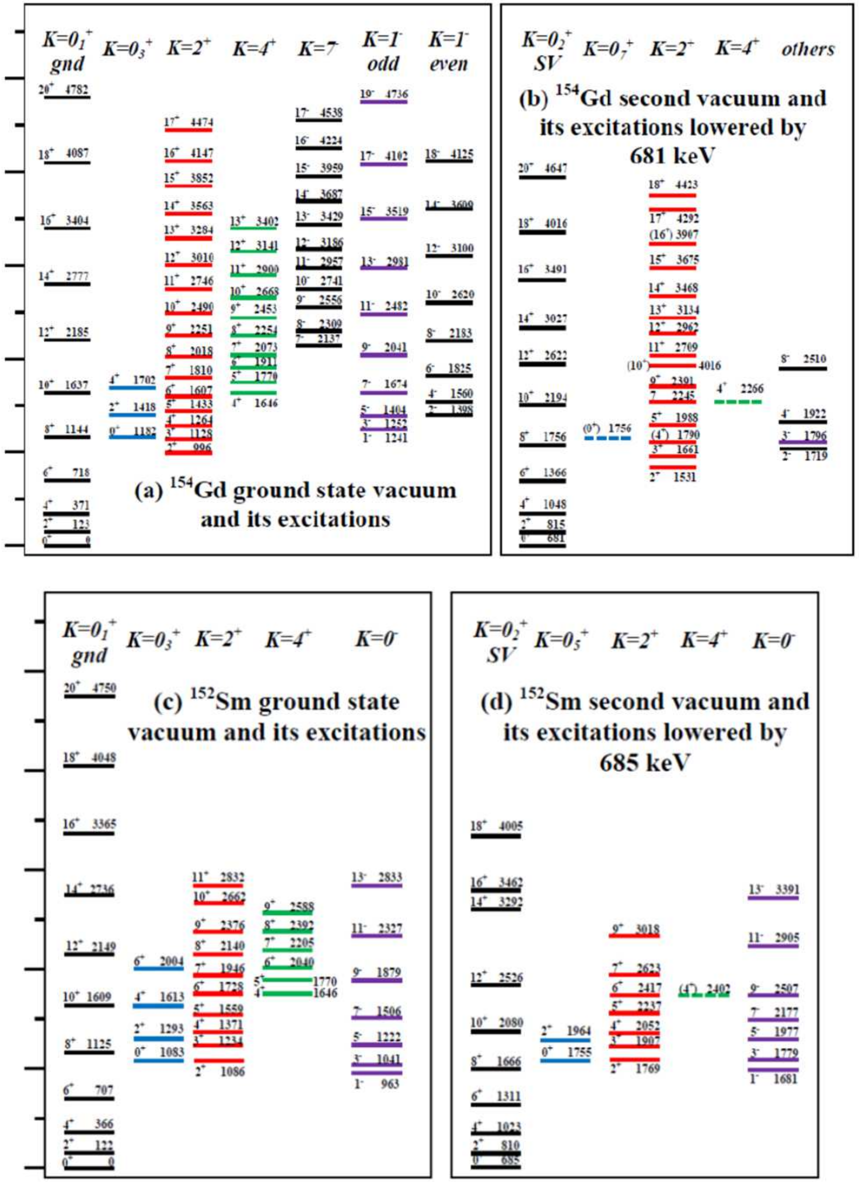


Fig. 1. A sample spectrum of the ^{152}Gd(t, p)^{154}Gd reaction at E_t = 15 MeV and θ_{lab} = 30°.

HENCE Monopole Pairing is NOT Sufficient



Similar Structures Built on the Ground state 0_1^+ and Second Vacuum 0_2^+ state in ^{154}Gd and ^{152}Sm

What is the $|0_2^+>$ Configuration ?

- (t,p) & (p,t) ⇨ $|0_2^+>$ **is $2p_n$- $2h_n$**

- **this gives J^π but <u>nothing</u> on the orbit.**

- **Single particle transfer would give l_n but does not populate $|0_2^+>$.**

⇨ **In { $|0_2^+>$ + neutron }, look to see which orbit does *<u>NOT</u>* couple to $|0_2^+>$.**

Configuration Dependent Pairing

R. E. Griffin, A. D. Jackson and A. B. Volkov, Phys. Lett. 36B, 281 (1971).

Suggested that $\Delta_{pp} \approx \Delta_{oo} >> \Delta_{op}$

for Actinide Nuclei where 0_2^+ states were observed in (p,t) that were not pairing- or β-vibrations.

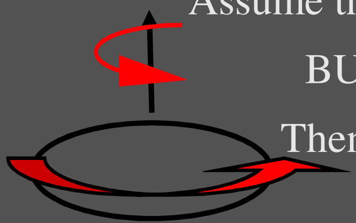
Suppose there are *n prolate* and *n oblate* degenerate levels at the Fermi Surface;

Assume that each pairing matrix element is the same for the same type *-a*

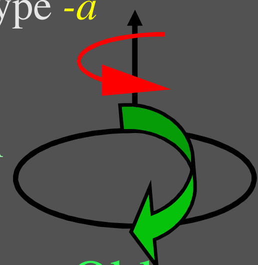
BUT the *prolate-oblate* matrix elements are very weak *–εa*

Then if the prolate *n*n* matrix is *A*, the oblate matrix is also *A*

The matrix for the total system is;

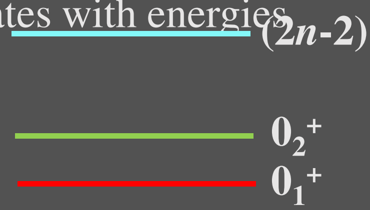

Prolate

$$\begin{bmatrix} A & \varepsilon A \\ \varepsilon A & A \end{bmatrix}$$


Oblate

Then there are (2*n*-2) states with ZERO energy and 2 states with energies

$$E_{1,2} = -(1 \pm \varepsilon)\, na$$



I. Ragnarsson and R. A. Broglia, Nucl. Phys. A263, 315 (1976).
coined the term "pairing isomers" for these 0_2^+ states

PHYSICAL REVIEW C VOLUME 15, NUMBER 6 JUNE 1977

Analysis of the (p,t) reaction on ^{158}Dy†

J. J. Kolata
Brookhaven National Laboratory, Upton, New York 11973

M. Oothoudt*
Princeton University, Princeton, New Jersey 08540

(Received 28 February 1977)

Jim Kolata and Mike Oothoudt
Phys. Rev. C15 (1977) 1947

N.B.
Log_{10}
Scale

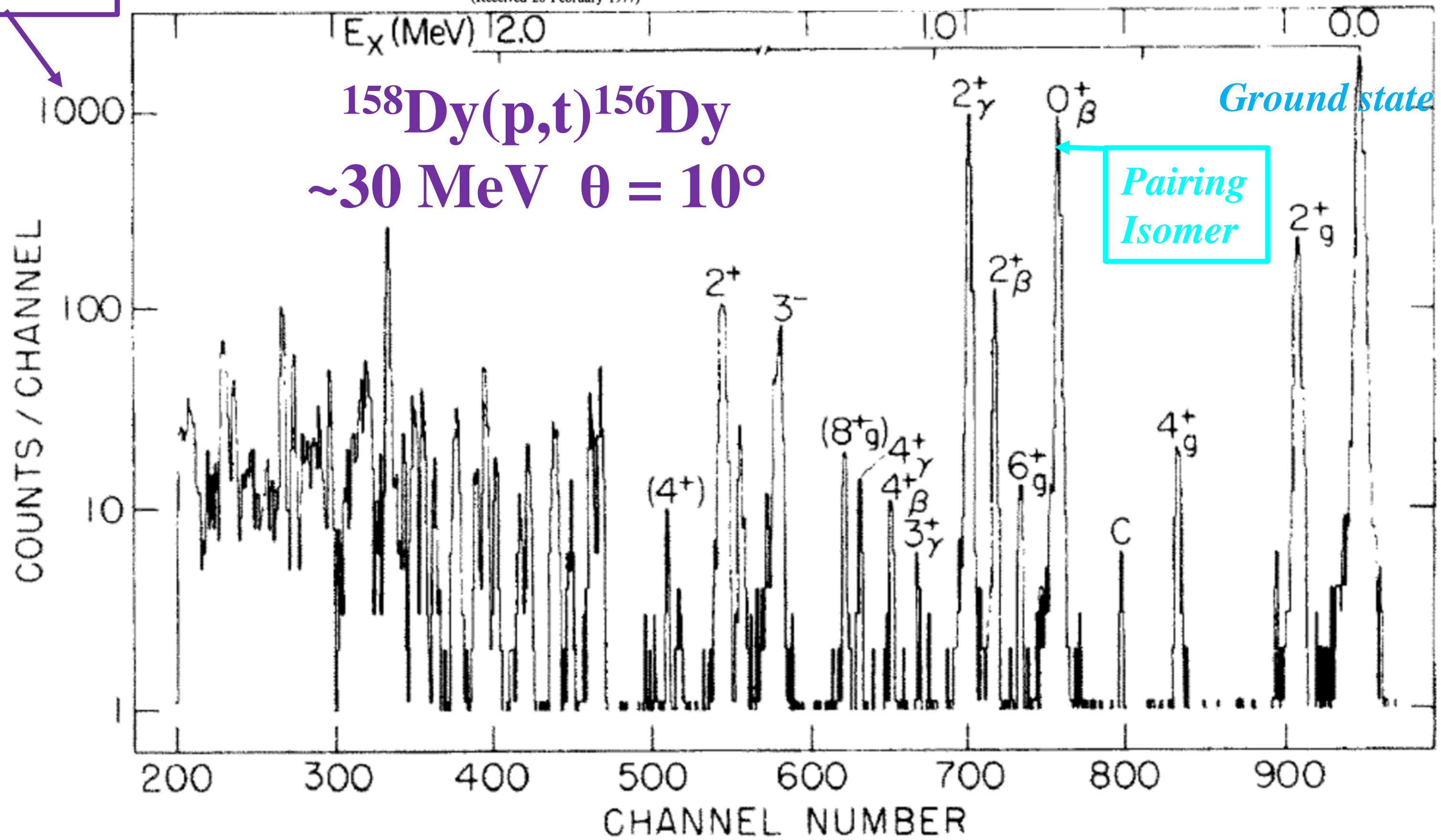
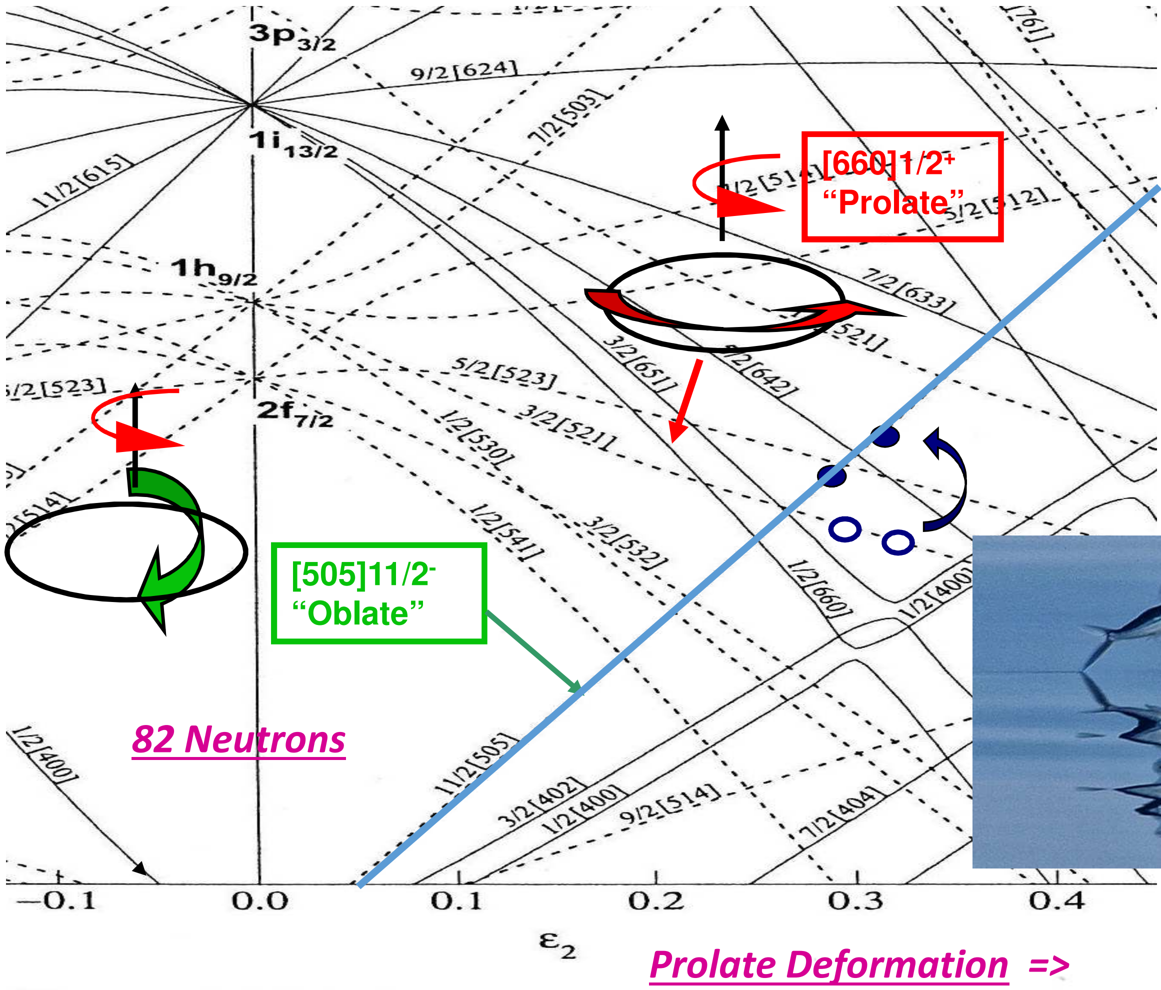


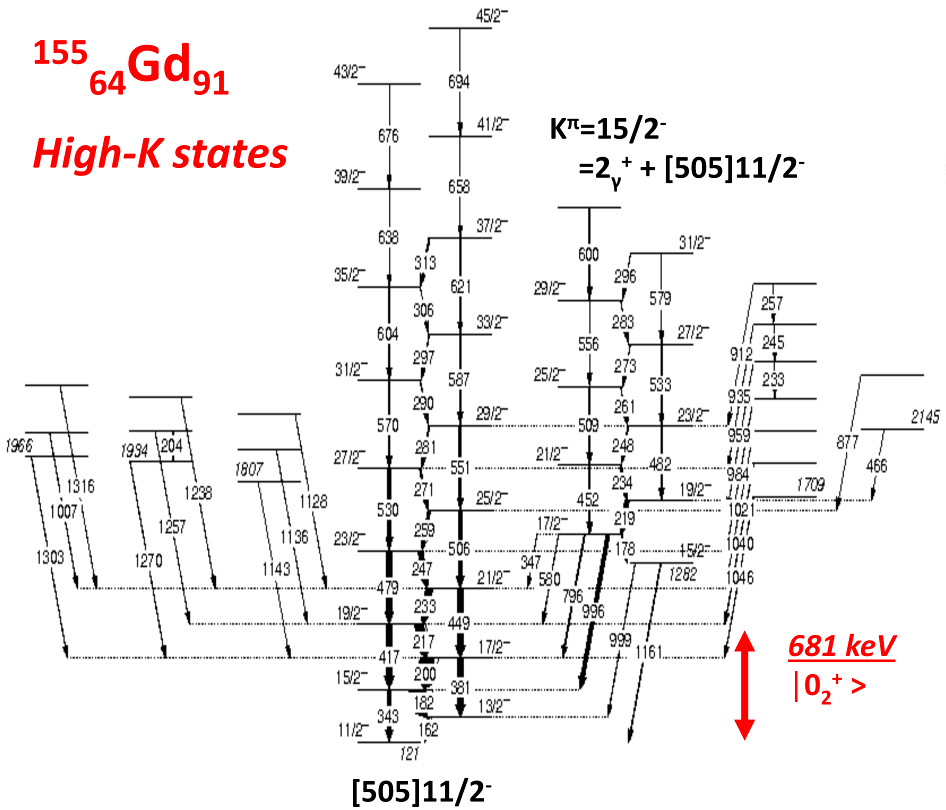
FIG. 1. Typical spectrum for the ^{158}Dy$(p,t)^{156}$Dy reaction at $E_p = 29.9$ MeV and $\theta_{lab} = 10°$. This is a com-



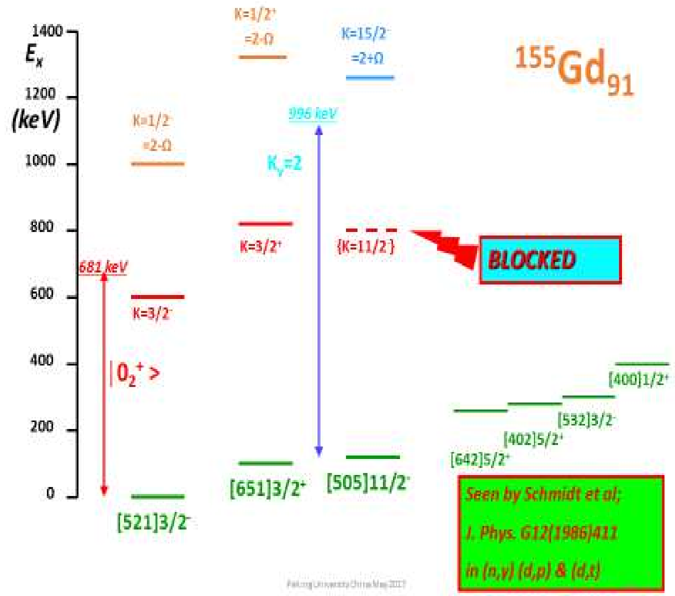
Configuration Dependent or Quadrupole Pairing; Assume $\Delta_{pp} \approx \Delta_{oo} >> \Delta_{op}$

$^{155}_{64}Gd_{91}$

High-K states



JFS-S et al. EPJ A47 (2011) 6



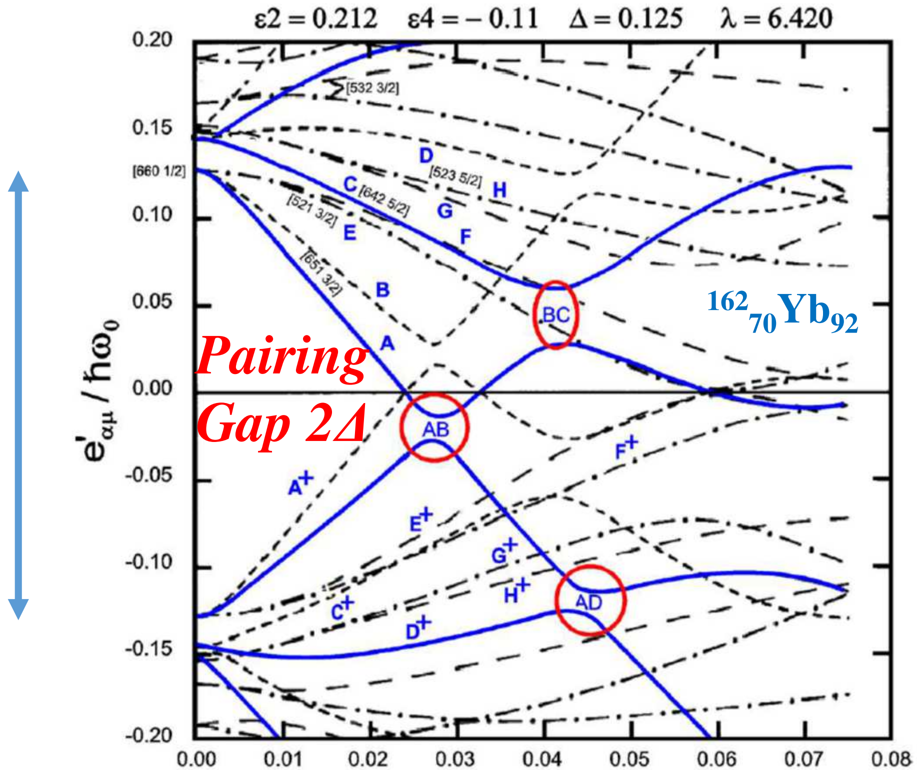
$\varepsilon \approx 0.05$??

What you really need is
SPLIT MONOPOLE PAIRING
so that

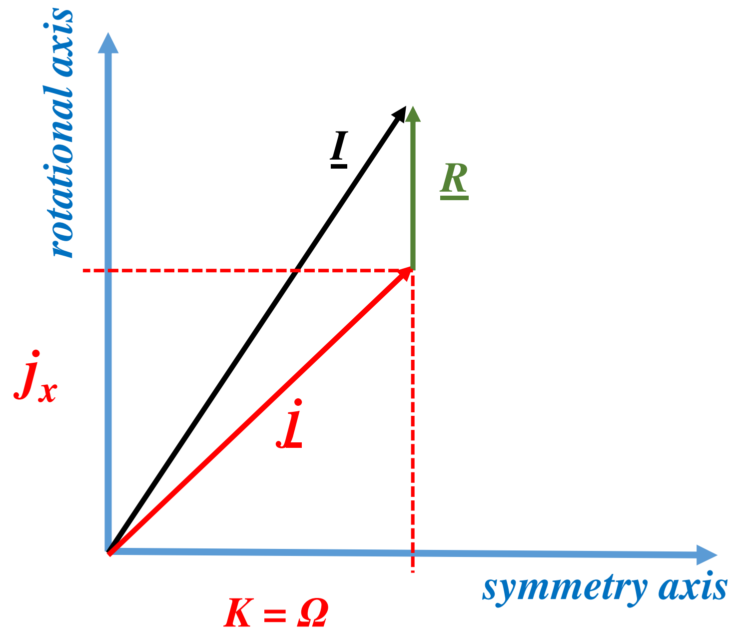
$$-\mathcal{H}_{pairing} = G_{p\text{-}p}\dot{P}_p^{\dagger}\dot{P}_p + G_{o\text{-}o}\dot{P}_o^{\dagger}\dot{P}_o + \varepsilon G_{p\text{-}p}\dot{P}_{po}^{\dagger}\dot{P}_{po}$$

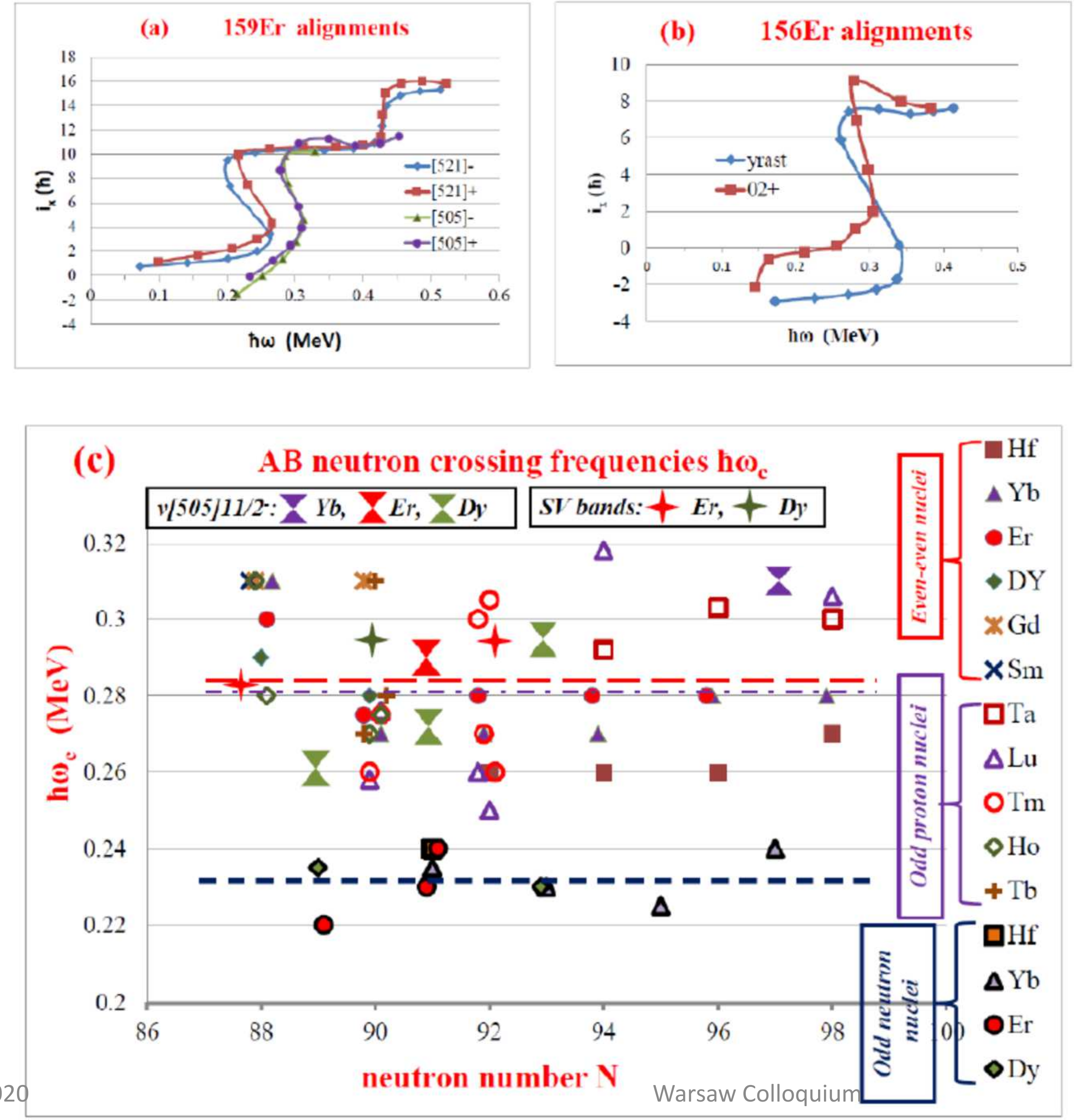
HOW TO MEASURE THE PAIRING ??

Use the Cranked Shell Model !!



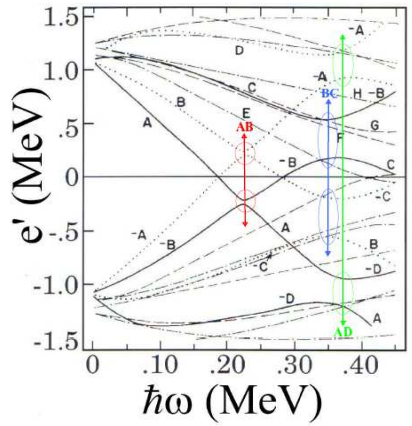
Coriolis term in the Hamiltonian = - j_x ω



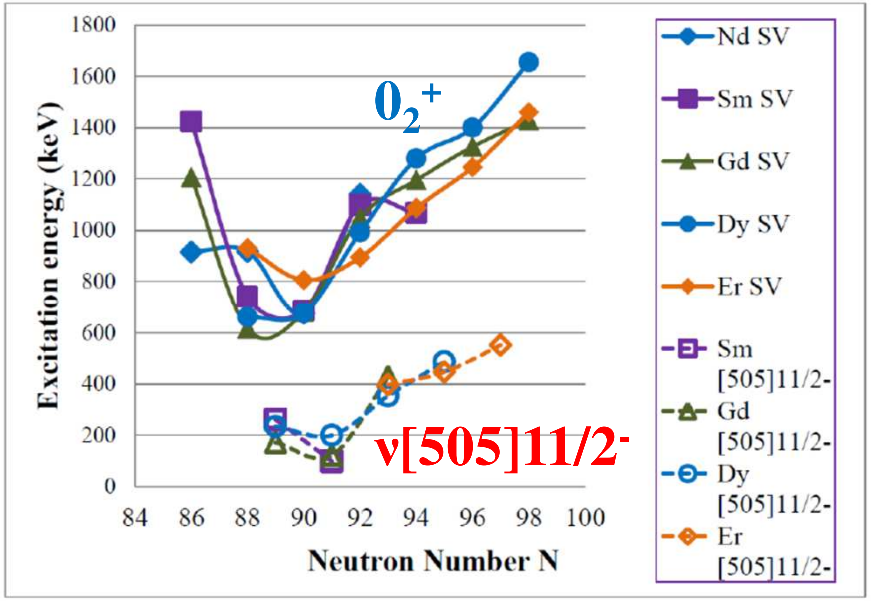


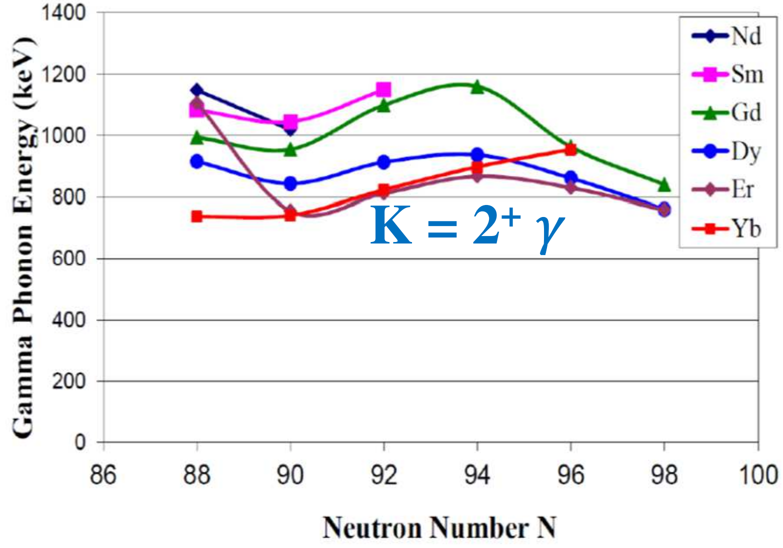
$i_{13/2}$ neutron AB alignments from N = 88 to 98 and Z = 62 to 72

See:
Jerry Garrett et al.
PL B118 (1982) 297



Cranked Shell Model Routhians *e'*

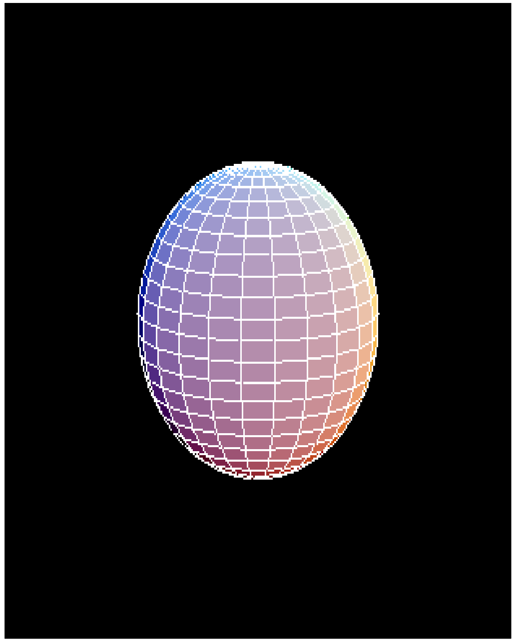


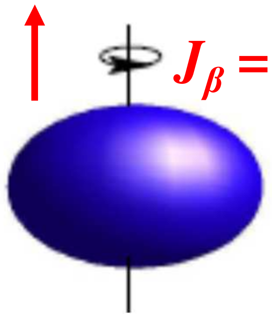


Excitation Energies of 0_2^+ band-heads and ν[505]11/2$^-$ isomers

Excitation Energies of K = 2^+ γ band-heads

Quantum Number $K = I_z$ ***the spin projection on the*** $\gamma = 0°$ ***symmetry axis***

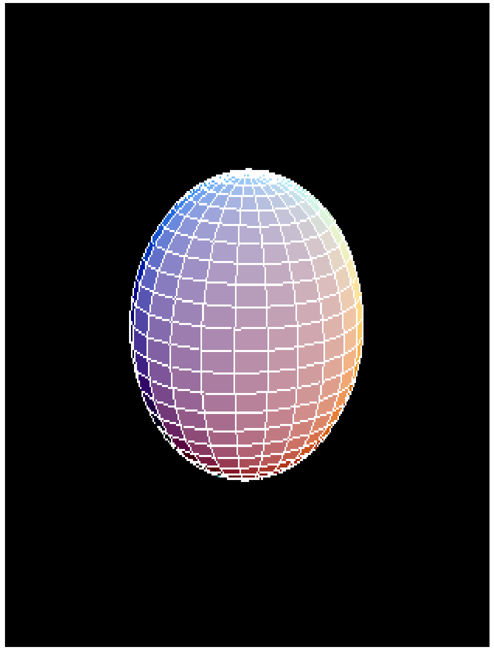


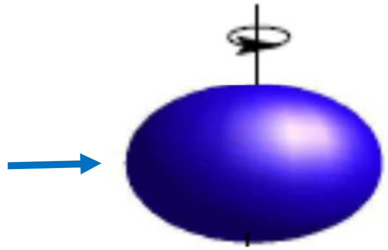


$J_\beta = 0$

$K_\beta = 0$

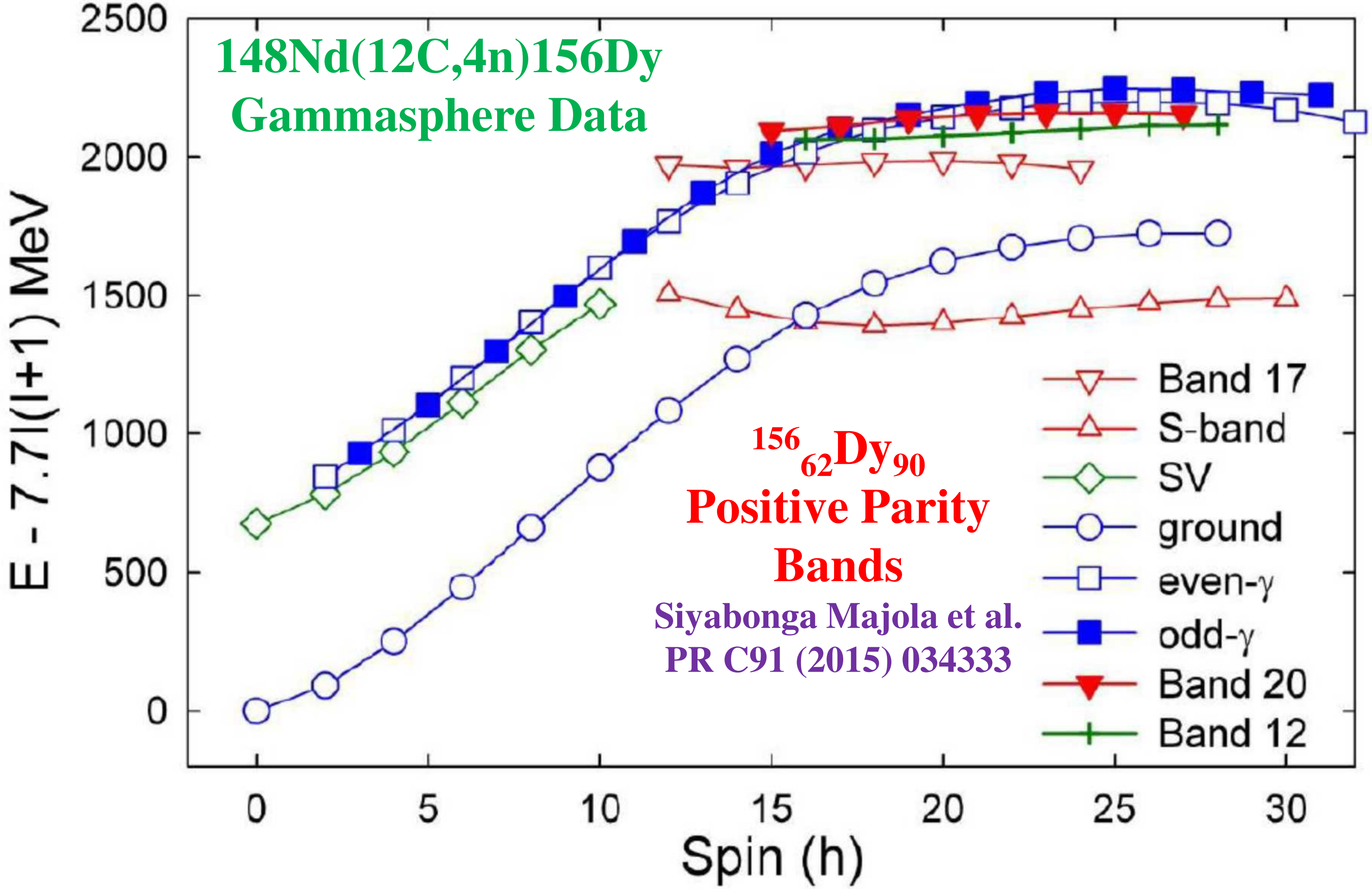
$\gamma=0°$



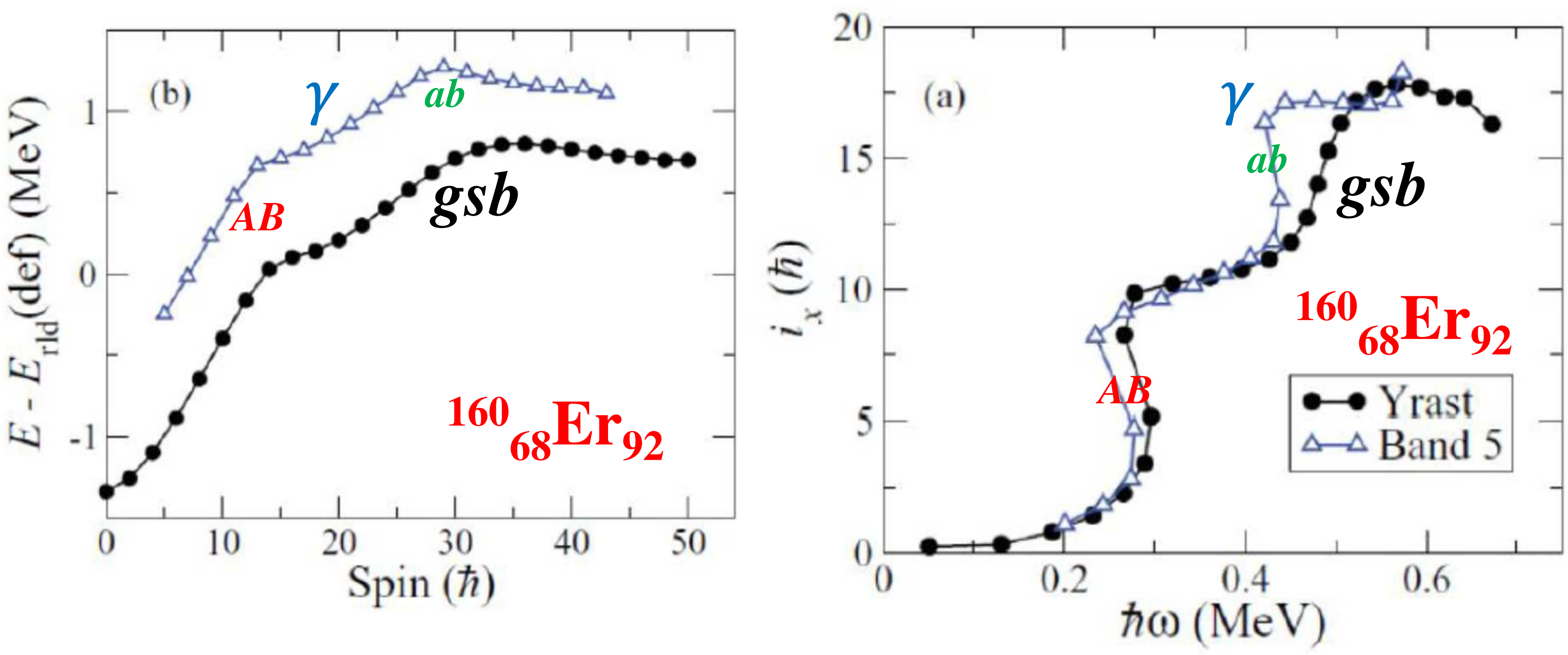


$J_\gamma = 2$

$K_\gamma = 2$



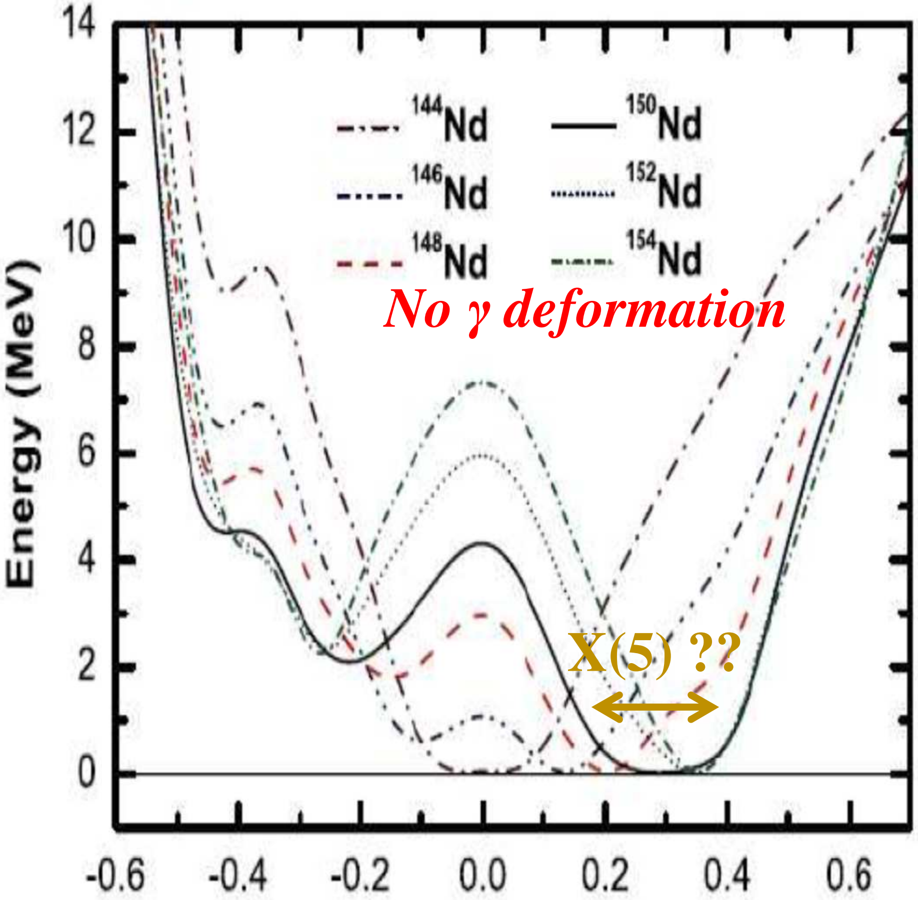
148Nd(12C,4n)156Dy
Gammasphere Data
156 62 Dy 90
Positive Parity
Bands
Siyabonga Majola et al.
PR C91 (2015) 034333
Band 17
S-band
SV
ground
even-γ
odd-γ
Band 20
Band 12
E - 7.7I(I+1) MeV
Spin (h)
2500
2000
1500
1000
500
0
0
5
10
15
20
25
30



Tracking of ***γ band*** ***through*** $i_{13/2}$ ***neutron AB alignment*** ***and*** $h_{11/2}$ ***proton alignment ab***

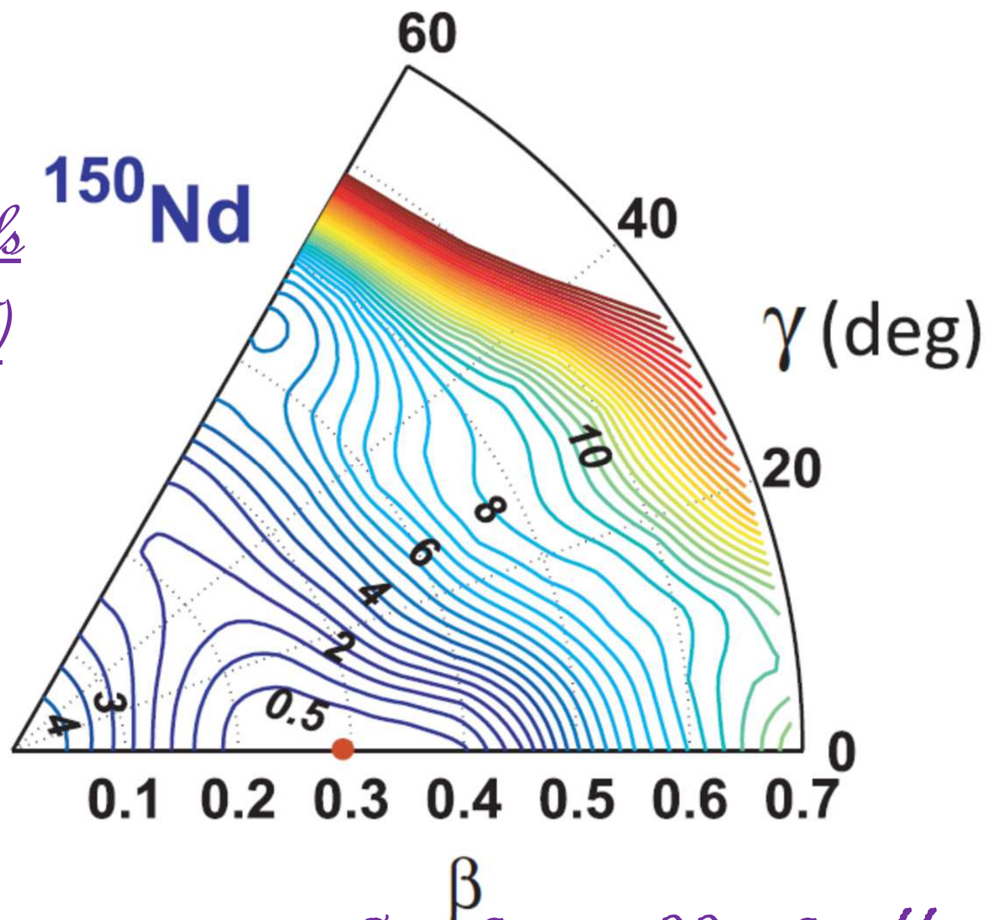
Ollier et al. PR C83 (2011) 044309

Nikšić et al., PRL99 (2007) 092502



Relativistic Energy-Density Functionals (REDF)

Li et al., PR C79 (2009) 054301



γ =60° ***Oblate*** ⟵ β ⟶ ***Prolate*** γ=0°

You cannot ignore the γ degree of freedom !!

Hope in Shell Models ??

DATA FROM

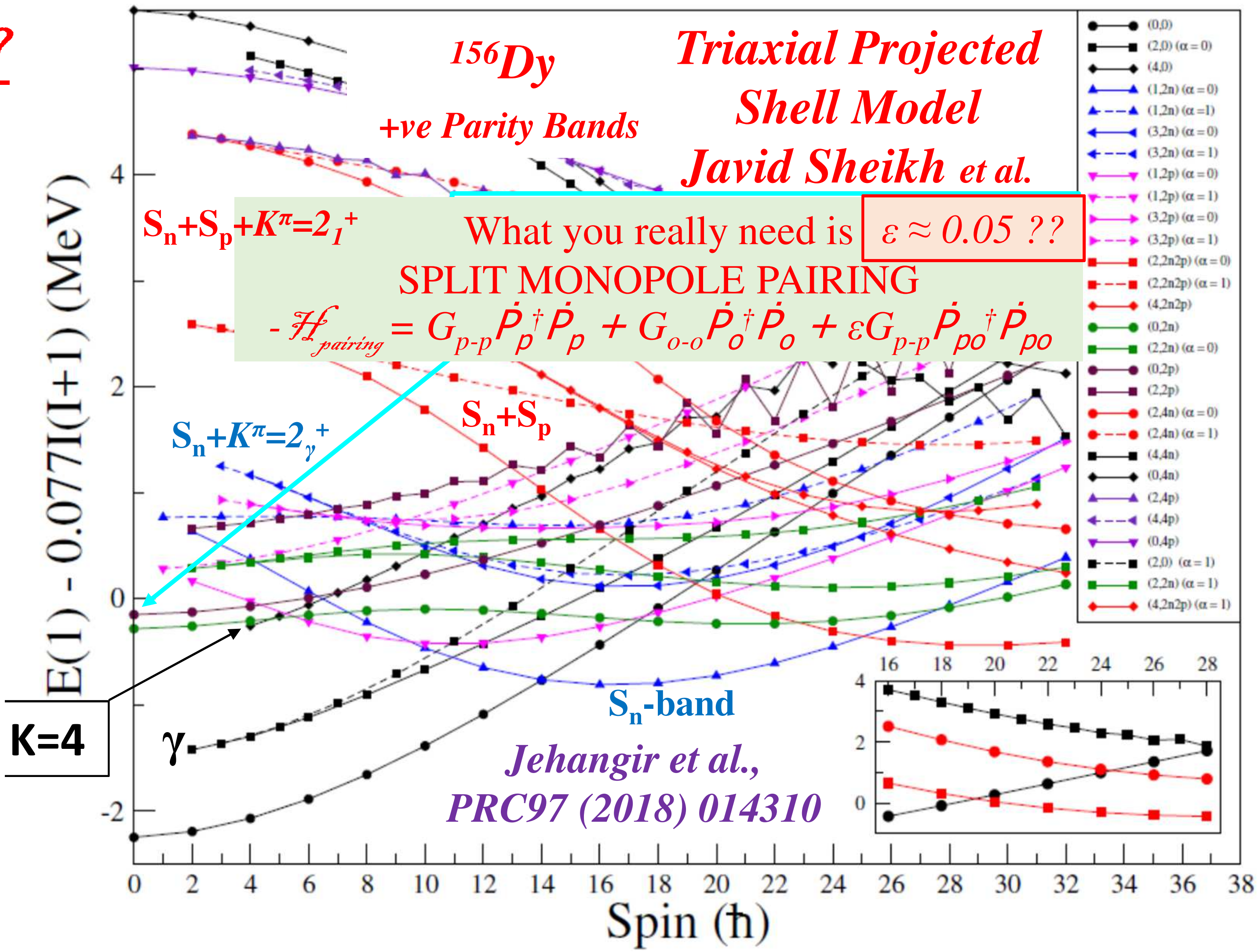
Majola et al. PRC91 (2015) 034330

TPSM Successes

1. *Predicts γ and γγ bands*
2. *Predicts S_n-band* and *S_n+S_p -band*
3. *Predicts observed γ band built on S_n-band*
4. *Predicts an S_n-band built on 0_2^+*
5. *Can show components of Wavefunctions*

TPSM Failures

1. *Pairing too crude, No Neutron Pairing Isomer 0_2^+ too high in Energy*
2. *Signature Splitting not spot on*



The Bohr Hamiltonian
Uses a 5-D Space ($\theta,\varphi,\psi,\beta,\gamma$) to
Characterize a Macroscopic Nuclear Drop
Rotating and Vibrating in Space

Quantization is achieved by the usual Pauli p

$$E(\alpha,\dot{\alpha}) = \tfrac{1}{2}C\alpha^2 + \tfrac{1}{2}D\dot{\alpha}^2$$

momentum $$\pi = \frac{\partial}{\partial\dot{\alpha}}(T - V) = D\dot{\alpha}$$

$$H = \tfrac{1}{2}D^{-1}\pi^2 + \tfrac{1}{2}C\alpha^2$$

$$E(n) = (n + \tfrac{1}{2})\hbar\omega \qquad \omega = \left(\frac{C}{D}\right)^{1/2}$$

quantization $$[\pi,\alpha] = -i\hbar$$

Bohr & Mottelson II Ch.6

Th

H

$$\hat{H} = \hat{T}_{\mathrm{vib}} + \hat{T}_{\mathrm{rot}} + V_{\mathrm{coll}},$$

with the vibrational kinetic energy:

$$\begin{aligned}\hat{T}_{\mathrm{vib}} = -\frac{\hbar^2}{2\sqrt{wr}}\Bigg\{&\frac{1}{\beta^4}\left[\frac{\partial}{\partial\beta}\sqrt{\frac{r}{w}}\beta^4 B_{\gamma\gamma}\frac{\partial}{\partial\beta} - \frac{\partial}{\partial\beta}\sqrt{\frac{r}{w}}\beta^3 B_{\beta\gamma}\frac{\partial}{\partial\gamma}\right] \\ &+ \frac{1}{\beta\sin 3\gamma}\left[-\frac{\partial}{\partial\gamma}\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\frac{\partial}{\partial\beta} + \frac{1}{\beta}\frac{\partial}{\partial\gamma}\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\frac{\partial}{\partial\gamma}\right]\Bigg\},\end{aligned} \quad (1)$$

wh
are
fra

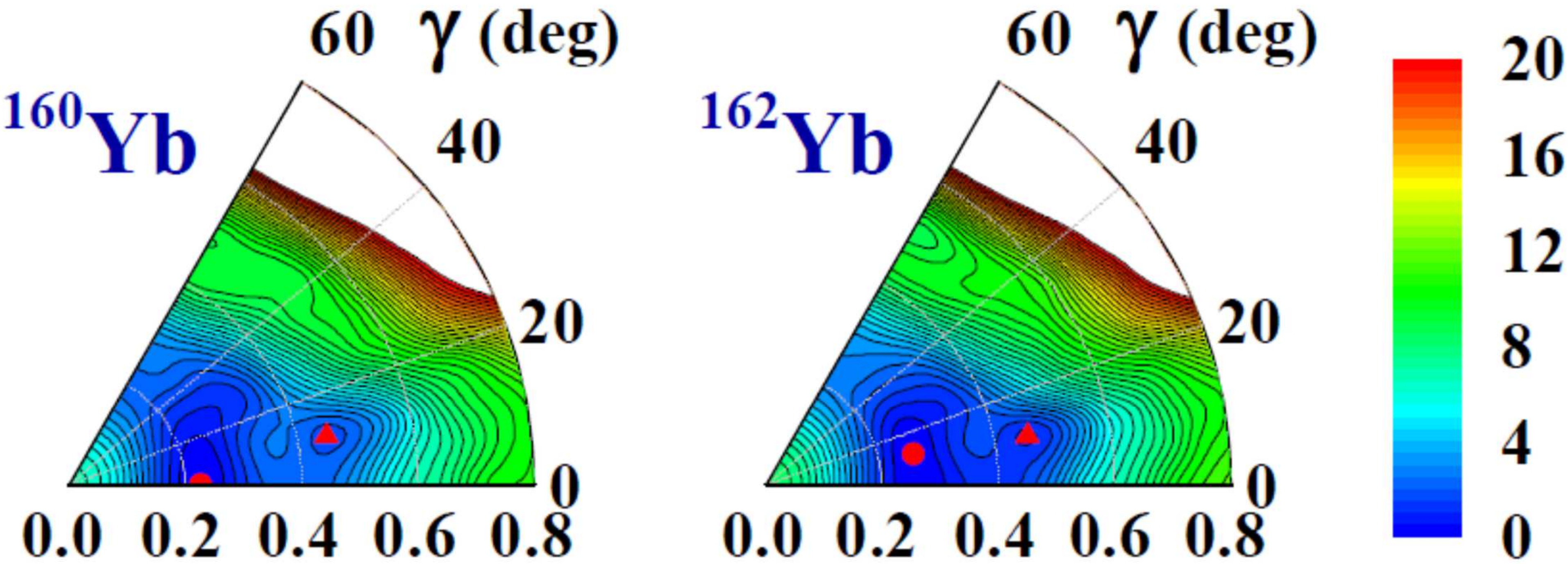
Q_k
sic

and rotational kinetic energy:

$$\hat{T}_{\mathrm{rot}} = \frac{1}{2}\sum_{k=1}^{3}\frac{\hat{J}_k^2}{\mathcal{I}_k}.$$

Li et al., PR C79 (2009) 054301

Can you calculate V(β,γ) and use Bohr-Type Hamiltonians to calculate the level structure ??

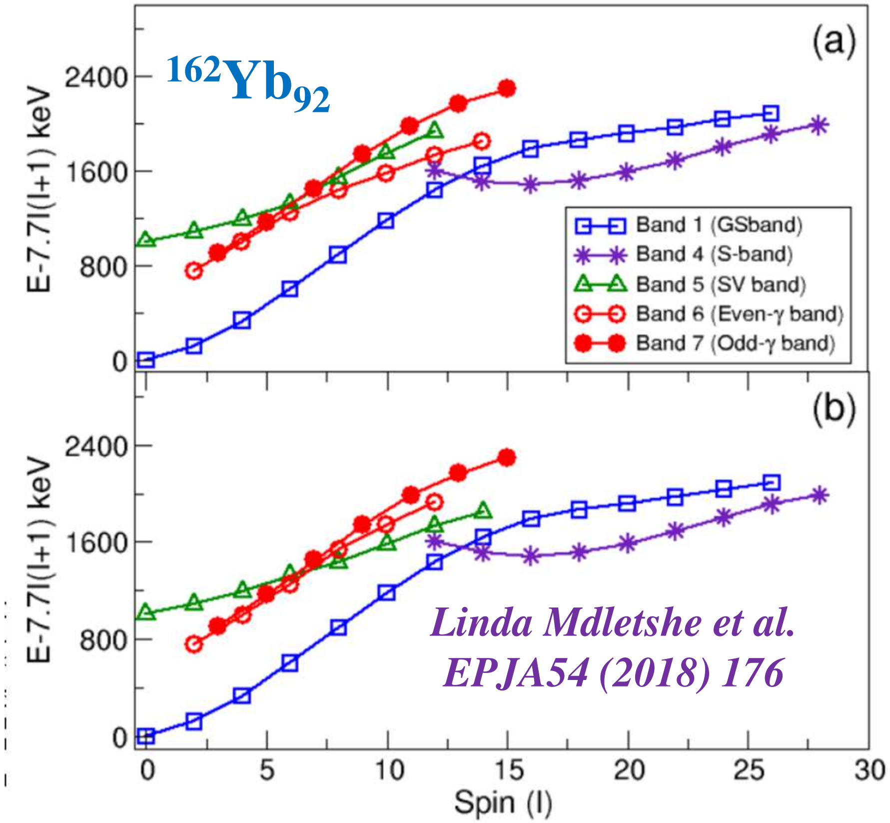


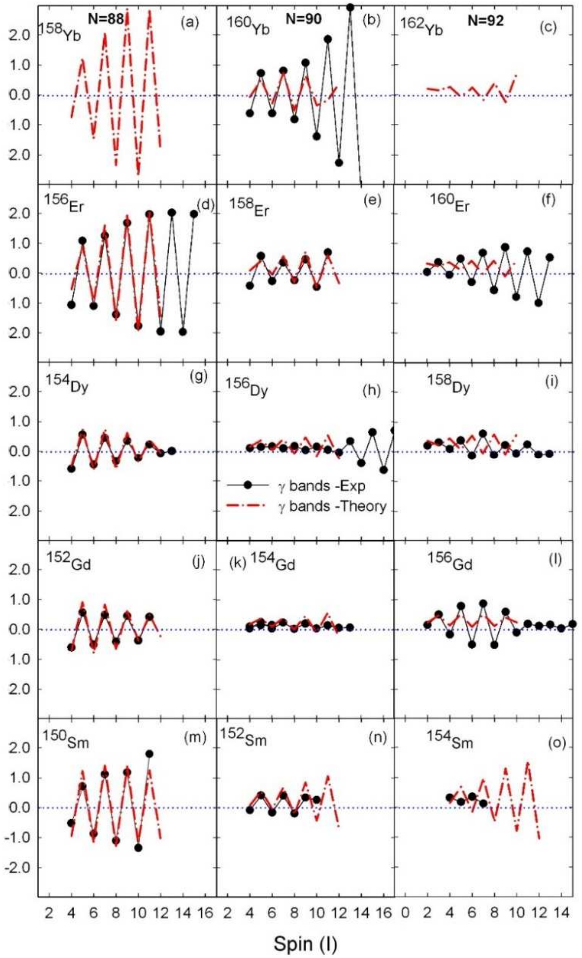
S. N. T. Majola et al., Phys. Rev. C100, 044321 (2019)

Majola et al., PRC100, 044324 (2019) +

Zhi Shi, Zhipan Li, Shuangquan Zhang et al.

South Africa (Experiments) – China (5-DCH+CDFT)

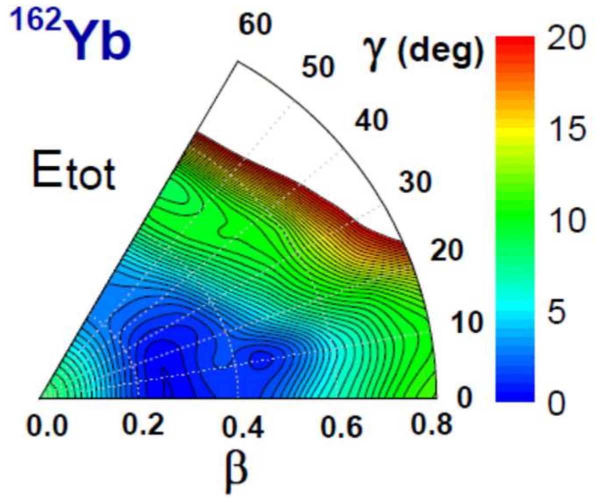


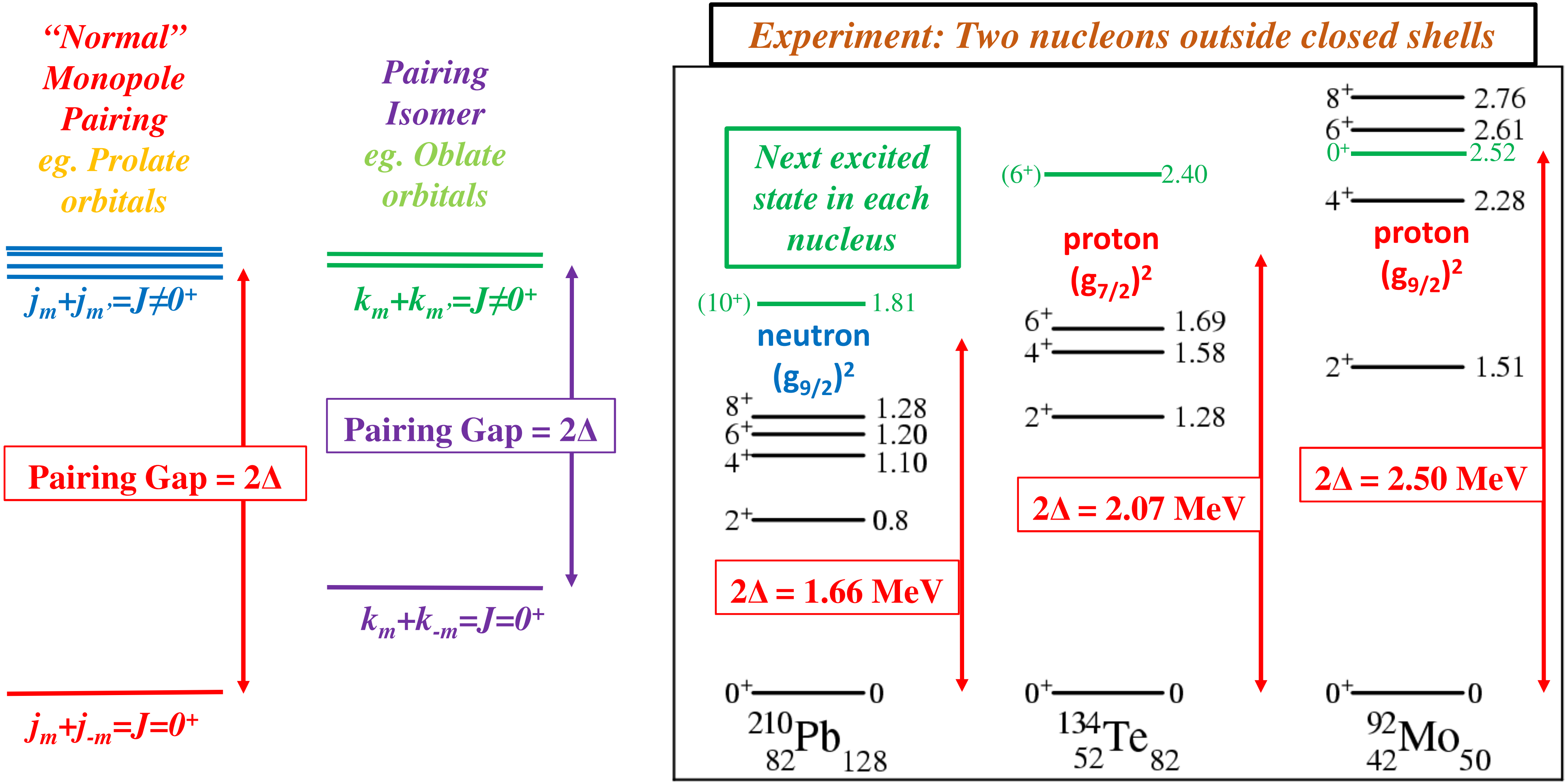


Signature Splitting

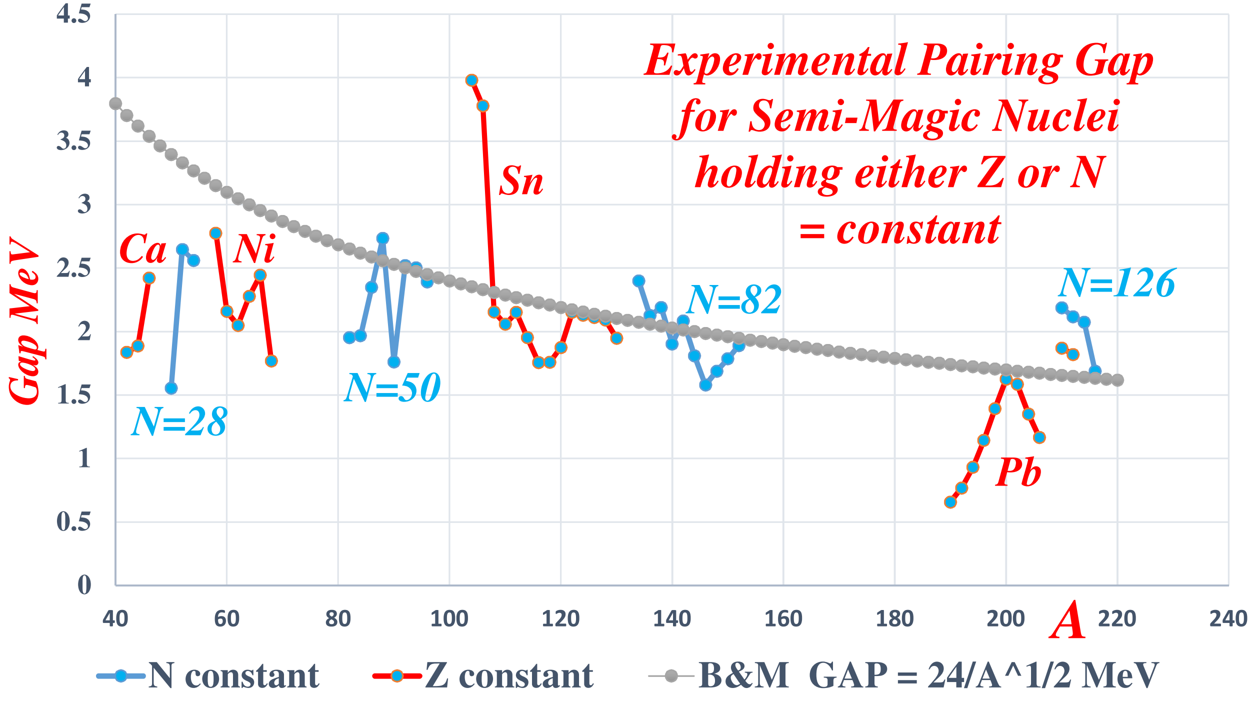
$$S(I) = \frac{\{[E(I) - E(I-1)] - [E(I-1) - (E(I-2)]\}}{E(2_1^+)}$$

Due to BAND MIXING !!



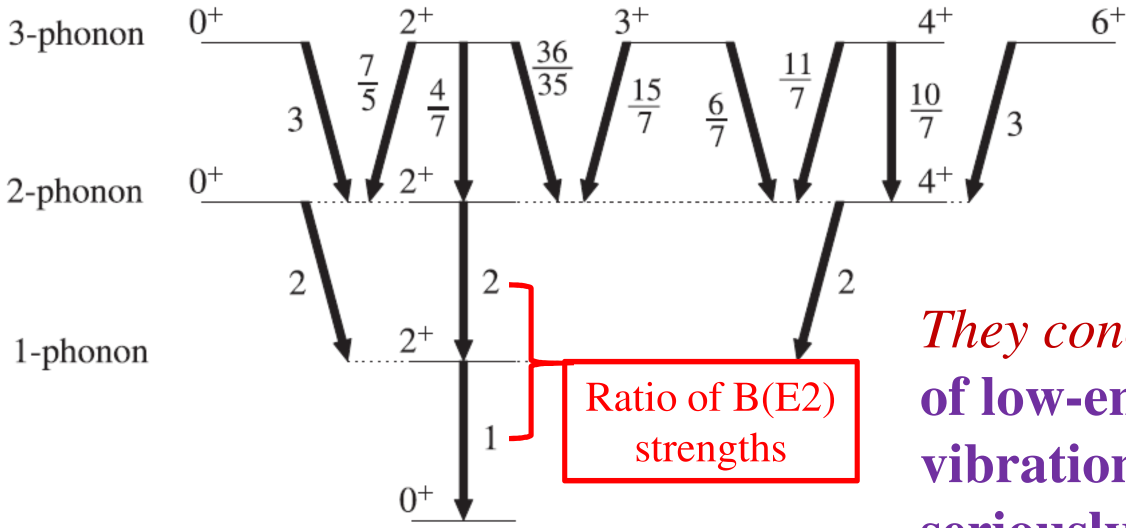


Pairing Energy $\Delta \approx 12/A^{1/2}$ MeV
From Bohr and Mottelson



Experimental Pairing Gap for Semi-Magic Nuclei holding either Z or N = constant
Gap MeV
A
Ca
Ni
Sn
Pb
N=28
N=50
N=82
N=126
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
40
60
80
100
120
140
160
180
200
220
240
N constant
Z constant
B&M GAP = 24/A^1/2 MeV

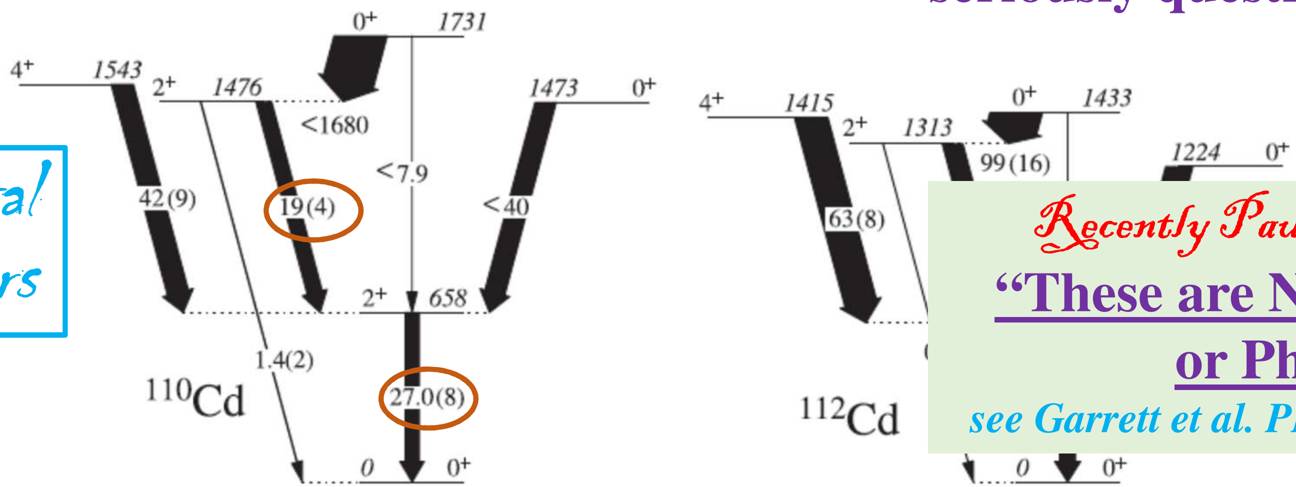
Ideal Spherical Vibrator



Taken from:
Garrett, Wood and Yates, Physica Scripta 93 (2018) 063001

They conclude: **"….the existence of low-energy quadrupole vibrations in nuclei must be seriously questioned."**

Best Experimental Spherical Vibrators



Recently Paul Garrett told us:

"These are NOT Vibrations or Phonons"

see Garrett et al. PRL123, 142502 (2019)

Advanced Monte Carlo Shell Model

TAKAHARU OTSUKA

NuSpin2018 Valencia

Otsuka et al., PRL123, 222502 (2019)

"Type II shell evolution is a simplest and visible case of

"QUANTUM SELF ORGANIZATION"

Atomic nuclei can "organize" their single-particle energies by taking particular configurations of protons and neutrons optimized for each eigenstate, thanks to orbit-dependences of monopole components of nuclear forces (*e.g.*, tensor force).

→ an enhancement of Jahn-Teller effect.

Nilsson-type effects can be enhanced by this optimization.

$$\text{deformation} = \frac{\text{quadrupole force}}{\text{resistance power}}$$

resistance power ← pairing force

resistance power ↖ single-particle energies

Monte Carlo Shell Model using $^{110}_{40}Zr_{70}$ Core, Single Particle Energies (SPE) from $^{123}_{51}Te_{82}$ and $^{123}_{50}Sn_{83}$

Calculates $^{154}_{62}Sm_{92}$ and $^{166}_{68}Er_{98}$

with Yusuke Tsunoda

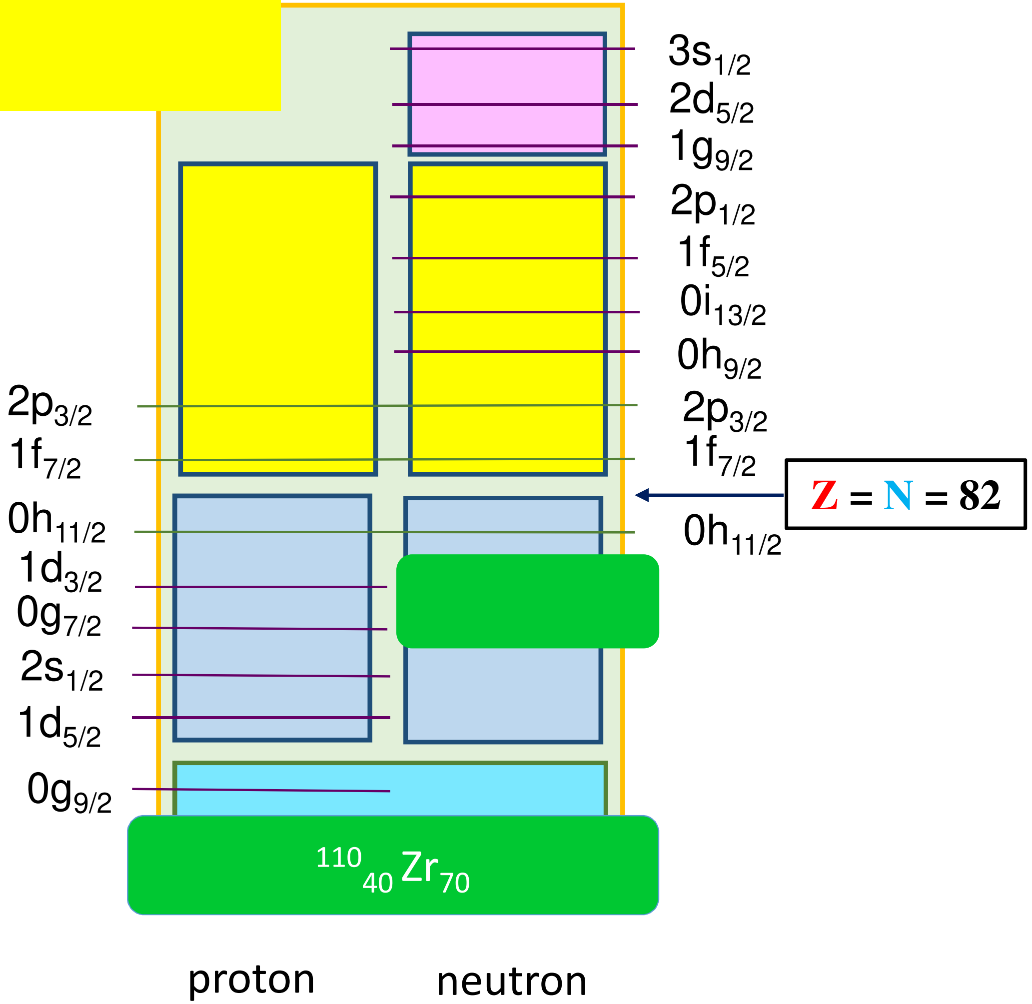
Otsuka et al., PRL123, 222502 (2019)

- Effective interaction: G-matrix* + V_{MU}

  * Brown, PRL 85, 5300 (2000)

Nucleons are excited fully within this model space (no truncation)

We performed Monte Carlo Shell Model (MCSM) calculations, where the largest case corresponds to the diagonalization of 3.9×10^{31} dimension matrix.



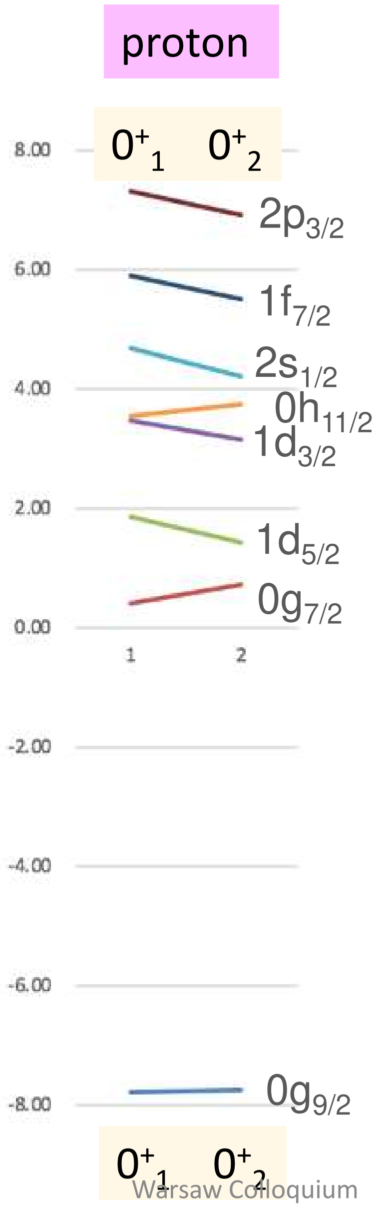
Effective Single-Particle Energies (ESPE)

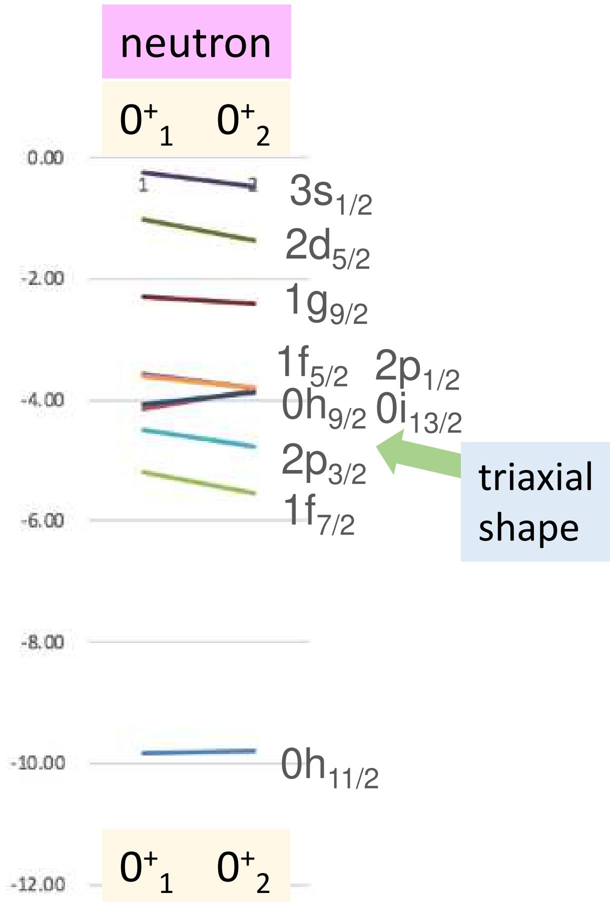
^{154}Sm

0^+_1 and 0^+_2 eigenstates

0^+_1 prolate
0^+_2 triaxial

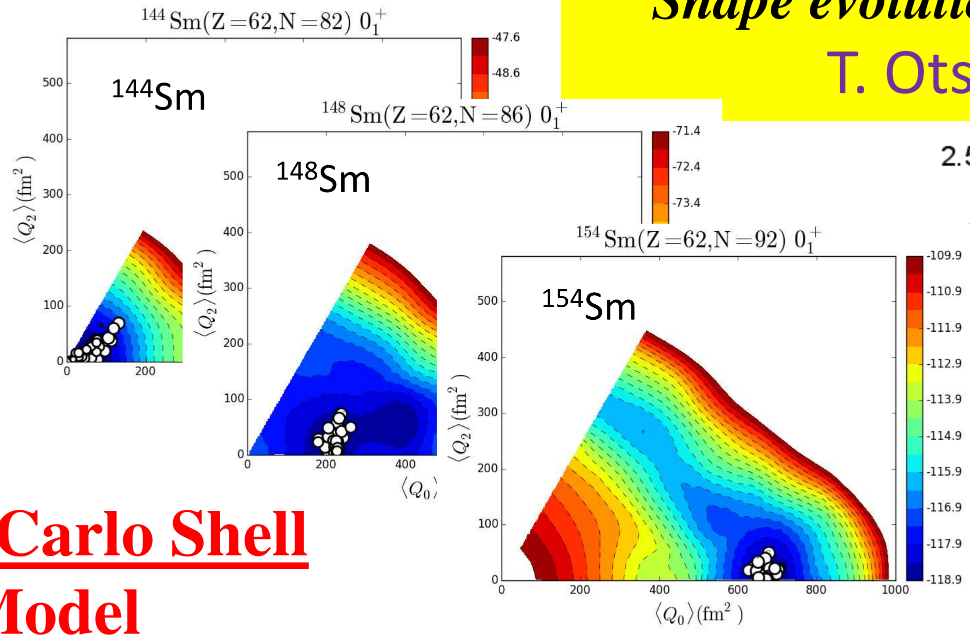
ESPEs show very different patterns between eigenstates

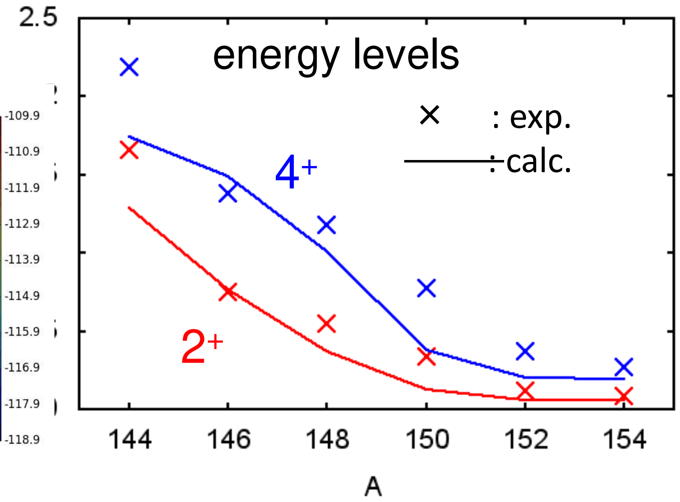




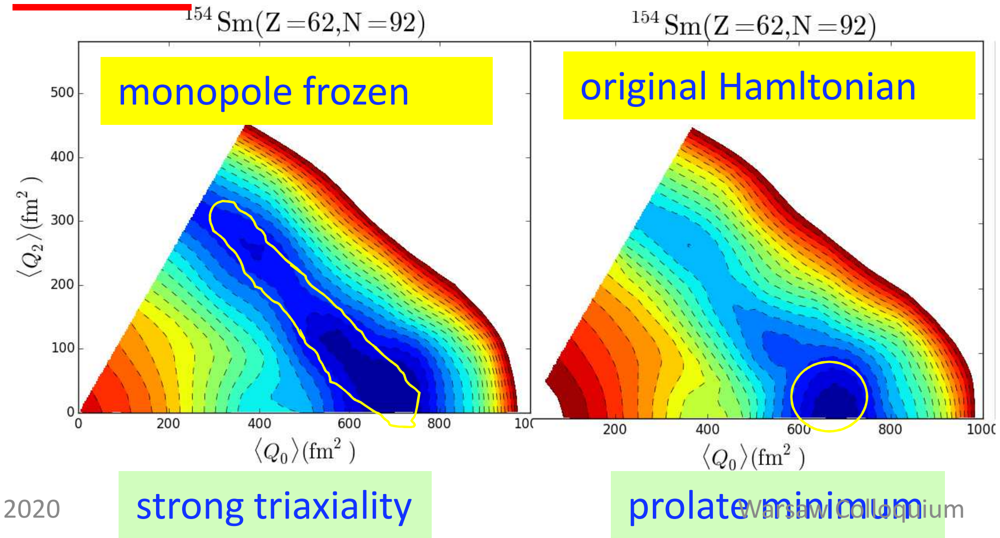
Shape evolution in Sm isotopes (very preliminary)

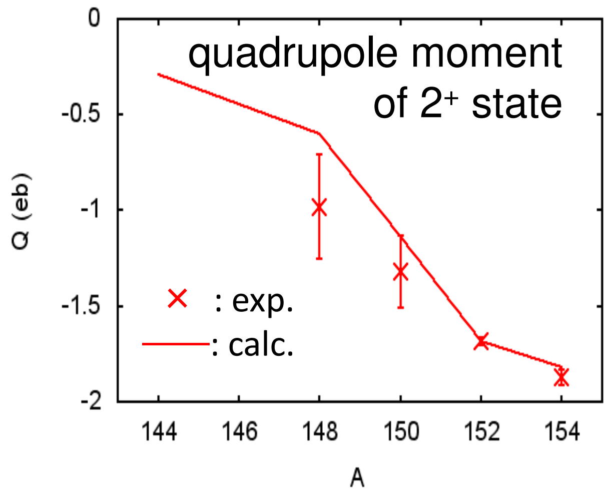
T. Otsuka NuSpin2018 Valencia

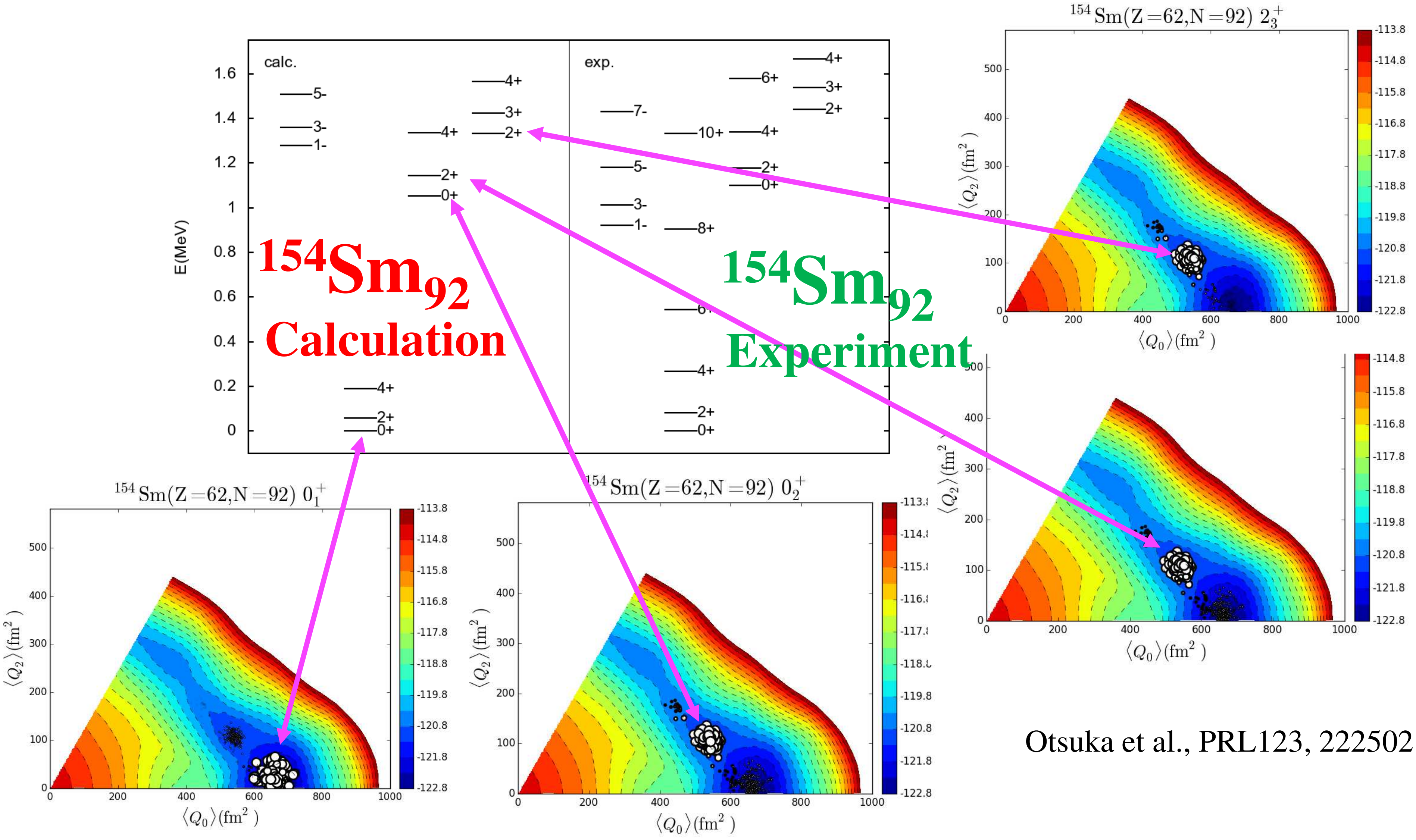




Monte Carlo Shell Model







Otsuka et al., PRL123, 222502 (2019)

Summary

Nuclear forces are rich enough to optimize single-particle energies for each eigenstate (especially in the cases of collective-mode states), as referred to as quantum self-organization. It produces sizable effects with

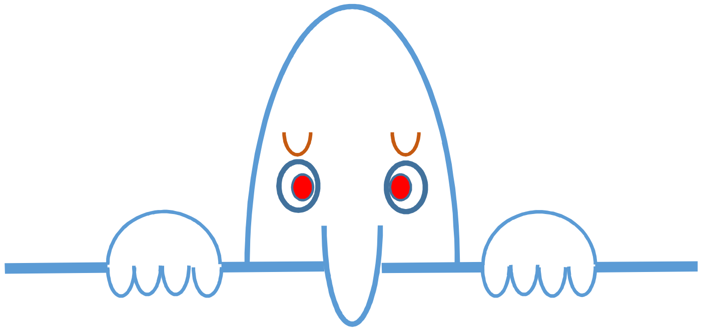
(i) two quantum fluids (protons and neutrons),
(ii) two major forces : *e.g.*, quadrupole interaction to drive collective mode
monopole interaction to control resistance

This feature fits well the general concept of the self organization.

"The 0^+_2 and $2^+_{2,\,3}$ may not be members of β or γ vibration, but are triaxially deformed states with stronger fluctuation."

Effective Single Particle Energies show different patterns to produce such shapes.

LASTLY



What, NO Vibrations ??

or Phonons or Bosons ??!!