

Theoretical investigations of three-nucleon systems

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(for the LENPIC Collaboration)



Warsaw, March 2, 2023

Outline

Introduction:

- A bit of history
- What is LENPIC and what is it for ?

Three generations of Bonn/Bochum chiral nuclear potentials

- Forces before 2014
- „Improved” (SCS) chiral potentials
- SMS chiral potentials

A quick tour of the results

- Nucleon-deuteron scattering studied with chiral forces
- Properties of many-nucleon systems
- Interactions of electroweak probes with two- and three-nucleon systems

Summary and outlook

The first three-nucleon force (3NF) model

360

Progress of Theoretical Physics, Vol. 17, No. 3, March 1957

Pion Theory of Three-Body Forces

Jun-ichi FUJITA and Hironari MIYAZAWA

Department of Physics, University of Tokyo, Tokyo

(Received October 27, 1956)

Three-body forces among three nucleons are calculated on the basis of the static pion theory. It is assumed that only one pion each is exchanged between nucleon (1) and (2), and between (2) and (3), although any number of pions may be emitted and absorbed by the same nucleon. The validity of this calculation is therefore limited to the case when every nucleon is well separated from the other. This three-body potential has a contribution of about 0.22 Mev to the triton binding. In a heavy nucleus the binding energy due to this potential is attractive and about ten percent of that due to two-body interaction. This effect can be roughly stated as follows. Within a nuclear matter the potential between two nucleons takes a different shape due to many body forces. The depth is increased by 9 percent and the range is decreased by 13 percent.

*Professor **Hironari Miyazawa**, Professor Emeritus of the University of Tokyo, passed away on January 14, 2023. He was 95 years old.*

Status of *ab initio* nuclear calculations in the 1980s

- Several models of NN potentials (Malfliet-Tjon, Reid, Paris, Bonn, their separable approximations are available in the mid 1980s)
- Satisfactory description of the deuteron properties and NN scattering data, **BUT ...**
- ... 3N bound states calculations (W. Glöckle, T. Sasakawa, S. Ishikawa, P. Sauer, G. L. Payne, J. L. Friar, B. F. Gibson, ...) with all available at that time NN interaction models left ${}^3\text{H}$ **underbound** \rightarrow 3N Hamiltonian contains a **three-nucleon potential**, which cannot be reduced to pairwise interactions

$$H = H_0 + V_{12} + V_{23} + V_{31} + V_{123}$$

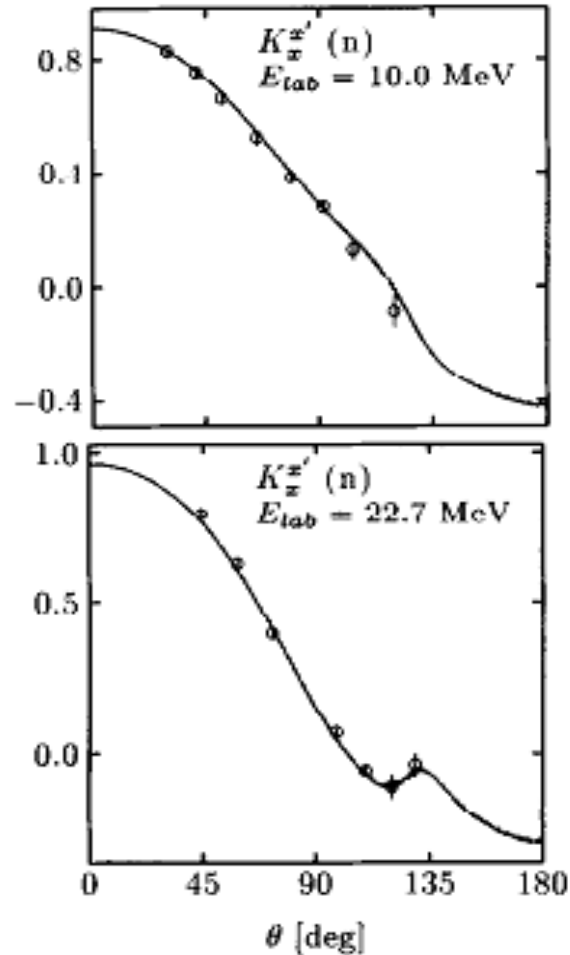
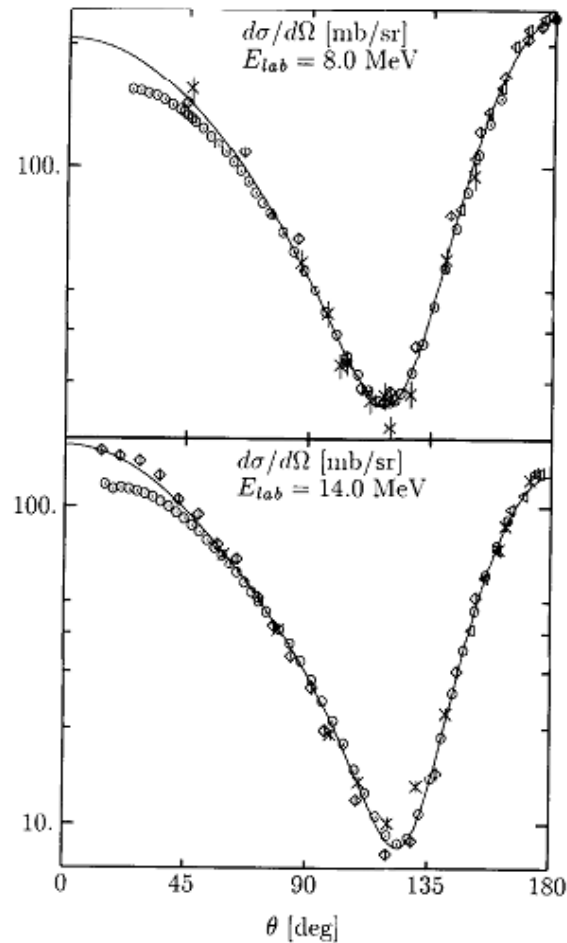
- In the late 1980s nucleon-deuteron scattering enters the game. Numerical framework for rigorous 3N continuum Faddeev calculations (H. Witała and W. Glöckle) opened the possibility of studying properties of different models of 2N and 3N potentials.

$$N + d \rightarrow N + d \quad (\text{elastic channel})$$

$$N + d \rightarrow N + p + n \quad (\text{breakup channel})$$

This triggered intensive collaboration with experimental groups !

Early results for low energies



Calculations employing realistic 2N potentials describe very well not only differential cross sections but also polarization observables (W. Glöckle et al., Phys. Rep. **274**, 107 (1996))

All potentials yield equivalent predictions !

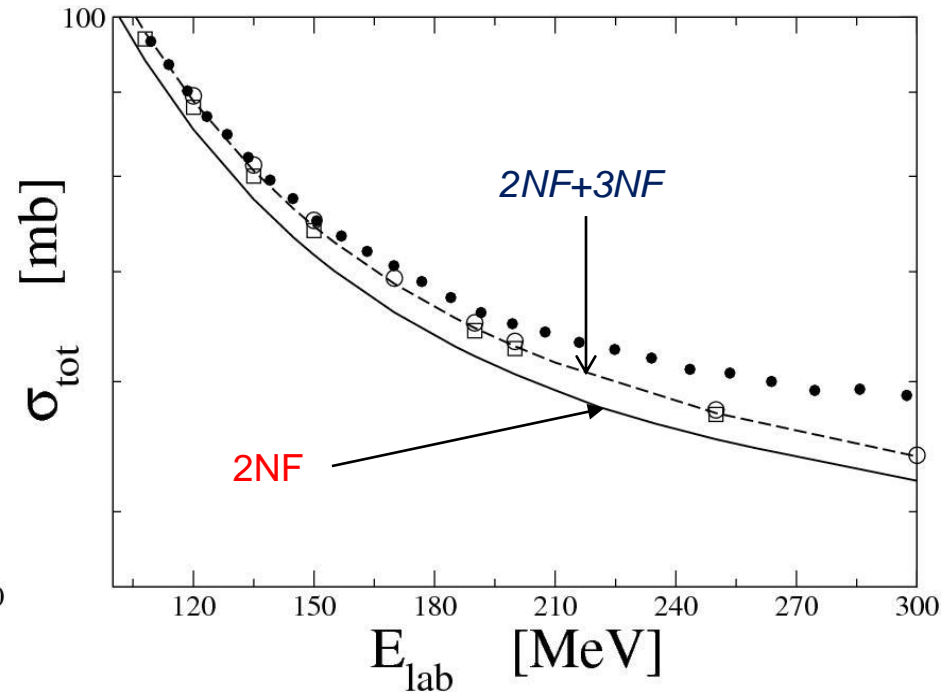
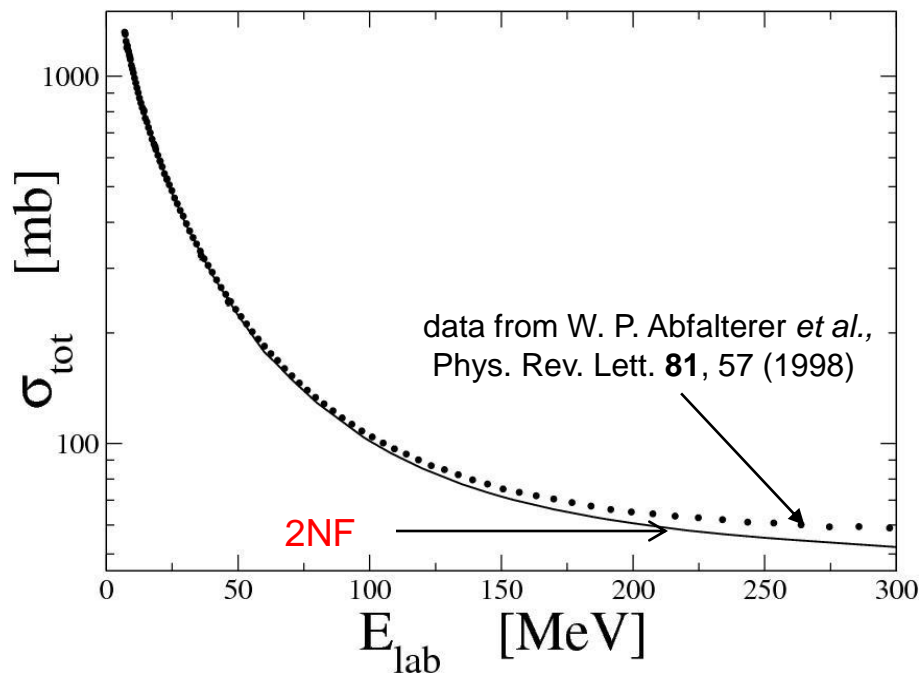
Problems for higher energies

Total neutron-deuteron scattering cross section:

Up to ~50 MeV good agreement with predictions based on 2NF (CD Bonn, AV18) only.

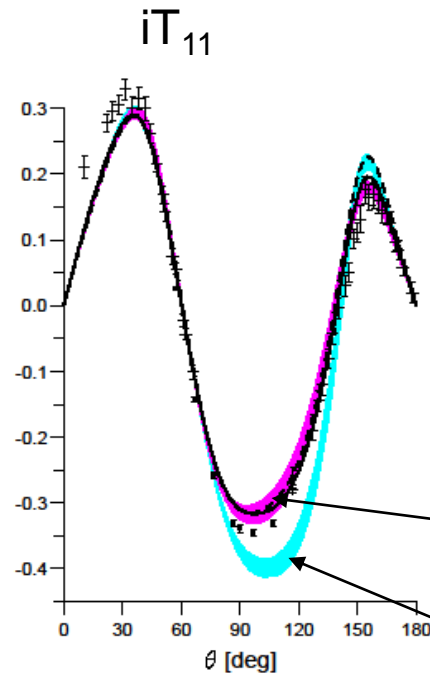
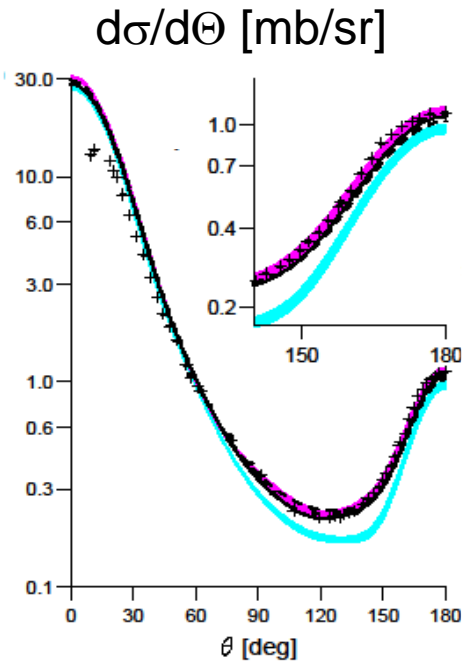
Adding 3NF helps up to ~150 MeV.

For still higher energies a disagreement between 2NF+3NF (CD Bonn+TM, CD Bonn+TM99, AV18+URIX) calculations and data grows with energy.



Elastic Nd scattering at 135 MeV

Here the 3N potentials clearly help describe the data



Experimental data from
H. Sakai et al., Phys. Rev. Lett. **84**, 5288 (2000)
N. Sakamoto et al., Phys. Lett. B **367**, 60 (1996)

Calculations from
H. Witała et al., Phys. Rev. C **63**, 024007 (2001)

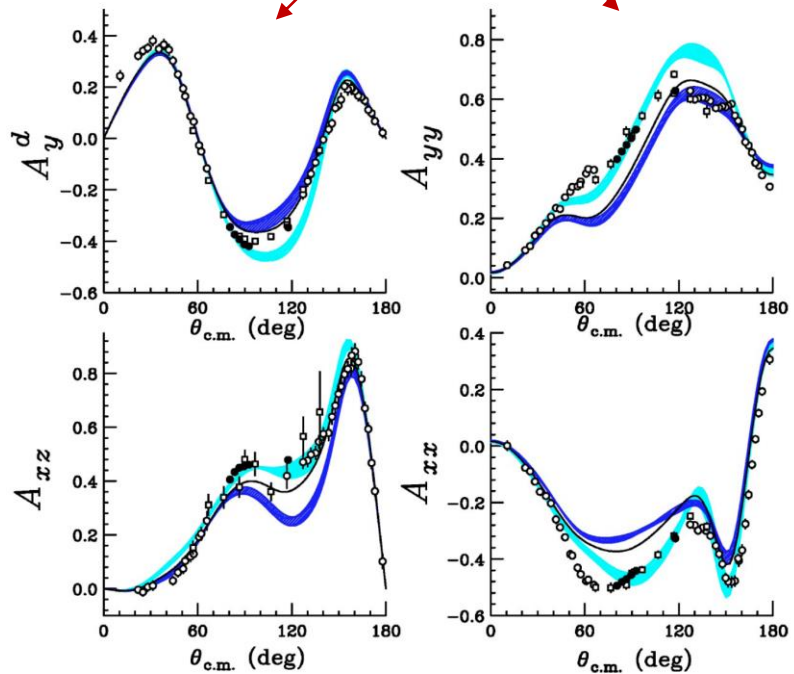
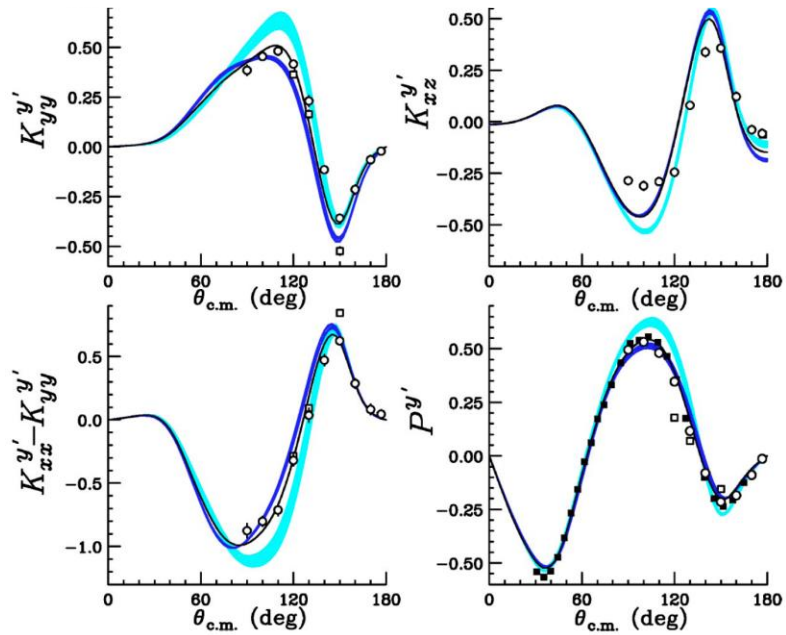
2NFs+TM99 3NF

2NFs

Elastic Nd scattering at 135 MeV

K. Sekiguchi *et al.*, Phys. Rev. C **70**, 014001 (2004)

Here 3NFs do NOT do a good job !



Nd scattering at high energies – relativistic formulation

Clear 3N force effects but experimental results are not properly described !

Are 3N potential models based on two-pion exchanges not sufficient ?
Or maybe relativity plays an important role ?

To answer these questions relativistic formulation of 3N dynamics was necessary. Our choice was the **relativistic Faddeev framework**, since it preserves the formal structure of the nonrelativistic Faddeev equation.

New ingredients

- Relativistic kinematics
- Relativistic NN potential in the 2N total momentum zero frame
- „Boosted” NN potential for the nonzero momentum of the 2N system
- Construction of spin states with inclusion of Wigner rotations
- Relativistic 3N potential

This ambitious program has been realized by H. Witała *et al.*,
Phys. Rev. C **71**, 054001 (2005), *ibid.* **77**, 034004 (2008), **83**, 044001 (2011) but ...

Nuclear forces from χ EFT

... relativistic framework was not a remedy, so the problems in elastic nucleon-deuteron scattering were caused by inadequate models of the nuclear forces !

Further progress required a **new approach to nuclear forces**.

The picture based on meson exchanges and inconsistent 2N and many-nucleon forces had to be replaced by a new framework – **chiral effective field theory (χ EFT)**.

- χ EFT is linked to QCD
- CONSISTENT 2N, 3N, 4N, ... forces are derived
- Information from the π - N system can be incorporated
- All the forces are analytically given !
- χ EFT explains (see S. Weinberg, Phys. Lett. B **295**, 114 (1992)), why 2N forces are more important than 3N forces, 3N forces are more important than 4N forces etc.
- CONSISTENT models of nuclear forces and electroweak transition operators (currents)

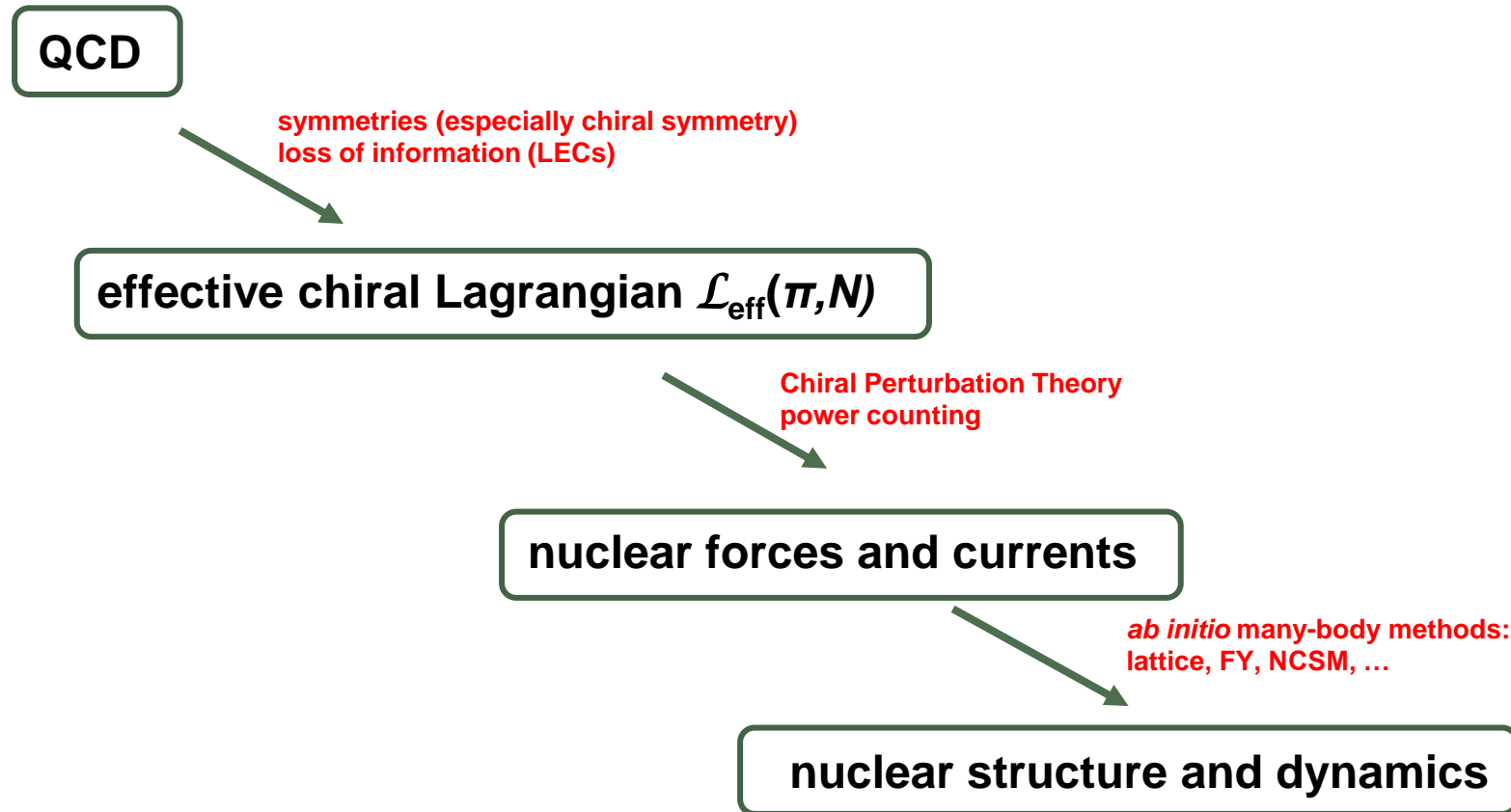
That was on the plus side ...

Nuclear forces from χ EFT

On the minus side:

- No unique choice of the initial degrees of freedom (nucleons+pions+Deltas)
- Derivation of forces and currents within this theory is very complicated
- Potentials contain many spin-isospin momentum-dependent operators (challenge for partial wave decomposition)
- Unknown constants in potentials; feedback from experiment is necessary
- High momentum components in potentials have to be eliminated; **regularization** introduces cutoff dependence of predictions
- Perturbative approach: various orders of the chiral expansion (LO, NLO, N2LO, ...)
- No unique prediction before getting to sufficiently high orders in the chiral expansion; theory gives you ***bands instead of lines***

From QCD to nuclear systems



Chiral nuclear forces from various groups ...

- Moscow (Idaho)-Salamanca

D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C **96**, 024004 (2017)

- Argonne

M. Piarulli *et al.*, Phys. Rev. C **91**, 024003 (2015) (minimally nonlocal)

M. Piarulli *et al.*, Phys. Rev. C **94**, 054007 (2016) (local)

- Oak Ridge












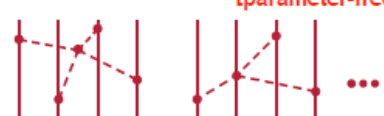



A. Ekström *et al.*, Phys. Rev. C **91**, 051301(R) (2015)

A. Ekström *et al.*, Phys. Rev. C **97**, 024332 (2015)

... however, with **NO** attempt to build consistent 3N potentials beyond N2LO !

The reason why our Cracow group uses mainly the Bochum-Bonn models of nuclear forces (described in next slides)

Chiral expansion of nuclear forces [Weinberg counting]

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)			
	Weinberg '90		
NLO (Q^2)			
	Ordonez, van Kolck '92		
N ² LO (Q^3)			
	Ordonez, van Kolck '92	van Kolck '94; EE et al. '02	
N ³ LO (Q^4)			
	Kaiser '00 - '02	Bernard, EE, Krebs, Meißner, '08, '11	EE '06
N ⁴ LO (Q^5)			
	Entem, Kaiser, Machleidt, Nosyk '15 EE, Krebs, Meißner '15	Girlanda, Klevsky, Viviani '11 Krebs, Gasparyan, EE '12, '13 (short-range loop contrib. still missing)	still have to be worked out

slide prepared by Evgeny Epelbaum

LENPIC (Low Energy Nuclear Physics International Collaboration): “to understand nuclear structure and reactions with chiral forces”



TECHNISCHE
UNIVERSITÄT
DARMSTADT

Sven Binder, Kai Hebeler,
Joachim Langhammer, Robert Roth

IOWA STATE
UNIVERSITY

Pieter Maris, Hugh Potter, James Vary



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LENPIC Co-Spokespersons

<http://www.lenpic.org/>



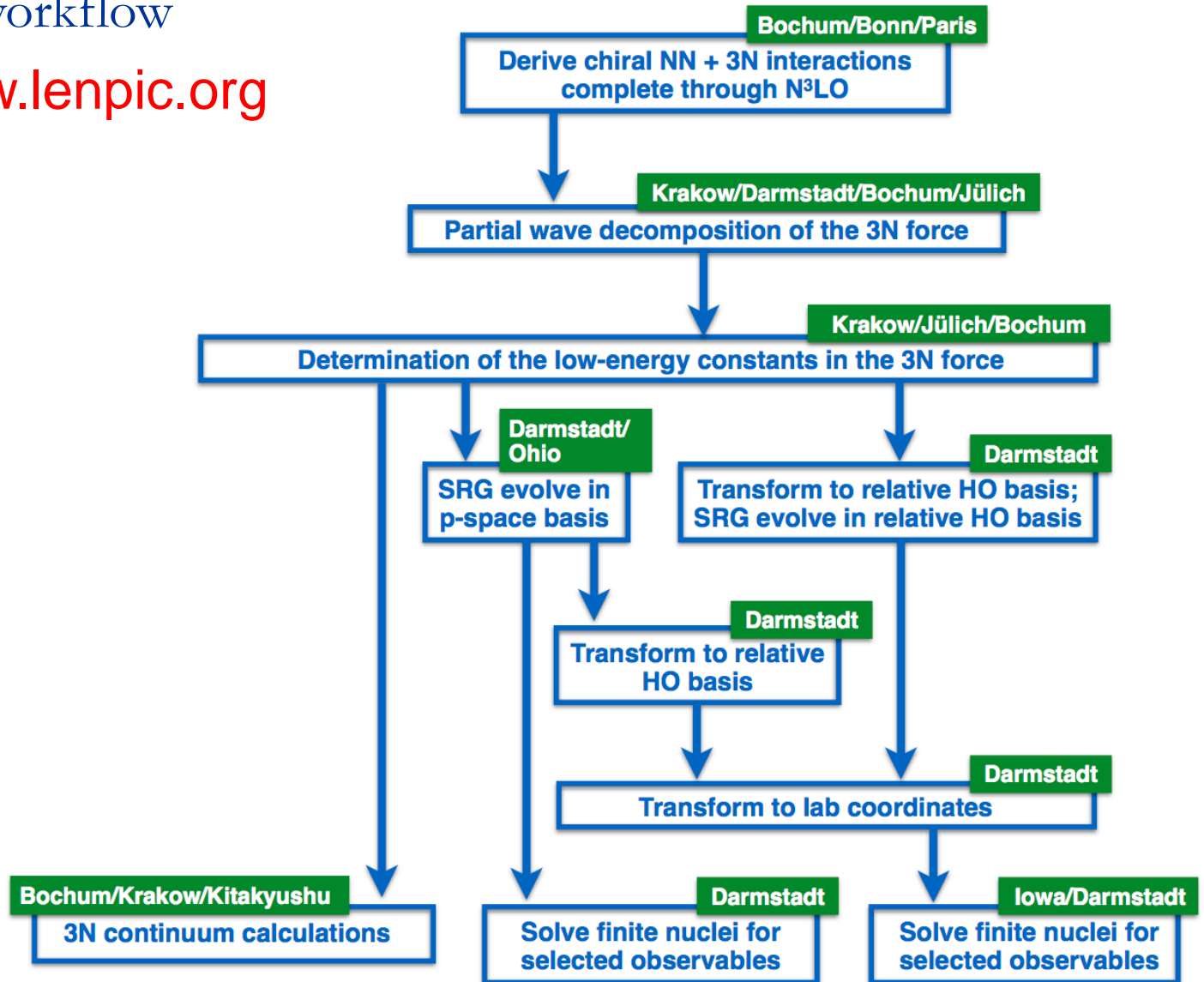
Evgeny Epelbaum
Ruhr-Universität Bochum



James Vary
Iowa State University

LENPIC workflow

<http://www.lenpic.org>



Chiral input for Nd scattering calculations in 2013

nonlocal regularization in momentum space

- Chiral 2N force available up to N3LO

- E. Epelbaum *et al.*, Nucl. Phys. A **747**, 362 (2005)
- E. Epelbaum, Prog. Part. Nucl. Phys. **57**, 654 (2006)

$$V(p', p) \rightarrow V(p', p)f(p', p), \text{ with } f(p', p) \equiv \exp\left(-\left(\frac{p'}{\Lambda}\right)^4 - \left(\frac{p}{\Lambda}\right)^4\right),$$

required additional SFR

$$\Lambda \in [450, 550] \text{ MeV}$$

- Chiral 3N force up to N3LO

- E. Epelbaum, Prog. Part. Nucl. Phys. **57**, 654 (2006)
- V. Bernard *et al.*, Phys. Rev. C **77**, 064004 (2008); **84**, 054001 (2011)

PWD accomplished with new methods

J. Golak *et al.*, Eur. Phys. J. A **43**, 241 (2010); R. Skibiński *et al.*, Eur. Phys. J. A **47**, 48 (2011)

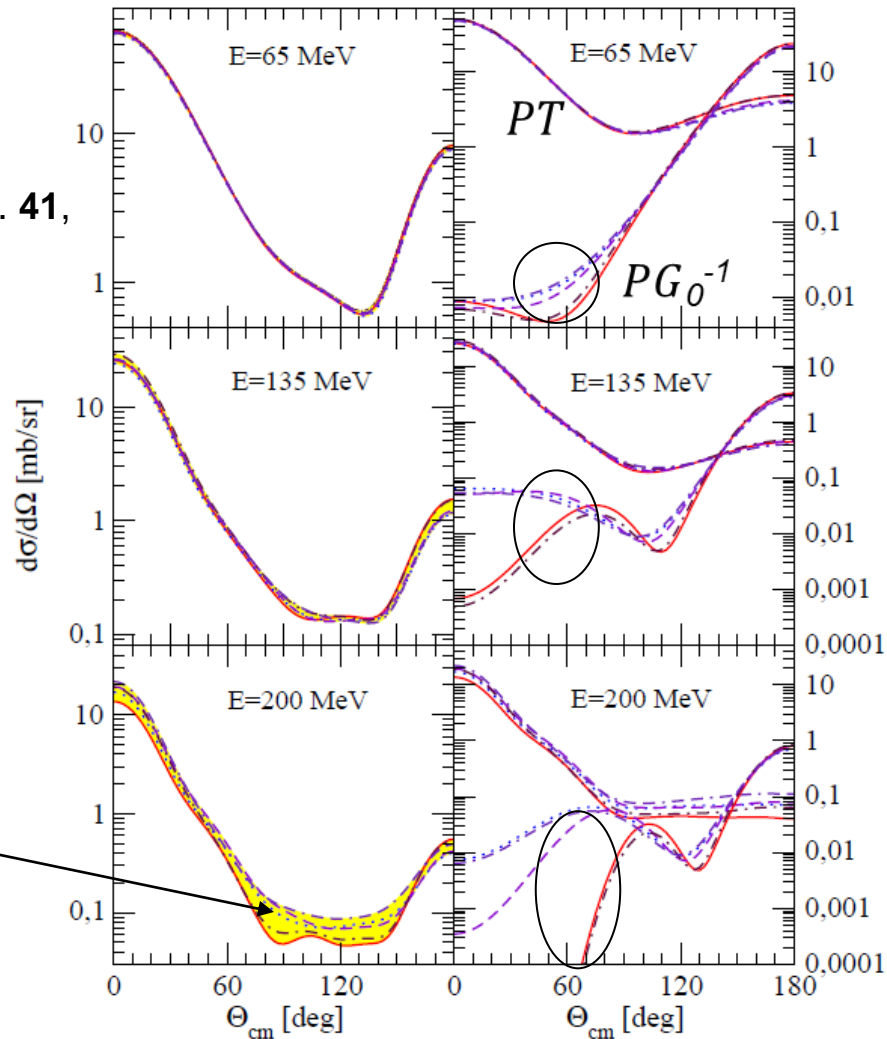
Forces seemingly ready for use but **3N calculations revealed problems in long range parts**

Nd scattering calculations with chiral potential in 2013

H. Witała *et al.*,
 J. Phys. G: Nucl. Part. Phys. **41**,
 094011 (2014)

$$\begin{aligned}
 T|\phi\rangle &= tP|\phi\rangle \\
 &+ tG_0PT|\phi\rangle \\
 U &= PT + PG_0^{-1}
 \end{aligned}$$

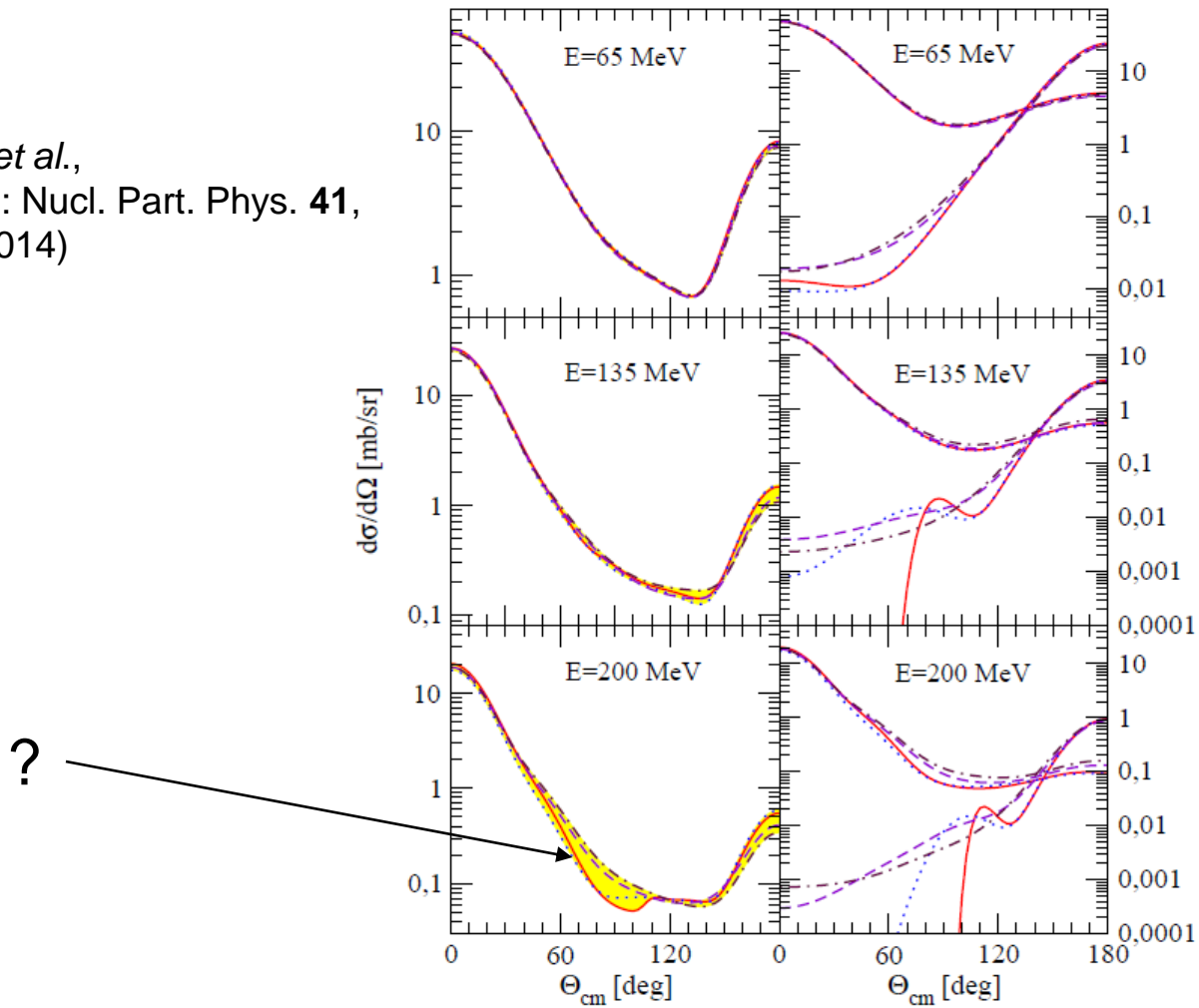
At N3LO loss of
 predictive power at
 higher energies ...



... caused by the
 deuteron wave
 function
 behaviour; can be
 traced back to the
 non-local
 regularization

Entem-Machleidt chiral NN forces pose similar problems ...

H. Witała *et al.*,
J. Phys. G: Nucl. Part. Phys. **41**,
094011 (2014)



Improved semilocal coordinate space regularized (SCS) chiral NN forces

(Sami)local regularization in coordinate space

E.Epelbaum, H.Krebs, U.-G.Meißner, Eur. Phys. J. A**51**, 53 (2015) – up to N3LO

E.Epelbaum, H.Krebs, U.-G.Meißner, Phys. Rev. Lett. **115**, 122301 (2015) – up to N4LO

$$V_{lr}(r) \rightarrow V_{lr}(r) f(r), \quad \text{with } f(r) \equiv \left(1 - e^{-r^2/R^2}\right)^n \quad n = 6, R \in [0.8, 1.2] \text{ fm}$$

- ❑ Preserves more long-range OPE and TPE physics
- ❑ No (unwanted) short-distance part of TPE force (thus no need for SFR)
- ❑ All LECs in the long-range part taken directly from π -N scattering
- ❑ Very good description of the deuteron properties, NN phase shifts etc.

But

- ❑ Still an ad hoc procedure
- ❑ (technically) difficult to apply to 3NF and exchange currents
→ consistent 3N forces only at N2LO !

Estimation of theoretical uncertainties due to truncation of the chiral expansion at a given order

S. Binder *et al.*, Phys. Rev. C **93**, 044002 (2016)

Let $X(p)$ be some observable with p denoting the corresponding momentum scale and $X^{(n)}(p)$, $n = 0, 2, 3, 4, \dots$ a prediction at order Q^n in the chiral expansion:

$$X^{(n)} = X^{(0)} + \Delta X^{(2)} + \dots + \Delta X^{(n)}$$

←←←
calculated in the chiral expansion

For the order- n contribution one expects $\Delta X^{(n)} \sim \mathcal{O}(Q^n X^{(0)})$ with $Q = \max\left(\frac{M_\pi}{\Lambda_b}, \frac{p}{\Lambda_b}\right)$

Theoretical uncertainty $\delta X^{(n)}$ estimated via the size of neglected higher-order contributions*

$$\delta X^{(0)} = Q^2 |X^{(0)}|,$$

$$\delta X^{(2)} = \max(Q^3 |X^{(0)}|, Q |\Delta X^{(2)}|),$$

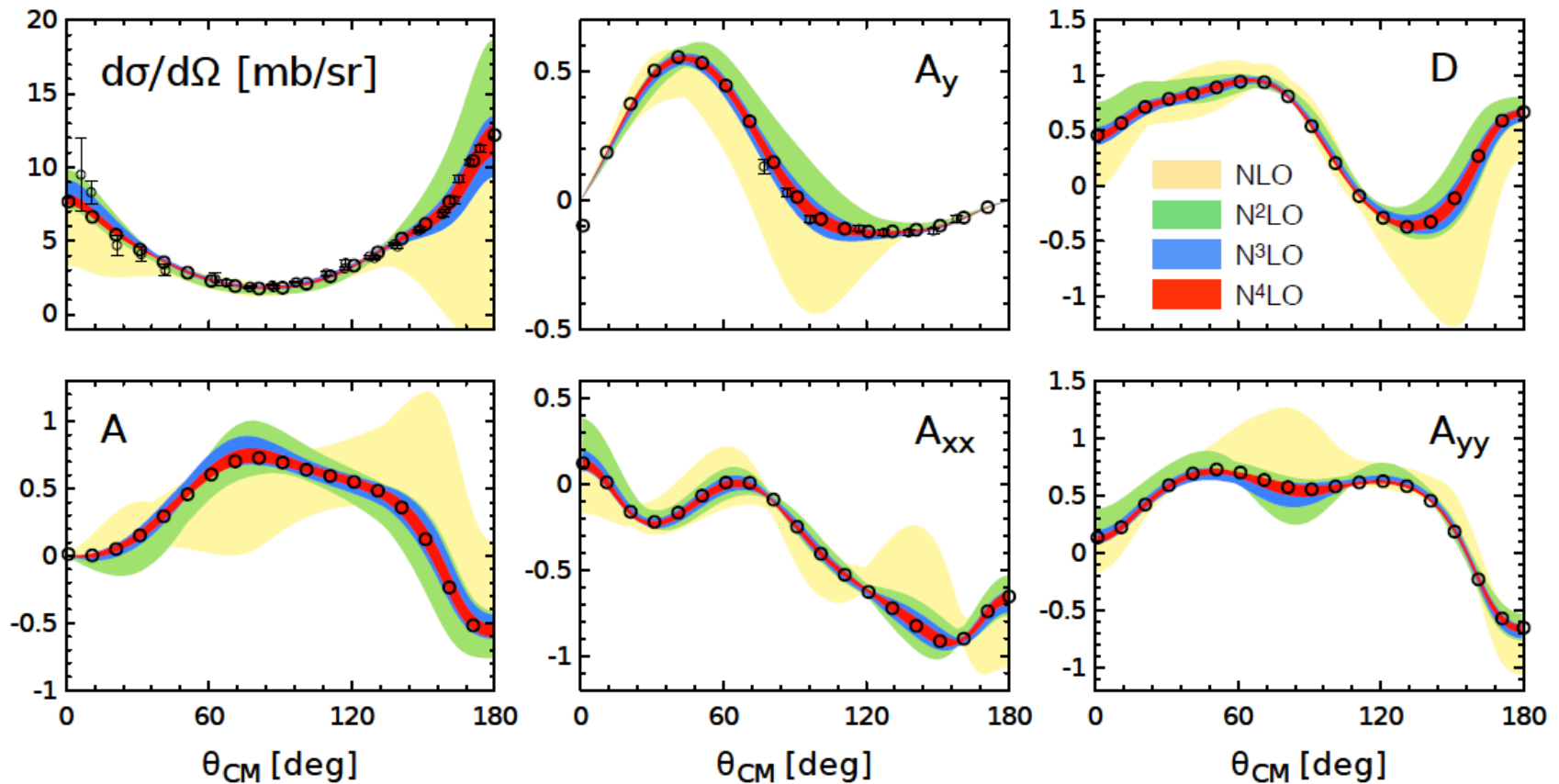
$$\delta X^{(3)} = \max(Q^4 |X^{(0)}|, Q^2 |\Delta X^{(2)}|, Q |\Delta X^{(3)}|),$$

...

(*Also demand that $\delta X^{(n)}$ is not smaller than the actual higher-order contributions whenever known)

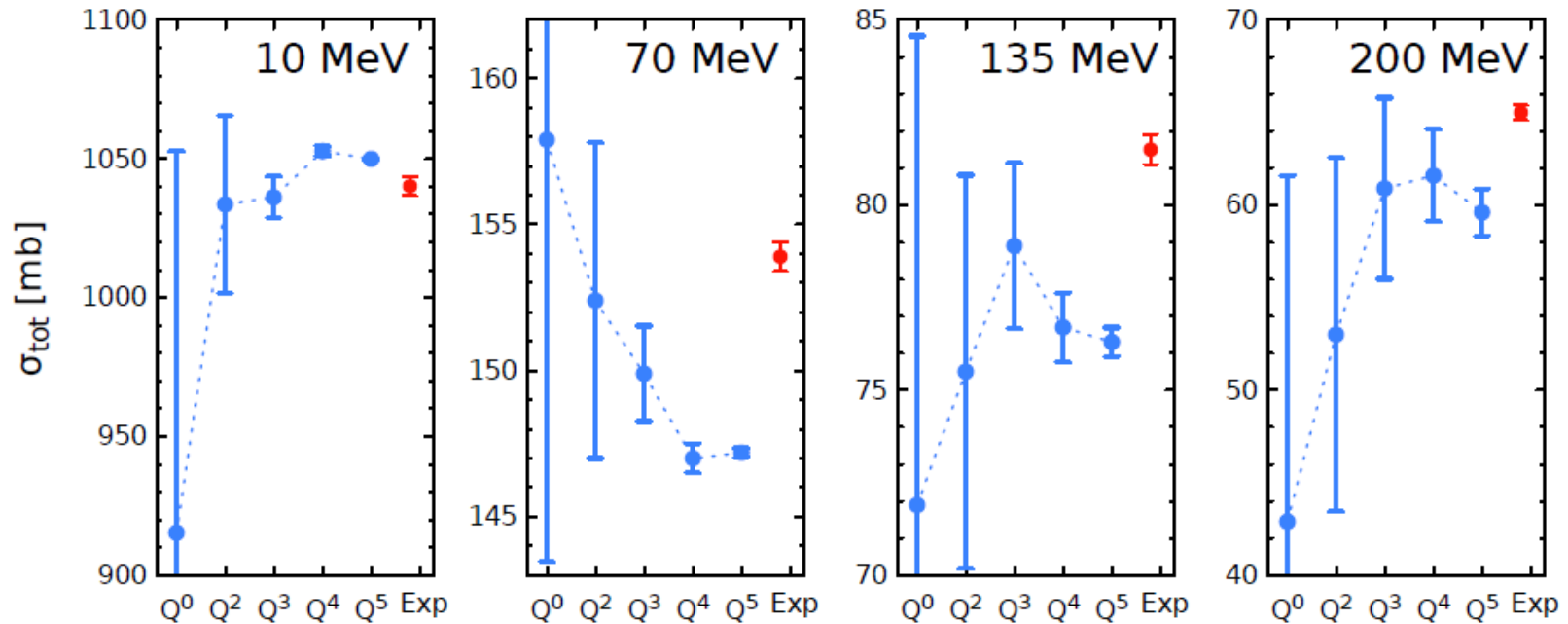
Improved chiral SCS forces: 2N system

Selected neutron-proton scattering observables at 200 MeV $R=0.9\text{fm}$



Improved chiral SCS forces: 3N system

Neutron-deuteron total cross section based on NN forces only R=0.9fm

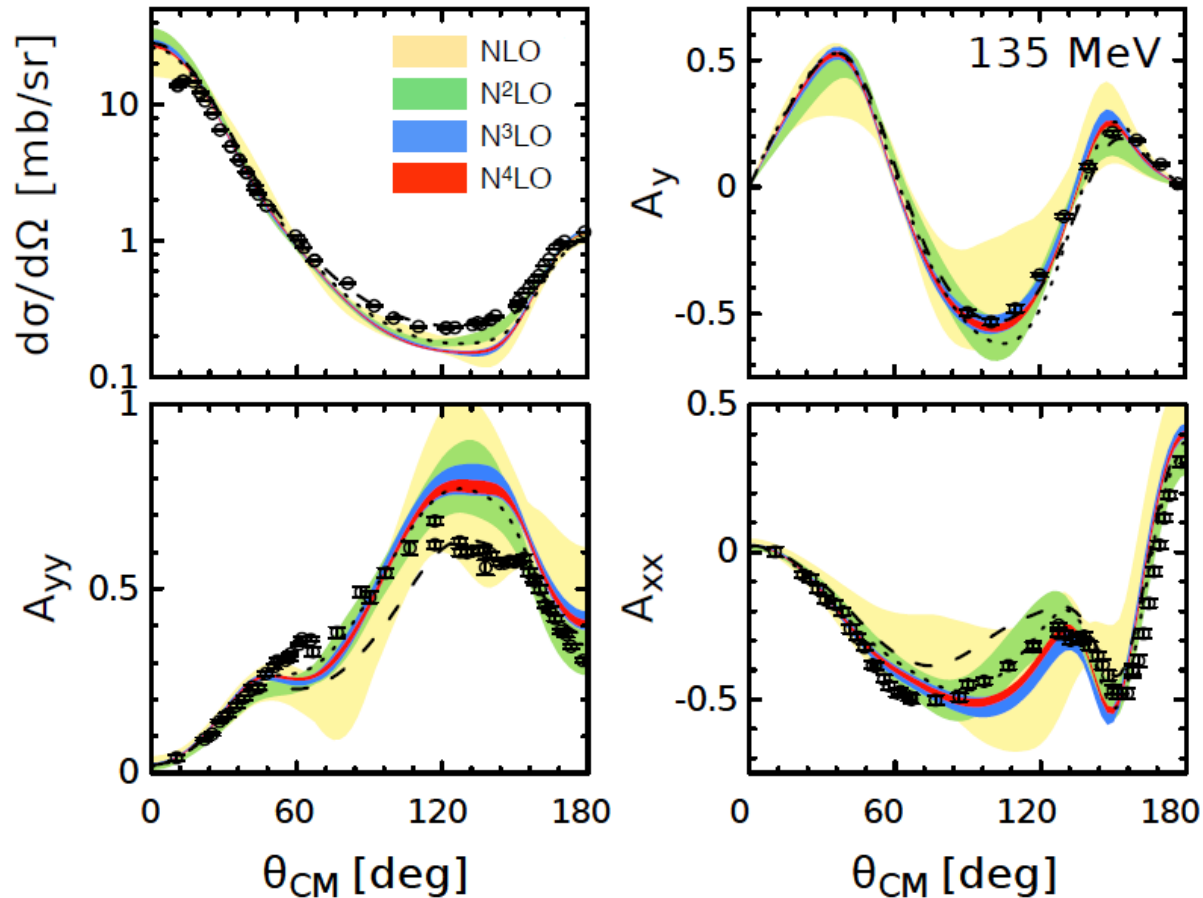


- Unambiguous evidence for missing three-nucleon forces (within our scheme)
- The size of the missing 3NF contribution agrees well with power counting ($N^2\text{LO}$)

Improved chiral SCS forces: 3N system

Phys. Rev. C **93**, 044002 (2016)

Elastic Nd scattering at N⁴LO at 135 MeV based on NN forces only $R=0.9\text{fm}$

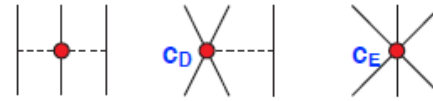


A lot of room
for 3NF !
N⁴LO
calculations
are needed

Improved chiral SCS forces: 3N system with 2N and 3N forces

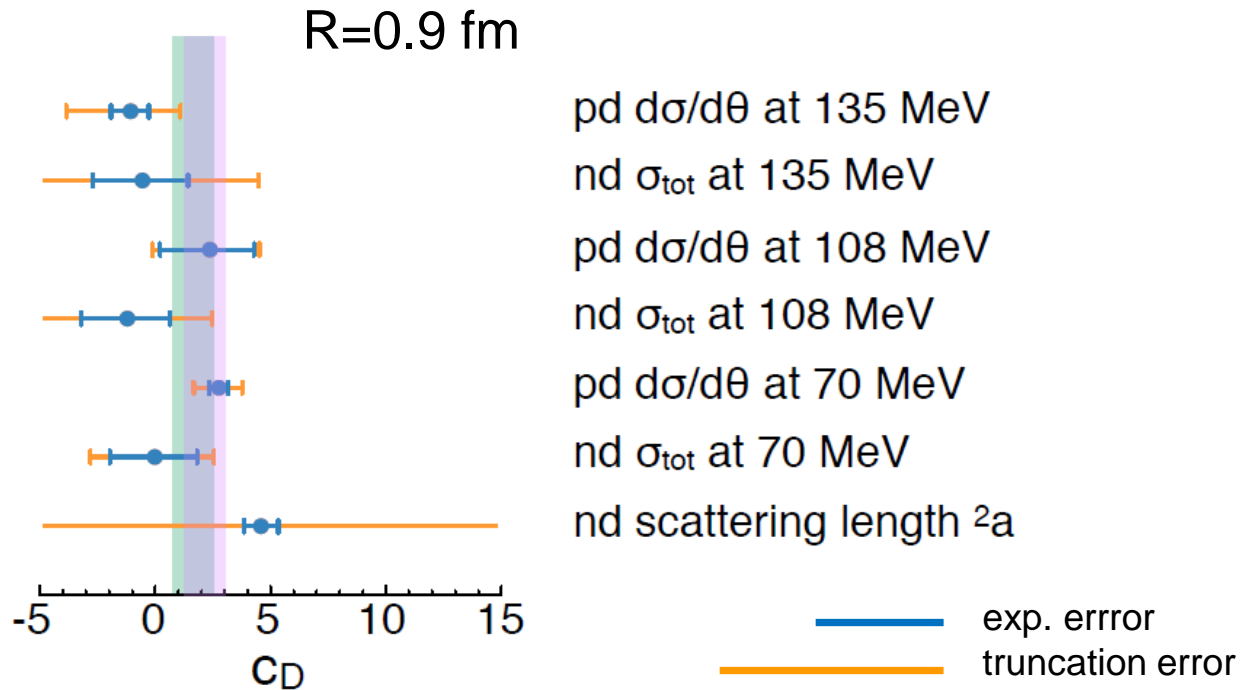
Phys. Rev. C **99**, 024313 (2019)

N²LO: tree-level graphs, 2 new LECs
van Kolck '94; EE et al '02



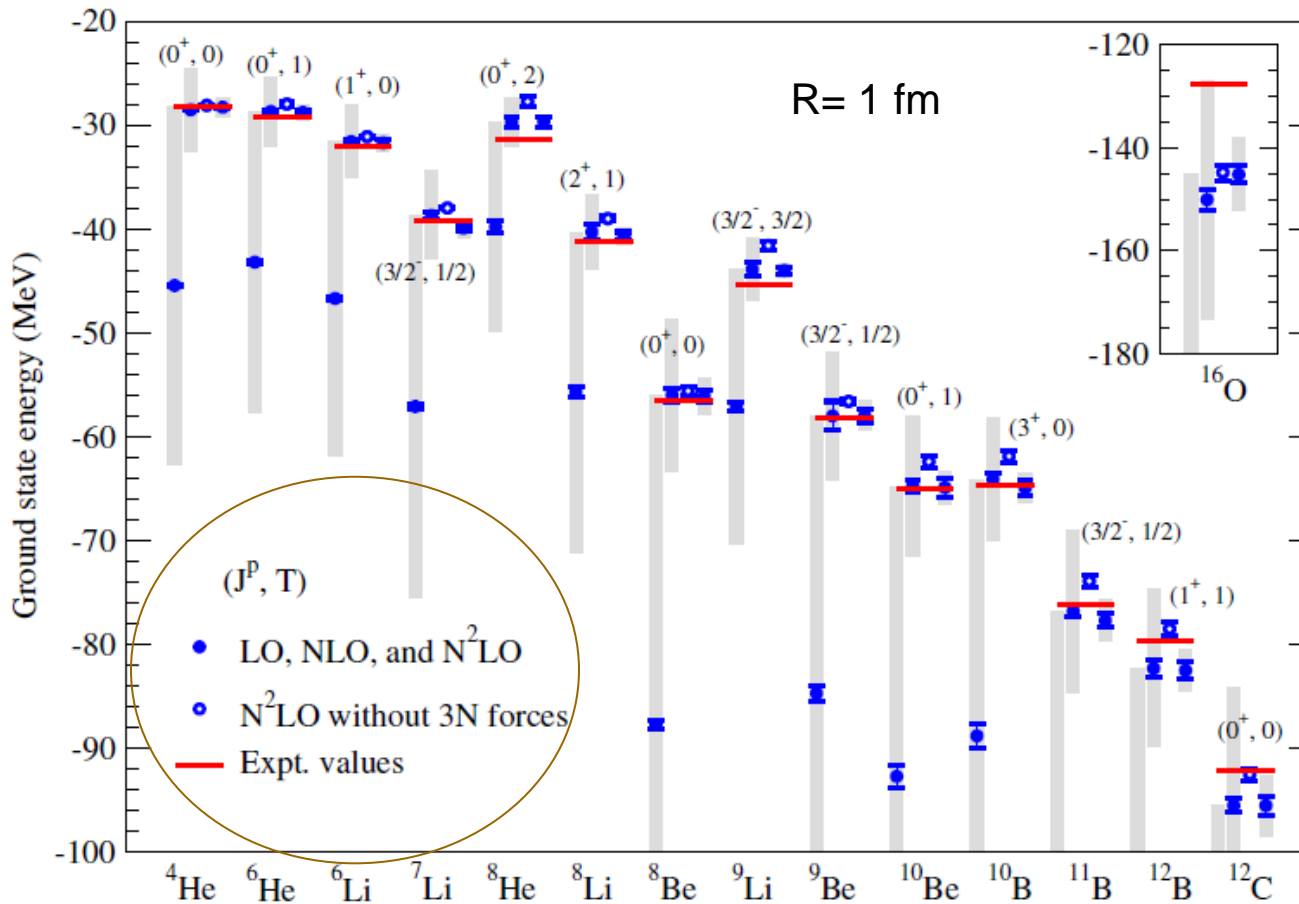
Determination of the LECs c_D , c_E

- Triton BE (c_D - c_E correlation)
- Explore various possibilities and let theory and/or data decide...



Improved chiral SCS NN + 3N forces at N2LO

Phys. Rev. C **99**, 024313 (2019)

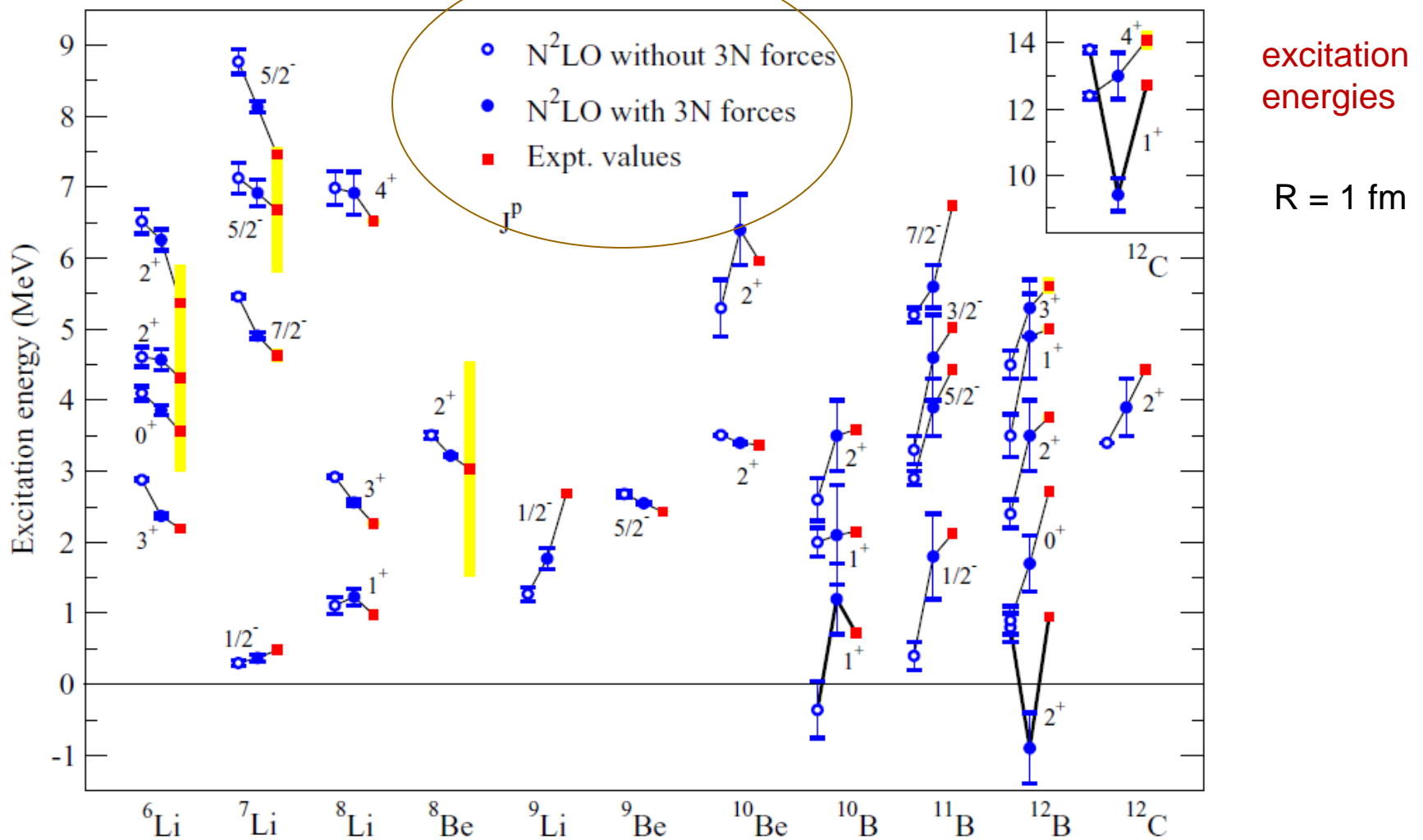


ground state energies of heavier light nuclei leave room for improvement !

Blue error bars indicate the no-core configuration interaction (NCCI) extrapolation uncertainty and, where applicable, an estimate of the similarity renormalization group (SRG) dependence. The shaded bars indicate the estimated truncation error at each chiral order.

Improved chiral SCS NN + 3N forces at N2LO

Phys. Rev. C **99**, 024313 (2019)



Semilocal momentum space regularized (SMS) chiral NN forces

P. Reinert *et al.*, Eur. Phys. J A **54**, 88 (2018)

(Sami)local regularization in momentum space

$$1/(l^2 + m_\pi^2) \rightarrow F(l^2)/(l^2 + m_\pi^2) \quad \text{with} \quad F(l^2) = e^{-\frac{(l^2 + m_\pi^2)}{\Lambda^2}} \quad \Lambda \in [400, 550] \text{ MeV}$$

Additional improvements

- ❑ pion-nucleon low energy constants taken directly from the pion sector
- ❑ remaining LECs fixed directly from data (the Granada database) and not from the Nijmegen PWA
- ❑ redundant operators at N³LO removed
- ❑ LO, NLO, ..., N⁴LO versions of the NN potential
- ❑ even “N⁴LO+” force obtained by including some contact terms from N⁵LO
- ❑ covariance matrix of the potential parameters available

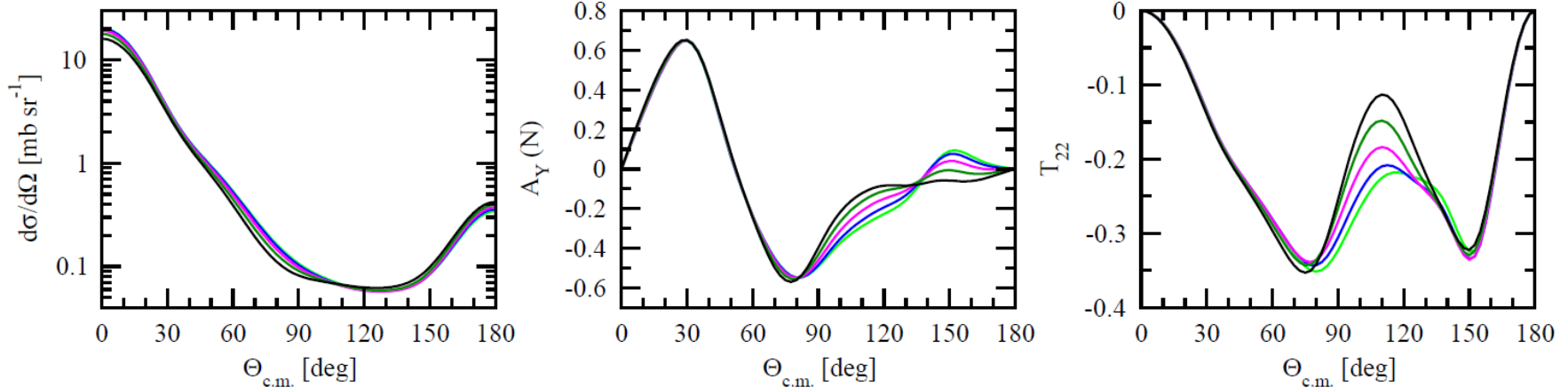
Excellent data description with $\chi^2/\text{datum} \approx 1.0$ for a small number of free parameters !

... For the first time, the chiral potentials match in precision and even outperform the available high-precision phenomenological potentials, while the number of adjustable parameters is, at the same time, reduced by about $\sim 40\%$...

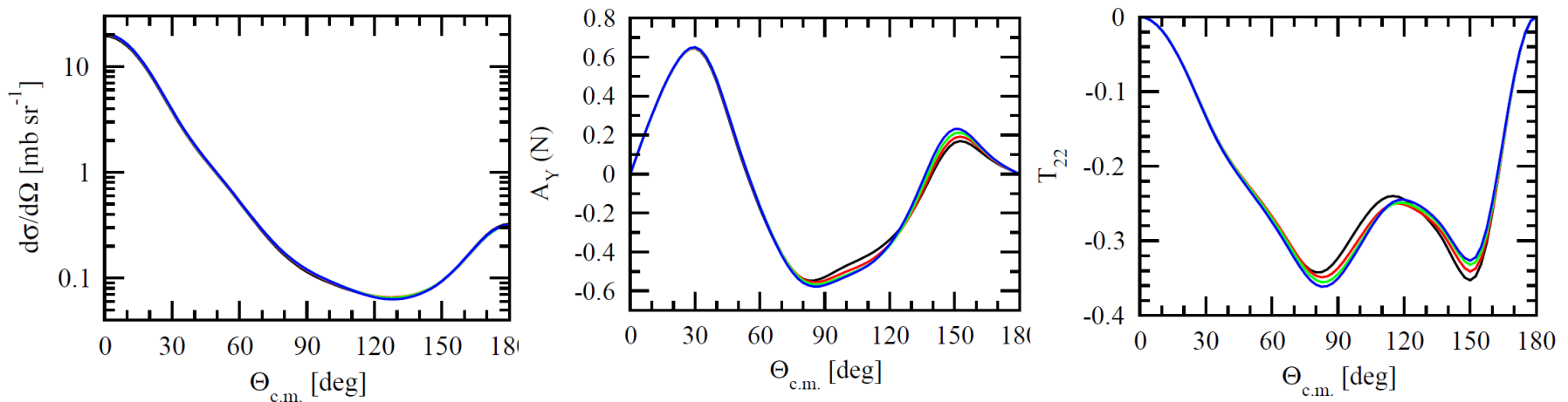
Test: regulator dependence at 200 MeV for elastic Nd scattering ...

SCS N^4LO , $R=0.8-1.2$ fm

NN forces only



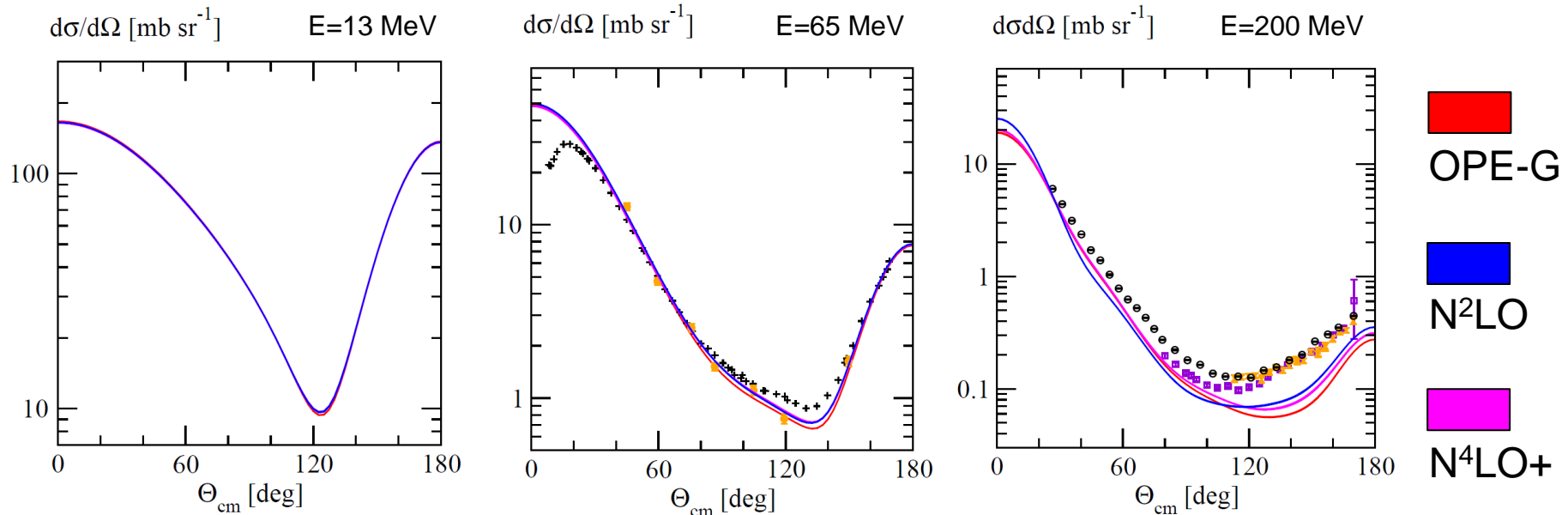
SMS, N^4LO+ , $\Lambda=400-550$ MeV



... is substantially smaller for the SMS force !

Statistical errors with chiral SMS forces for elastic Nd scattering

SMS potential allows us to study propagation of uncertainties from 2N potential parameters to 3N observables

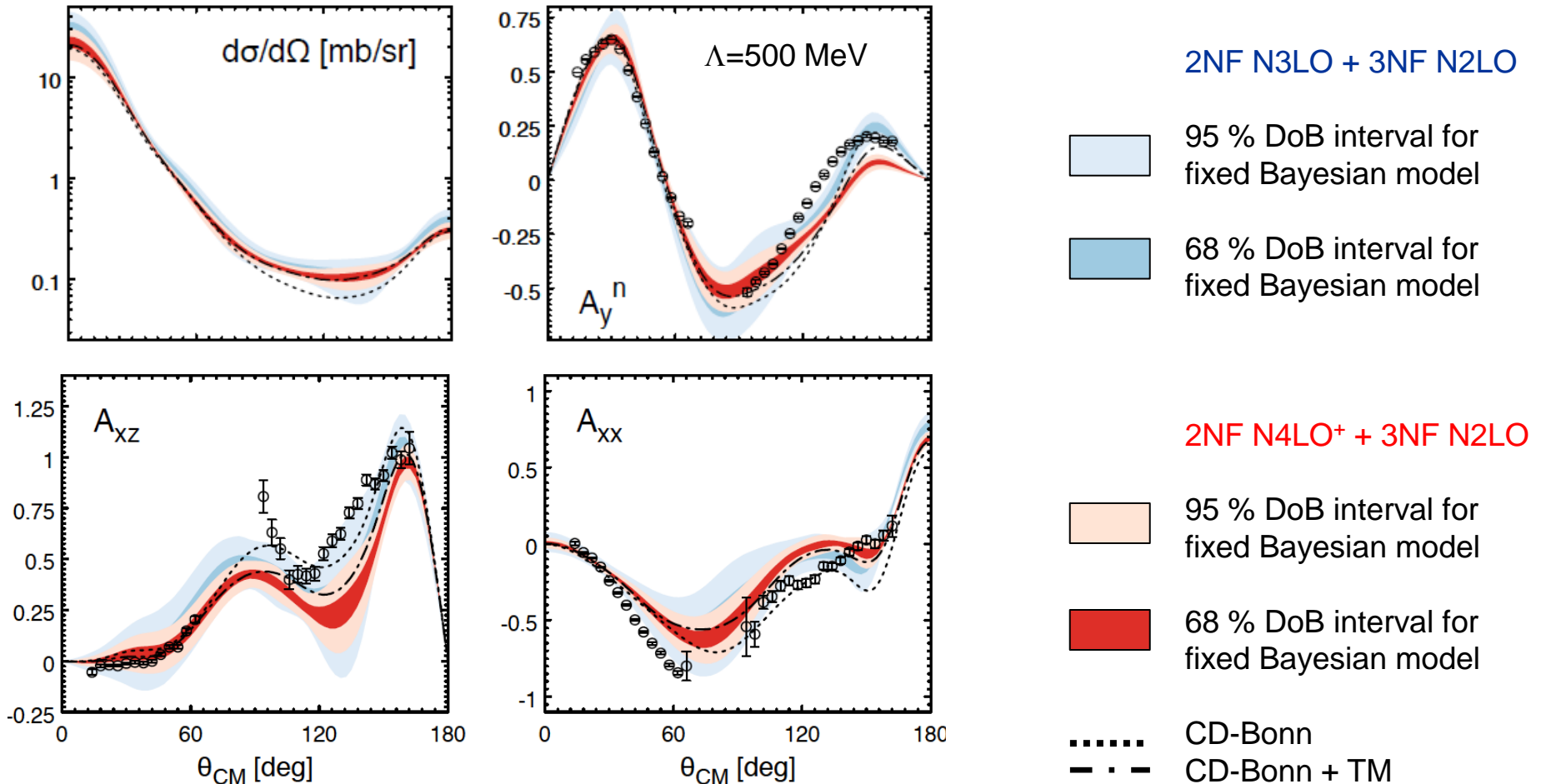


- Statistical errors are small, also at $E=200$ MeV.
- Statistical errors for the SMS force are of similar magnitude as the ones for the OPE-Gaussian potential from Phys. Rev. C **89**, 064006 (2014)
- Similar magnitudes at N²LO and N⁴LO+
- Errors due to the truncation of the chiral expansion order more important

Elastic Nd scattering at 200 MeV: SMS+3NF

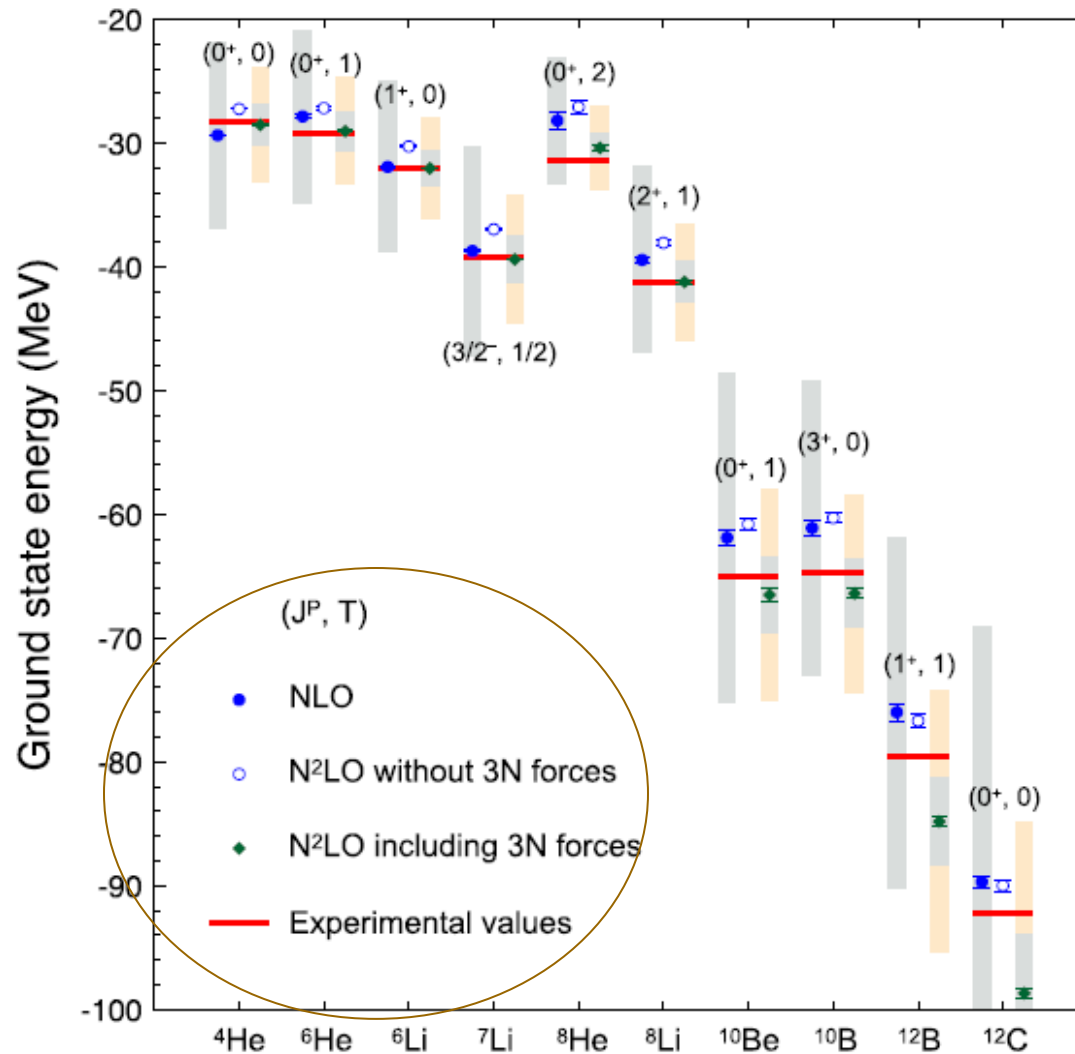
Bayesian approach to truncation errors

E.Epelbaum et al., Eur. Phys. J. A **56**, 92 (2020)

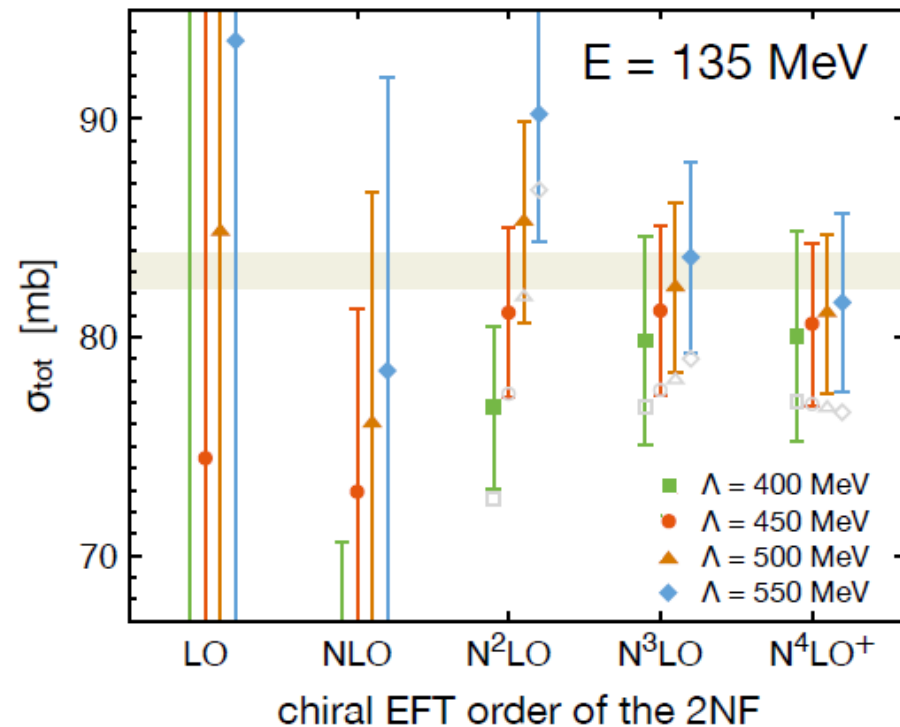


Ground state energies of light nuclei: SMS+3NF

P. Maris et al., Phys. Rev. C **103**, 054001 (2021)



Total nd cross section with SMS 2NF (various orders)+3NF (N2LO)

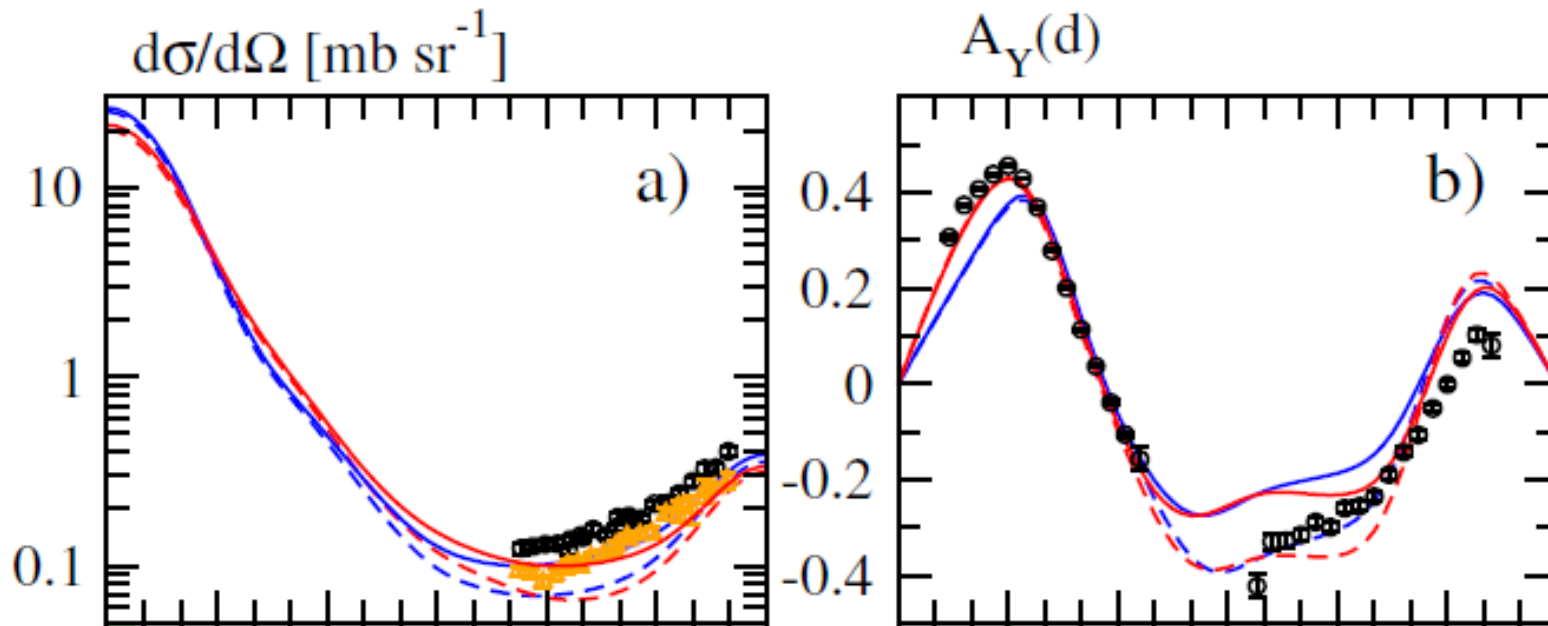


P. Maris *et al.*, Phys. Rev. C **106**, 064002 (2022)

Predictions for the neutron-deuteron total cross-section at 135 MeV based on the semilocal momentum-space regularized chiral interactions at different orders (shown by solid symbols with error bars). Three nucleon force is included at N²LO only. Error bars show the EFT truncation uncertainty calculated using a Bayesian model (68% DoB intervals). For the incomplete calculations at N³LO and N⁴LO, the quoted errors correspond to the N²LO truncation uncertainties. Gray open symbols without error bars show the results based on the two-nucleon forces only. Horizontal band represents experimental data.

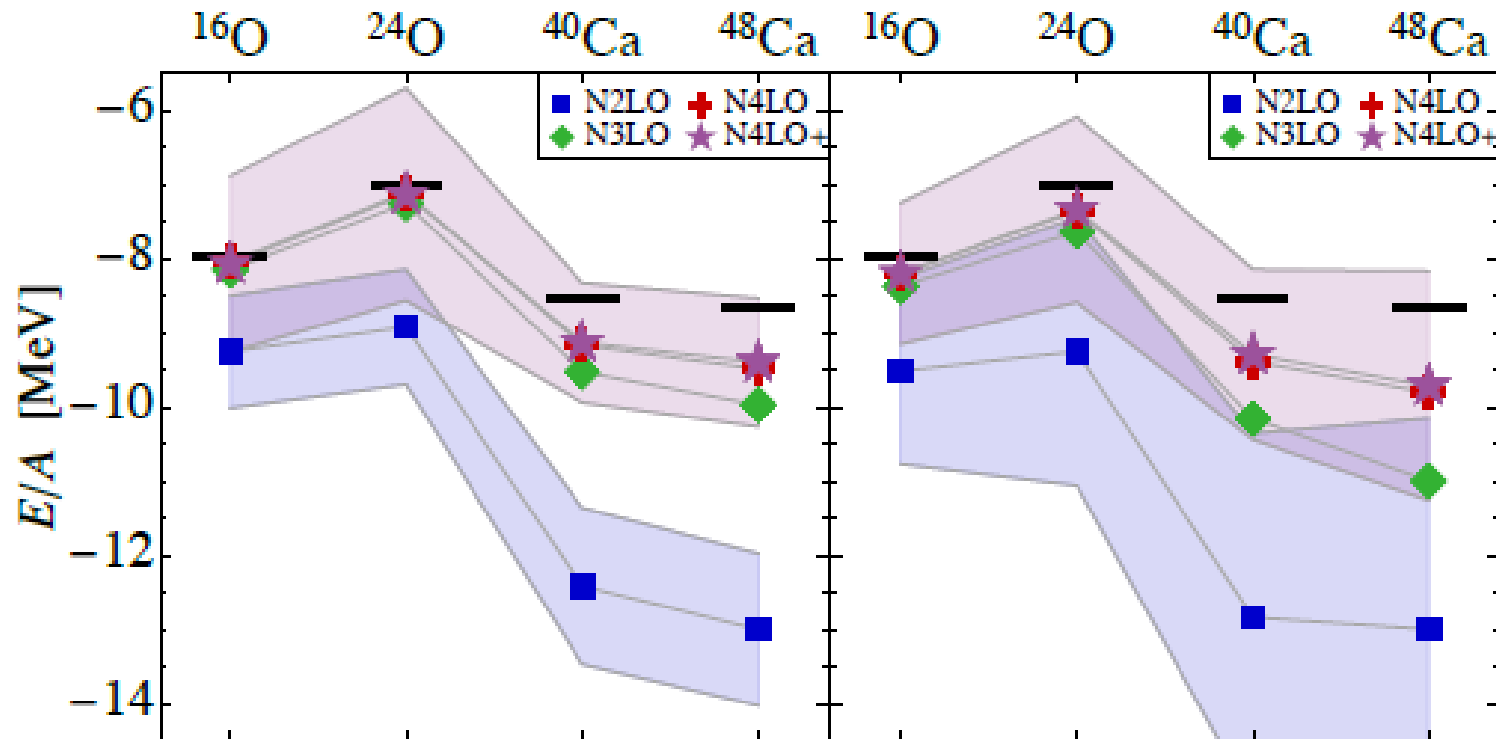
Other 3N observables with SMS 2NF (various orders)+3NF (N2LO)

P. Maris *et al.*, Phys. Rev. C **106**, 064002 (2022)



The center-of-mass differential cross section and the deuteron vector analyzing power $A_Y(d)$ for the neutron-deuteron elastic scattering at incoming neutron lab. energy $E = 200$ MeV. The dashed blue (red) curve represents predictions based on the two-nucleon N2LO (N4LO $^+$) forces. The solid blue curve represents complete results at N2LO and the solid red curve stands for predictions of N4LO $^+$ NN interaction supplemented by N2LO 3NF. In all cases, the cutoff $\Lambda = 450$ MeV is used.

„Real” nuclei with SMS 2NF (various orders)+3NF (N2LO)



Ground-state energies for doubly-magic oxygen and calcium isotopes obtained in the IM-SRG with SMS interactions from NLO to N4LO⁺ for $\Lambda = 450$ MeV (left) and $\Lambda = 500$ MeV (right) with SRG. The error bands show the chiral truncation uncertainties at the 95% confidence level obtained using a Bayesian model for N2LO and N4LO⁺.

A few words about applications to electroweak processes

We investigate possible applications of our momentum space framework to processes, where 3N scattering states appear either in the initial or final state:

$$e^- + {}^3\text{He} \rightarrow e^- + p + d, \quad e^- + {}^3\text{He} \rightarrow e^- + p + p + n, \dots$$

$$\gamma + {}^3\text{He} \leftrightarrow p + d, \quad \gamma + {}^3\text{He} \rightarrow p + p + n, \dots$$

$$\mu^- + {}^3\text{He} \rightarrow \nu_\mu + n + d, \quad \mu^- + {}^3\text{He} \rightarrow \nu_\mu + p + n + n, \dots$$

$$\bar{\nu}_e + {}^3\text{He} \rightarrow e^+ + n + d, \quad \bar{\nu}_e + {}^3\text{He} \rightarrow e^+ + p + n + n, \dots$$

$$\nu_e(\bar{\nu}_e) + {}^3\text{He} \rightarrow \nu_e(\bar{\nu}_e) + p + d, \quad \nu_e(\bar{\nu}_e) + {}^3\text{He} \rightarrow \nu_e(\bar{\nu}_e) + p + p + n, \dots$$

$$\pi^- + {}^3\text{He} \rightarrow \gamma + n + d, \quad \pi^- + {}^3\text{He} \rightarrow \gamma + p + n + n, \dots$$

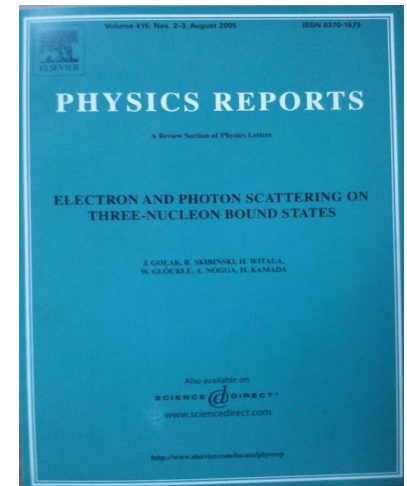
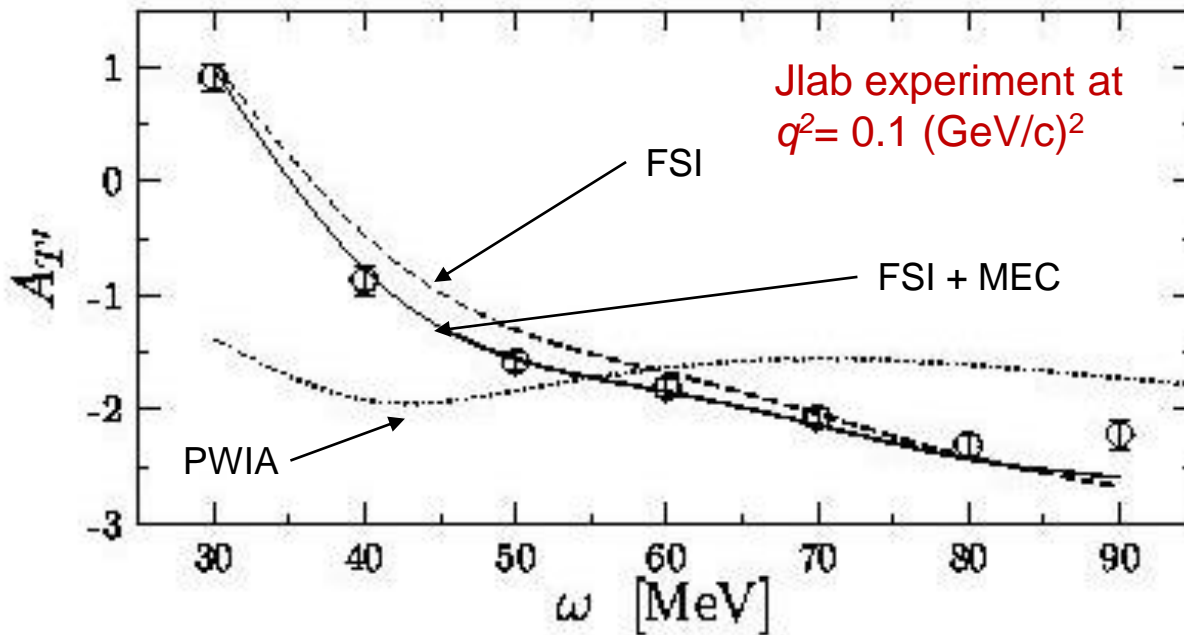
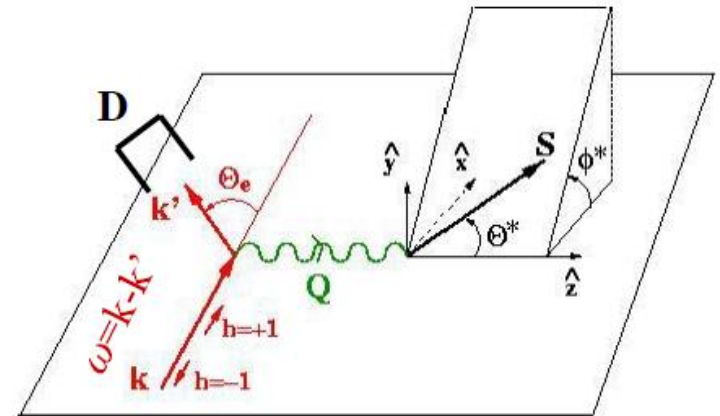
$$\pi^- + {}^3\text{He} \rightarrow n + d, \quad \pi^- + {}^3\text{He} \rightarrow p + n + n, \dots$$

LENPIC will provide consistent nuclear forces and current operators !

Polarized electron scattering on polarized ^3He

Inclusive electron scattering on ^3He provides information about the neutron magnetic form factor \mathbf{G}_M^n

AV18+UrbanalX and SNC with π - and ρ -like currents

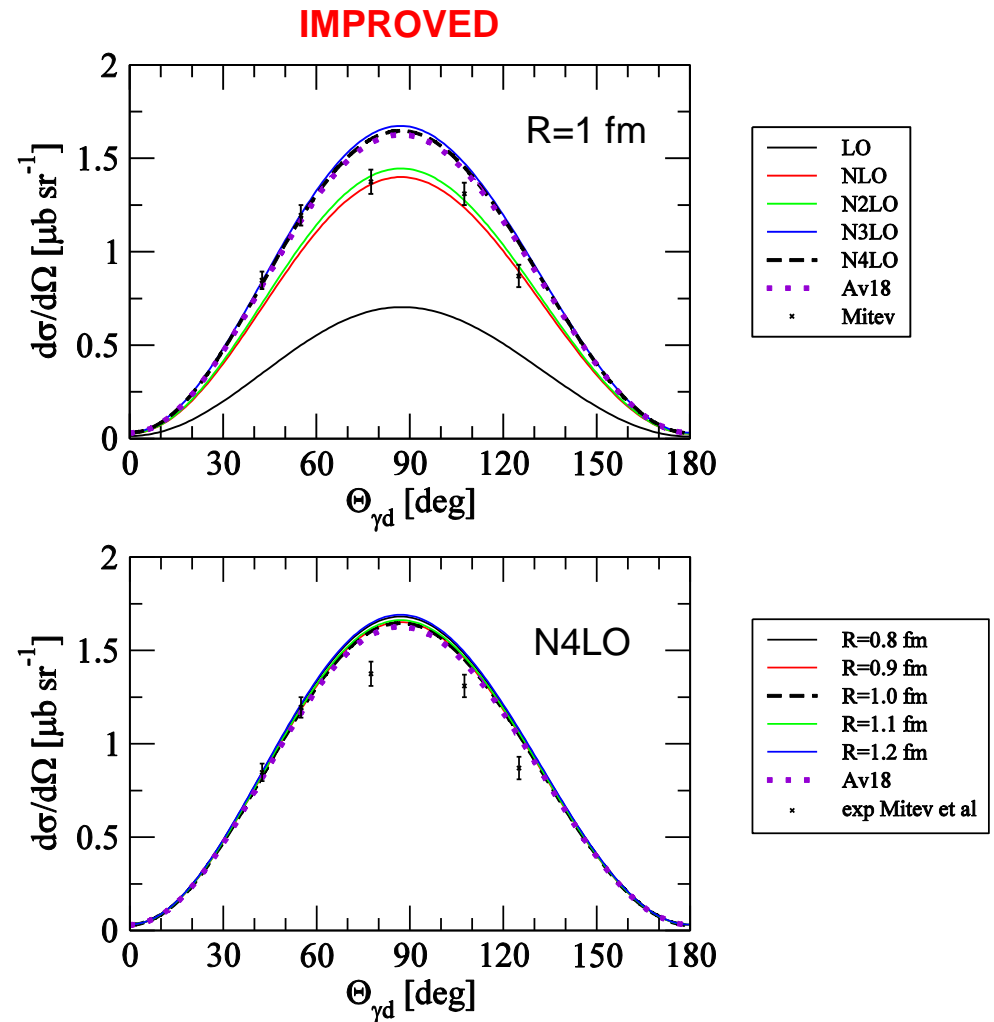
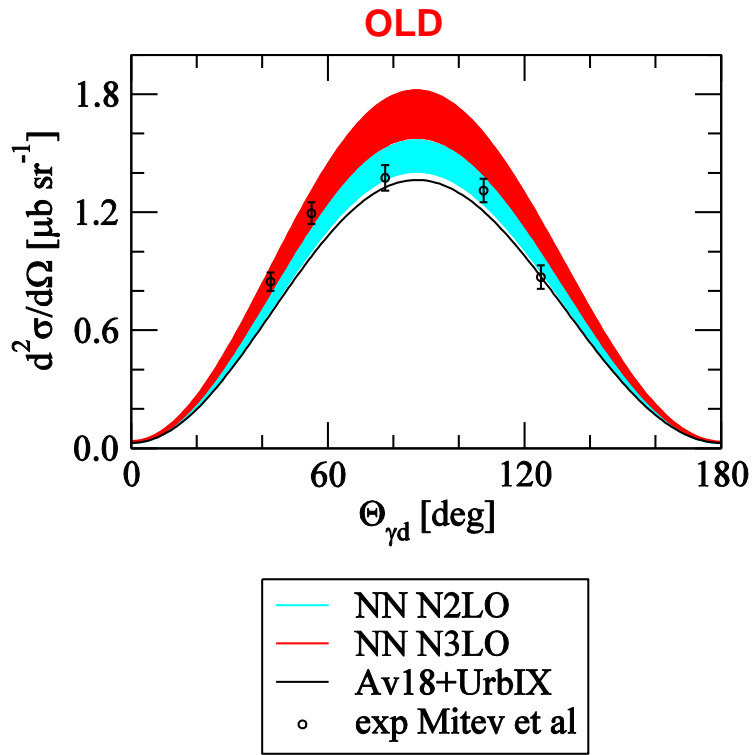


Phys. Rep. **415**, 89 (2005)

Radiative capture with chiral SCS forces and Siegert approximation



The c.m. neutron-deuteron capture cross section at $E_n^{\text{lab}} = 9 \text{ MeV}$



Muon capture on ^3He with chiral SCS forces: predictions for breakup channels

Total capture rates in s^{-1} calculated with the improved chiral potentials and the single nucleon current operator with RC

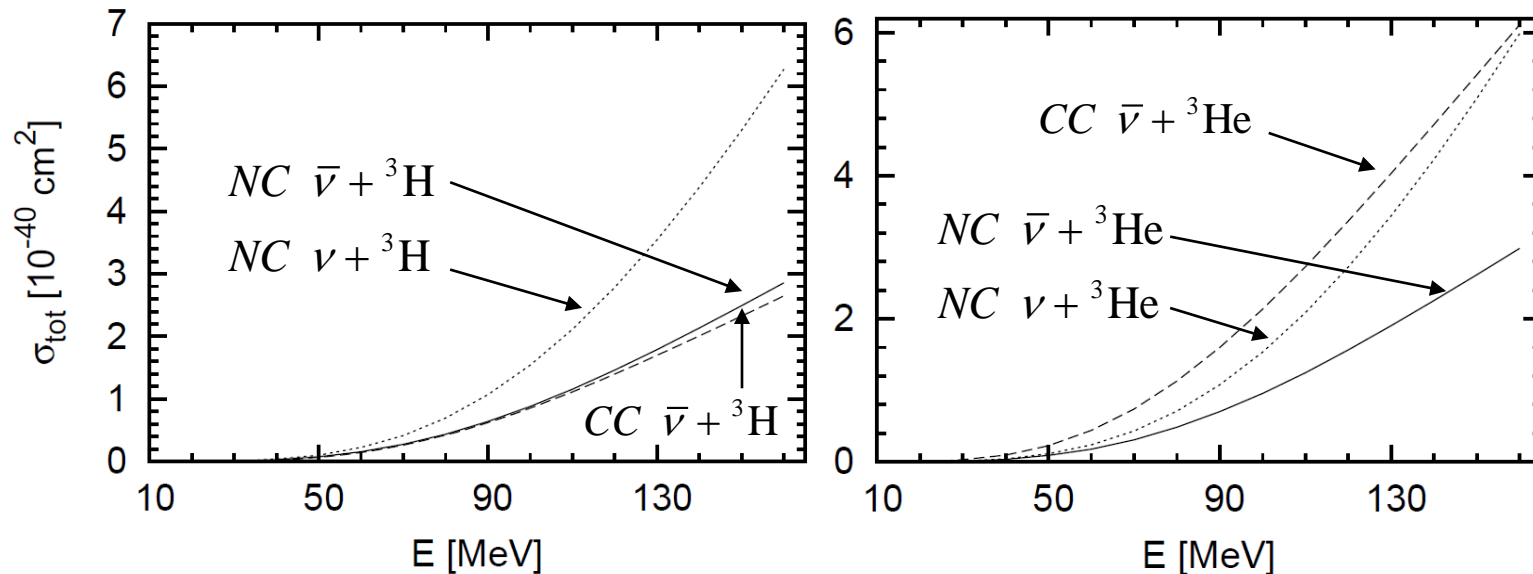
Chiral order	R=0.8 fm	R=0.9 fm	R=1 fm	R=1.1fm	R=1.2 fm	$\Gamma_{\text{max}} - \Gamma_{\text{min}}$
LO	357	381	417	463	512	155
NLO	695	682	669	655	640	55
N2LO	708	698	686	672	658	50
N3LO	753	755	763	768	778	25
N4LO	760	753	752	757	768	16

very weak dependence on the regulator parameter R

AV18 773

Neutrino scattering off ${}^3\text{He}$ and ${}^3\text{H}$

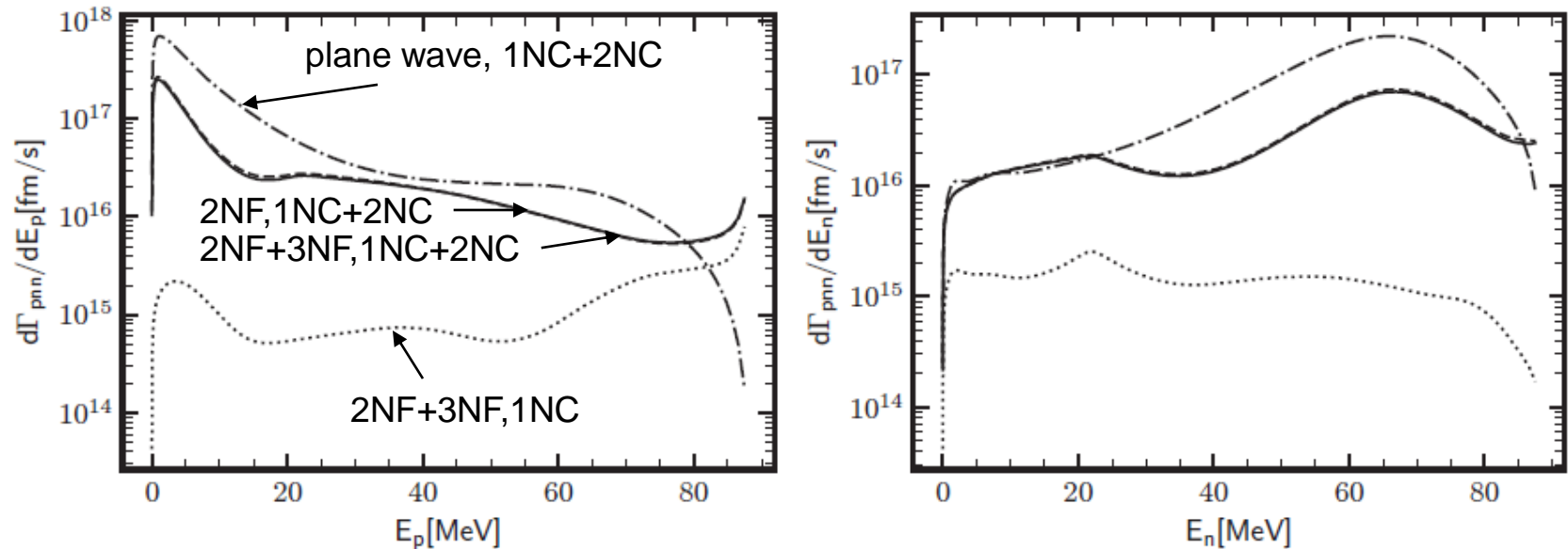
Total breakup cross sections calculated with the AV18 NN potential



J. Golak *et al.*, Phys. Rev. C **98**, 015501 (2018); Phys. Rev. C **100**, 064003 (2019)

Negative pion absorption on ${}^2\text{H}$, ${}^3\text{He}$ and ${}^3\text{H}$

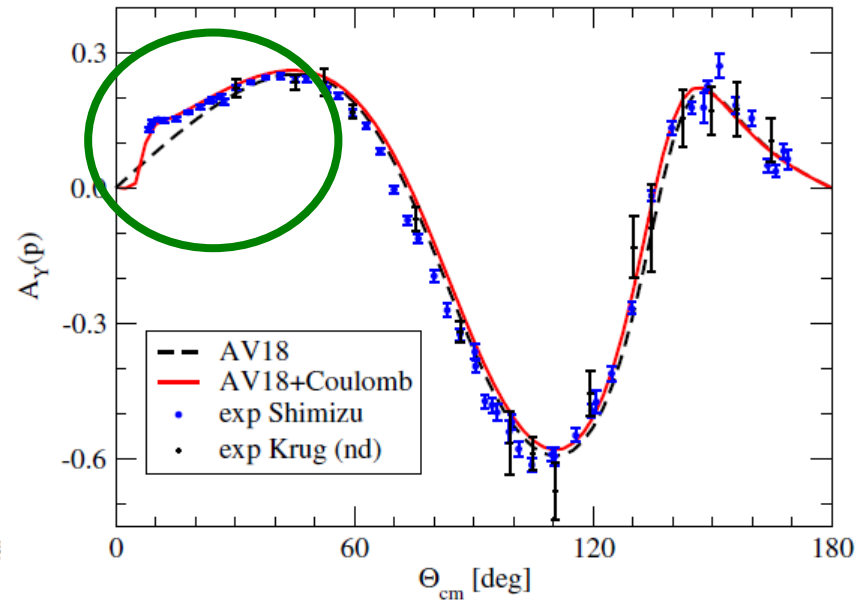
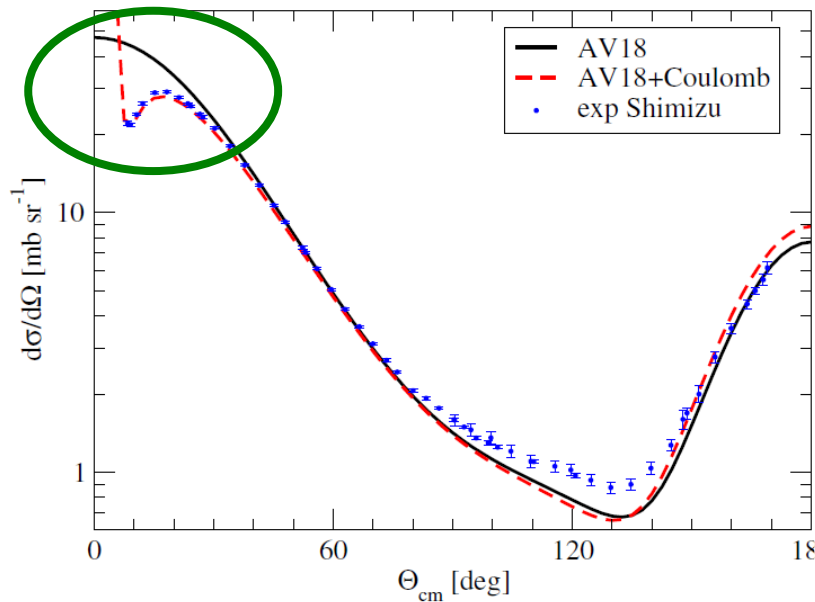
Differential and total absorption rates calculated with chiral SMS forces and transition operators J. Golak *et al.*, Phys. Rev. C **106**, 064003 (2022)]



plane-wave results insufficient
large effects of 2N transition operators (2NC)
relatively small effects of 3N forces (3NF)

Elastic Nd scattering revisited (with Coulomb pp forces)

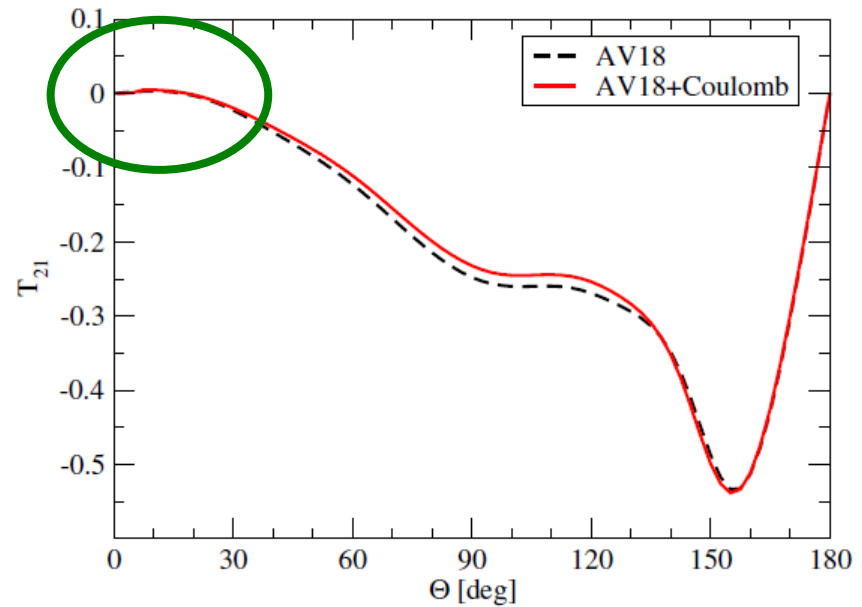
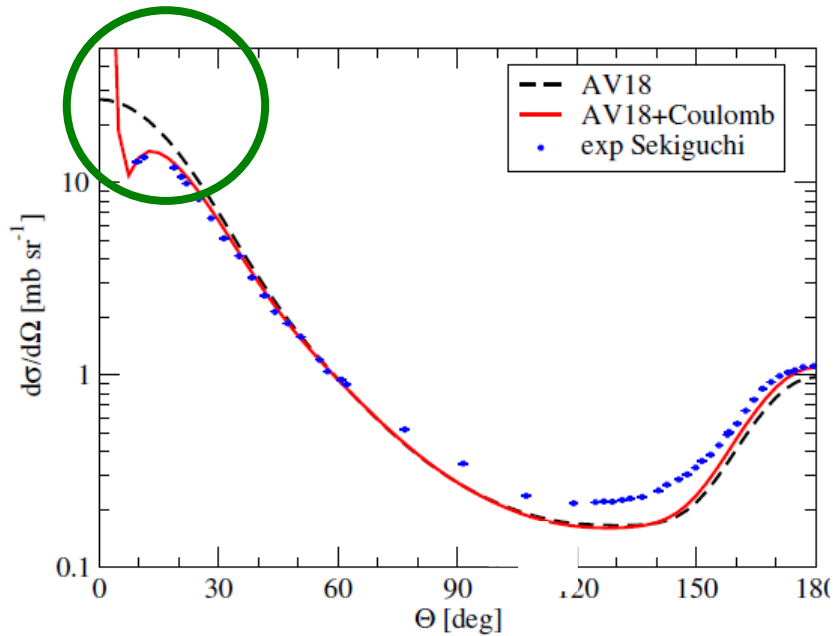
$E = 65 \text{ MeV}$



NEW! Coulomb effects taken into account !
converged results with screened Coulomb potentials

Elastic Nd scattering (revisited)

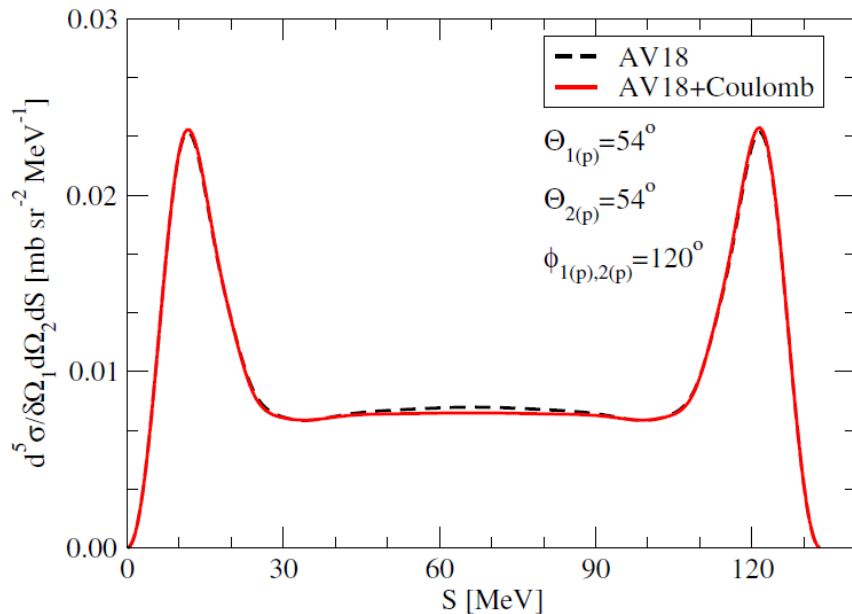
E= 135 MeV



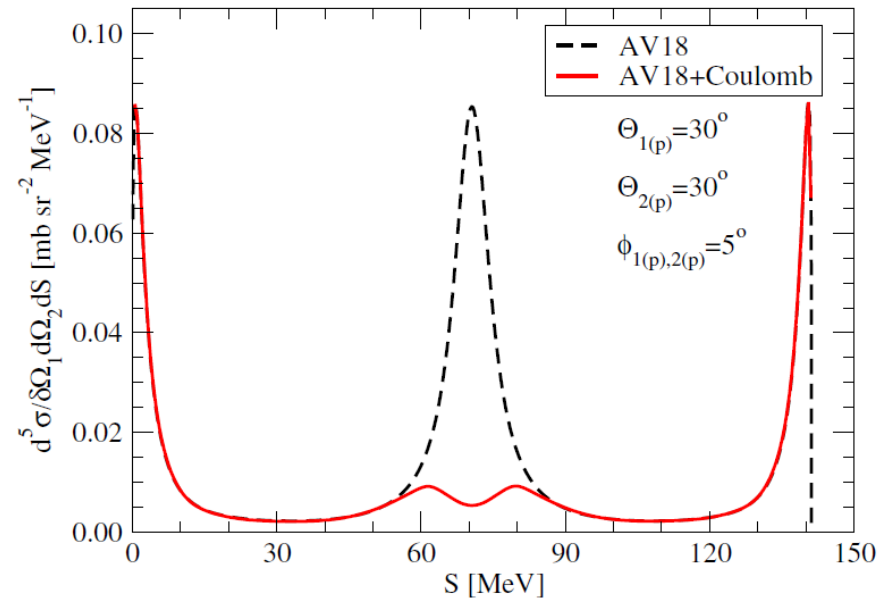
Smaller Coulomb effects at higher energies, especially for polarization observables !

Nucleon-induced deuteron breakup

E= 135 MeV



For some configurations Coulomb effects are hardly visible



For some other configurations they can be dramatic !

How much are inclusive observables distorted ?

Summary

LENPIC was established “to understand nuclear structure and reactions with chiral forces”. How far are we from achieving this goal ?

Nuclear Hamiltonian

2N forces

- problem of cut-off artefacts in 3N scattering definitely solved
- chiral SMS potential at N4LO⁺ yields nearly perfect description of np and pp data up to 300 MeV. No significant improvement can be expected by going to even higher orders.
- fine tuning for charge independence and charge dependence breaking effects
Number of potential parameters matters !

3N forces

- derivation of contributions up to N3LO announced in 2011; derivation of N4LO corrections accomplished recently (new numerous LECs)
- promising results for few-N systems based on 2NF+3NF at N2LO
- improved (SCS and SMS) 2N potentials combined with 3NF at N²LO give predictions of similar quality as semi-phenomenological interactions **but none of 3N puzzles solved**
- truncation errors large so N3LO (maybe even N4LO) potentials are needed
- nontrivial regularization (**maintaining χ -symmetry**) in progress

Summary (cont.)

Electroweak current operators

- derived up to N3LO
- some unknown πN LECs in 1π axial charge at N3LO have to be fixed from other sources (lattice QCD? neutrino-induced π -production? resonance saturation?)
- nontrivial regularization (**maintaining χ -symmetry**) in progress
- promising results for the deuteron form factors
- 2N charge density operators important for the **radii of nuclei !**

Next steps:

- precision tests of the theory for triton beta decay, muon capture on ^2H and ^3He (accurate data)
- other processes, heavier nuclei, N4LO, explicit Δ 's, ...

3N calculations are an important part of the quest for the nuclear Hamiltonian !



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Andrzej Mleczek's drawing

Thank you for your attention !